Semiempirična masna formula

$$M(A,Z) = Zm_p + \overset{N}{\boxed{(A-Z)}}m_n - W_b(A,Z)/c^2$$

$$m_p \approx m_n \approx 939\ MeV/c^2$$

$$m_n - m_p = 1{,}3\ MeV/c^2$$

$$W_b = w_0 A - w_1 A^{2/3} - w_2 \frac{Z^2}{A^{1/3}} - w_3 \frac{(A-2Z)^2}{A} - w_4 \frac{\delta^{ZN}}{A^{1/2}}$$

$$\delta^{LL} = 1 \quad \delta^{SL} = 0 \quad \delta^{SS} = -1$$

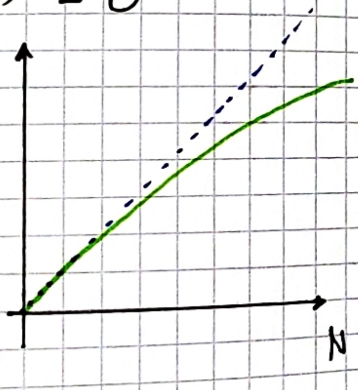
1. [Določi izobar]

Za dani A določi izobar (enaka masa) z največjo energijo W_b.

Predpostavimo A = lih

$$\frac{\partial W_b}{\partial Z} = -2w_2 \frac{Z}{A^{1/3}} - 2w_3 \frac{(A-2Z)}{A}(-2) = 0$$

$$\Rightarrow \quad Z = \frac{A/2}{1 + \underset{0{,}08}{\boxed{\frac{w_2}{4w_3}}} A^{2/3}}$$



A so zelo velika jedra lahko vezana?

$A >> 1$

$$Z = \frac{\frac{A}{2}}{\underset{\text{Zanemarimo}}{\boxed{1+}} \frac{w_2}{4w_3} A^{2/3}} \simeq \frac{1}{2 \cdot 0{,}08} A^{1/3}$$

To vstavimo: (ker razvijemo po A)

$$W_b = \underset{15{,}7\ MeV}{\boxed{w_0}} A - \underset{23.3}{\boxed{w_3}} A < 0$$

Torej ne, zelo velika jedra ne morejo biti vezana.

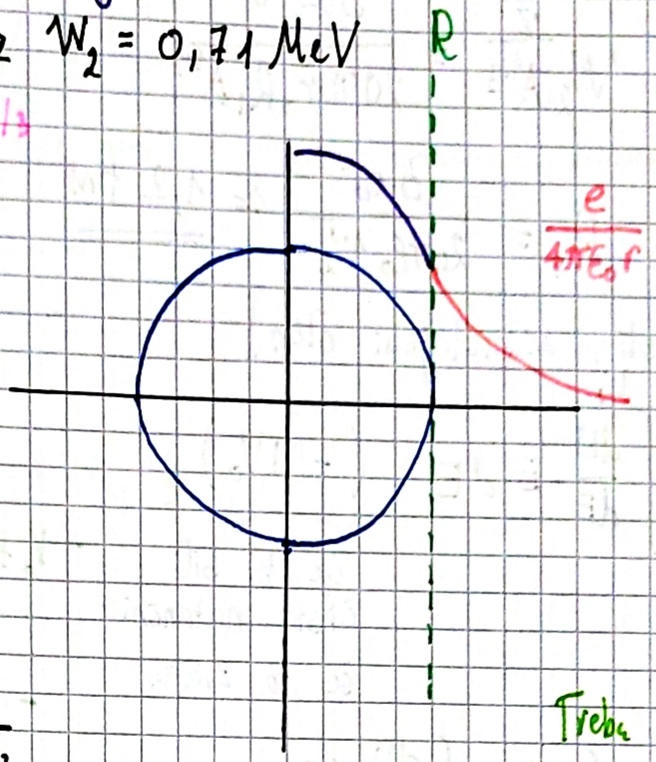
2. [Osmislimo člen Coulombskega odboja]

Določi R_1 (ki je 1,2 fm) iz $W_2 = 0{,}71\ MeV$

$R = R_1 A^{1/3}$

$W_p =$

↑ Potencial enakomerno nabite krogle z radijem R in e nabojem



Treba pravilno nastaviti za zveznost

$$E(r<R) = e\frac{r^3}{R^3 4\pi\varepsilon_0 r^2}$$

$$\phi(r<R) = -\int E(r<R)\,dr = -\frac{er^2}{8\pi\varepsilon_0 R^3} + C$$

$$= \frac{e}{8\pi\varepsilon_0 R}\left(3 - \frac{r^2}{R^2}\right)$$

$$W_p = \frac{1}{2}\int_0^{R_1} \phi(r<R)\,de = \frac{1}{2}\int_0^{R_1} \phi(r<R)\,\rho\,dV =$$

$$= \frac{1}{2}\int_0^{R_1} \frac{e}{8\pi\varepsilon_0 R}\left(3 - \frac{r^2}{R^2}\right)\frac{3e}{\cancel{4\pi}R^3}\cancel{4\pi}r^2\,dr =$$

$$= \frac{1}{2}\int_0^{R} \frac{3e^2 r^2}{8\pi\varepsilon_0 R^4}\left(3 - \frac{r^2}{R^2}\right)dr = \frac{3e^2}{16\pi\varepsilon_0 R^4}\int_0^R \left(3 - \frac{r^2}{R^2}\right)r^2\,dr =$$

$$= \frac{3e^2}{16\pi\varepsilon_0 R^4}\left[\frac{3R^3}{3} - \frac{R^5}{5R^3}\right] = \frac{3e^2}{20\pi\varepsilon_0 R}$$

$$W_2 \frac{\cancel{Z^2}}{\cancel{A^{1/3}}} = \frac{3\cancel{Z^2} e_0^2}{20\pi \varepsilon_0 R_1 \cancel{A^{1/3}}}$$

$$R_1 = \frac{3 e_0^2}{20\pi\varepsilon_0 W_2} \simeq \underline{\underline{1{,}2\ \mathrm{fm}}}$$

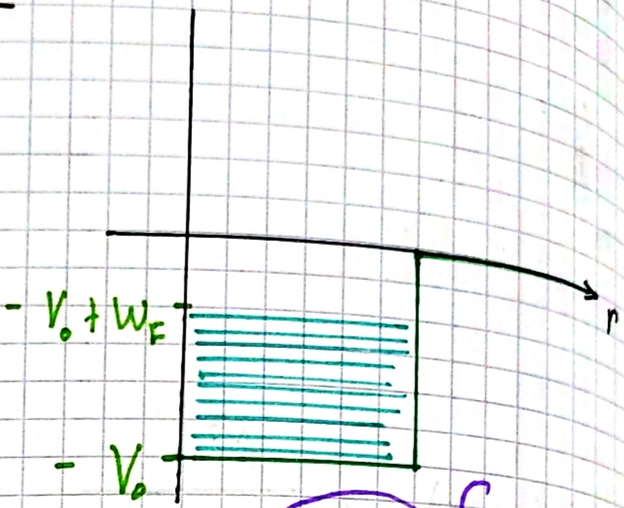
3. [Osmislimo asimetrijski člen]

$$E = \int_0^{W_F} \boxed{\frac{dN}{dE}} E\, dE \quad \underbrace{(-NV_0)}_{\text{če bi bili čisto natančni še to zraven}}$$

gostota stanj

$g(E)$

št. protonov
in št. nevtronov
Ali



$$\boxed{N} = \int dN = \int \frac{dN}{dE} dE =$$

$$g(E) = V \underbrace{\boxed{\frac{\sqrt{2}\, m_n^{3/2}}{\pi^2 \hbar^3}}}_{C} \sqrt{E}$$

$$= \int g(E)\, dE =$$

$$= C \int_0^{W_F} V \sqrt{E}\, dE = VC \frac{2}{3} W_F^{3/2} \Rightarrow$$

$$\frac{N}{V} = n = \frac{2}{3} C W_F^{3/2}$$

$$\Rightarrow W_F = (n)^{2/3} \frac{1}{C^{2/3}} \left(\frac{3}{2}\right)^{2/3}$$

$$V = \frac{4\pi R^3}{3} = \frac{4\pi R_1^3 A}{3}$$

$$E = \int_0^{W_F} \frac{dN}{dE} E\, dE = VC \int_0^{W_F} \sqrt{E}\, E\, dE = VC \frac{2}{5} W_F^{5/2} =$$

$$= VC \frac{2}{5} n^{5/3} \left(\frac{13}{C \cdot 2}\right)^{5/3} = VC \frac{2}{5} \left[\frac{N}{V} \frac{3}{2C}\right]^{5/3}$$

$$\Rightarrow \quad E \propto \frac{N^{5/3}}{V^{2/3}}$$

Isti postopek še za protone z Z namesto N.

$$E_{tot} = b\left(\frac{N^{5/3}}{V^{2/3}} + \frac{Z^{5/3}}{V^{2/3}}\right) =$$

$$= \tilde{b}\left(\frac{N^{5/3} + Z^{5/3}}{A^{2/3}}\right)$$

Sedaj fiksiramo A in iščemo izbor Z, pri katerem imamo minimalno energijo:

$$E_{tot}(A,Z) = \tilde{b}\,\frac{(A-Z)^{5/3} + Z^{5/3}}{A^{2/3}}$$

$$\frac{\partial E_{tot}}{\partial Z} = 0 = \frac{\tilde{b}}{A^{2/3}}\left(-\frac{5}{3}(A-Z)^{2/3} + \frac{5}{3} Z^{2/3}\right)$$

$$A - Z = Z \quad \Rightarrow \quad A = 2Z$$

okoli minimuma

oz.

$$N = Z$$

$$\left.\frac{\partial^2 E_{tot}}{\partial Z^2}\right|_{Z=A/2} = \frac{\tilde{b}}{A^{2/3}}\,\frac{5}{3}\left[+\frac{2}{3}(A-Z)^{-1/3} + \frac{2}{3} Z^{-1/3}\right]\Bigg|_{Z=A/2} =$$

$$= \frac{\tilde{b}}{A^{2/3}}\,\frac{10}{9}\left[(A-Z)^{-1/3} + Z^{-1/3}\right]\Bigg|_{Z=A/2}$$

$$= \frac{\tilde{b}}{A^{2/3}}\,\frac{20}{9}\left(\frac{2^{1/3}}{A^{1/3}}\right) \propto 1/A$$

ker smo v minimumu

Razlaga teh odvodov:

$$E_{tot}(A,Z)\Big|_{Z=A/2} = \tilde{b}\,\frac{2\left(\frac{A}{2}\right)^{5/3}}{A^{2/3}} + \overset{\overset{0}{\|}}{\left.\frac{\partial E_{tot}}{\partial Z}\right|_{Z=A/2}}\left(Z - \frac{A}{2}\right) +$$

$$+ \frac{1}{2}\,\frac{\partial^2 E_{tot}}{\partial Z^2}(A,Z)\Big|_{Z=A/2}\left(Z - A/2\right)^2$$

$$\tau \propto e^{2G}$$

$$\frac{R}{R'} \ll 1$$

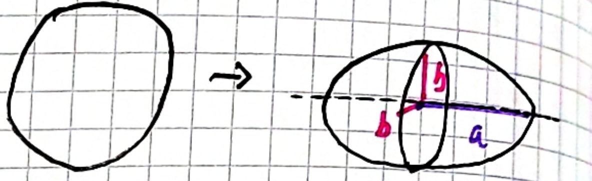
$$G = \frac{\alpha}{\sqrt{2}} \sqrt{m_\alpha c^2}\, \pi Z_\alpha \left(\frac{Z'}{\sqrt{Q}} - \frac{4}{\pi} \sqrt{\frac{Z' R}{Z_\alpha \alpha \hbar c}} \right)$$

$$\ln \frac{\tau'}{\tau} = 2(\overset{G'}{\cancel{\gamma}} - \overset{G}{\cancel{\gamma}})$$

$$\log_{10} \frac{\tau}{\tau'} = 1{,}72\, \mathrm{MeV}^{1/2} \cdot \underset{Z-2}{\boxed{Z'}} \left(\frac{1}{\sqrt{Q}} - \frac{1}{\sqrt{Q'}} \right)$$

[Kdaj postane sfericno jedro nestabilno?]

Predpostavimo $V_0 = \text{konst.}$



$$W_0(A,Z) = -w_1 A^{2/3} - w_2 \frac{Z^2}{A^{1/3}} \; ; \; A = 2Z$$

pariter zanemarimo

Volumen konstanten

Majhno !

Poglejmo spremembo povrsine! $\quad a = R(1+\boxed{\varepsilon})$

$$V_0 = \frac{4\pi a b^2}{3} = \frac{4\pi R^3}{3} = \frac{4\pi}{3} R(1+\varepsilon) b^2$$

$$\Rightarrow \quad b^2 = \frac{R^2}{1+\varepsilon} \qquad\qquad \frac{z^2}{a^2} + \frac{r^2}{b^2} = 1$$

$$S = 2\pi \int_{-a}^{a} 2\pi r(z) \sqrt{dz^2 + dr^2} = \ldots = 4\pi b a \frac{1}{2} \left(\sqrt{1-e^2} + \frac{1}{e} \arcsin e \right)$$

$e = \sqrt{1 - b^2/a^2}$ ... ekscentricnost $\qquad (1+\varepsilon)^{-1/2} = 1 - \frac{1}{2}\varepsilon + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}\varepsilon^{2}$

To razvijemo po ε:

$$S \approx 2\pi R^2 (1+\varepsilon)\left(1 - \frac{1}{2}\varepsilon + \frac{3}{8} \cdot \varepsilon^2\right)\left(\sqrt{1-e^2} + \frac{1}{e} \arcsin(e)\right)$$

Še ekscentričnost:

$$\sqrt{1-\frac{R^2}{R^2(1+\epsilon)^3}} = \sqrt{\frac{(1+\epsilon)^3-1}{(1+\epsilon)^3}} = (1+\epsilon)^{-\frac{3}{2}}\sqrt{3\epsilon+3\epsilon^2+\epsilon^3} =$$

$$= \sqrt{3\epsilon}\,(1+\epsilon)^{-3/2}\sqrt{1+\epsilon+\frac{1}{3}\epsilon^2} =$$

$$(1+r)^x = 1+rx+\frac{r(r-1)}{2!}x^2 \ldots$$

$$= \sqrt{3\epsilon}\left(1-\frac{3}{2}\epsilon+\frac{15}{8}\epsilon^2\right)\left(1+\frac{1}{2}\left(\epsilon+\frac{1}{3}\epsilon^2\right)-\frac{1}{8}\left(\epsilon+\frac{1}{3}\epsilon^2\right)^2\right) =$$

$$= \sqrt{3\epsilon}\left(1+\epsilon\left(\frac{1}{2}-\frac{3}{2}\right)+\epsilon^2\left(\frac{15}{8}+\frac{1}{6}-\frac{1}{8}-\frac{3}{4}\right)\right) =$$

$$= \sqrt{3\epsilon}\left(1-\epsilon+\frac{7}{6}\epsilon^2\right)$$

To še damo nazaj in razvijemo pride:

$$\ldots = \ldots = 4\pi R^2\left(1+\frac{2}{5}\epsilon^2\right)$$

Poglejmo še elektrostatsko energijo elipsoida:

$$W_p = \overset{A}{\boxed{\frac{3\overset{\text{naboj}}{q^2}}{10\cdot 4\pi\epsilon_0}}}\int_0^\infty \frac{ds}{(b^2+s)\sqrt{a^2+s}} =$$

To bi razvili in dobili integral za 0. red in 2. red.

$$= A\int_0^\infty \frac{ds}{\left(\frac{R^2+s(1+\epsilon)}{1+\epsilon}\right)\sqrt{(1+\epsilon)^2R^2+sR^2}} =$$

$$\boxed{\frac{e_0}{4\pi\epsilon_0}} = \alpha$$

$$= \ldots = \frac{3q^2}{5\cdot 4\pi\epsilon_0}\left(1-\frac{\epsilon^2}{5}\right) = \frac{3Z^2\alpha\hbar c}{5}\left(1-\frac{\epsilon^2}{5}\right)$$

$$W_0 = -W_1\left(1+\frac{2}{5}\epsilon^2\right)A^{2/3} - W_2\frac{Z^2}{A^{1/3}}\left(1-\frac{\epsilon^2}{5}\right)$$

Odštejemo energijo krogle:

Če je $\Delta W_p > 0$ bo jedro nestabilno in lahko razpade

$$\Delta W_p = -W_1\frac{2}{5}\epsilon^2A^{2/3} - W_2\frac{Z^2}{A^{1/3}}\left(-\frac{\epsilon^2}{5}\right) > 0$$

$$\Rightarrow \frac{Z^2}{A}\,\frac{1}{2}\,\frac{W_2}{W_1} > 1 \quad \Rightarrow \text{Ob upoštevanju } Z = \frac{A}{2}$$

$$\Rightarrow A \gtrsim 193$$

Torej velika jedra lahko razpadejo z fizijo.

Razpad $\beta^{\pm}$

močna, EM

Razpadi: α, γ, fisija se ohranja št. Nevtronov, št. protonov, št. elektronov. Pr beta razpadu pa to ne velja:

$$\beta^{+} : p \rightarrow n\, e^{+}\, \nu_e$$
$$\beta^{-} : n \rightarrow p\, e^{-}\, \bar{\nu}_e$$

} šibke interakcije

Ohranja pa se:

- B... barionsko število
- L... leptonsko število

\+

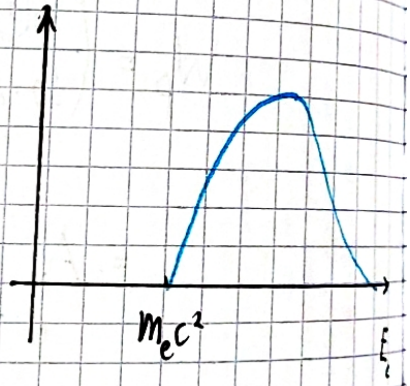
- Naboj
- Energija in gibalna količina
- Vrtilna količina

	p	n	${}^{A}_{Z}X$	e^{-}	e^{+}	ν_e	$\bar{\nu}_e$	$\bar{p}$
B	1	1	A	0	0	0	0	-1
L	0	0	0	1	-1	1	-1	0

V jedru zgleda to kot:

$$\beta^{-} : {}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + e^{-} + \bar{\nu}_e$$

$$\beta^{+} : {}^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + e^{+} + \nu_e$$



Sproščena energija:

jedrske mase

$$\beta^{-} : Q = (m_X - m_Y - m_e)\, c^2$$

$$Q = (m_X^{at} - m_Y^{at} + m_e - m_e)\, c^2 \Rightarrow Q = (m_X^{at} - m_Y^{at})\, c^2$$

$$\beta^{+} : Q\, (m_X - m_Y - m_e)\, c^2$$

$$Q = (m_X^{at} - m_Y^{at} - 2m_e)\, c^2 \Rightarrow Q = (m_X^{at} - m_Y^{at} - 2m_e)\, c^2$$

Ujetje elektrona (Electron capture):

$$p e^- \rightarrow n \nu_e$$

$$^{A}_{Z}X^{atom} \rightarrow {}^{A}_{Z-1}Y^{atom} \, \nu_e$$

DN [1.5.2]

Klasifikacija razpadov beta:

$$P = P'(-1)^{l}$$

Parnost

$$^{A}_{Z}X(J^{P})$$

Vk/ spin

1.5.3

~~$^{22}Na(3^+) \rightarrow {}^{22}$~~

$$^{22}Na(3^+) \rightarrow {}^{22}Ne(2^+)\, e^+ \nu_e \qquad 0{,}99944$$

$$\phantom{^{22}Na(3^+)} \rightarrow {}^{22}Ne(0^+)\, e^+ \nu_e \qquad 1-0{,}99944$$

Ugotovi a je Gammow-Teller ali Fermijev razpad

$$W_{fi} = \frac{2\pi}{\hbar} |V_{fi}|^2 \rho_f(E)$$

$$f_e = \frac{1}{\sqrt{V}} e^{i\frac{\vec{p}_e \cdot \vec{r}}{\hbar}}$$

$$f_\nu = \frac{1}{\sqrt{V}} e^{i\frac{\vec{p}_\nu \cdot \vec{r}}{\hbar}}$$

$$\vec{q} = \vec{p}_e + \vec{p}_\nu$$

$$V_{fi} = \boxed{G_F} \int \psi_f^* \psi_i \boxed{f_e^* f_\nu^*} \, d^3r = \quad \propto e^{i\frac{\vec{q}\cdot\vec{r}}{\hbar}}$$

Fermijeva sklopitvena konstanta.

je majhno

$$= G_F \int \psi_f^* \psi_i \Big(\underset{l=0}{1} + \underset{l=1}{\frac{i\vec{q}\cdot\vec{r}}{\hbar}} + \underset{l=2}{\frac{(i\vec{q}\cdot\vec{r})^2}{2\hbar^2}} + \dots \Big) d^3r$$

Fermijev razpad: $\vec{S}_e + \vec{S}_{\nu_e} = 0$ (spinski singlet)

$$\vec{J} = \vec{J}' + \vec{l}$$

$$|J - J'| \le l \le J + J'$$

Gamow-Teller razpad: $|\vec{S}_e + \vec{S}_{\bar{\nu}_e}| = 1$: (Spinski triplet)

$$|J-J'|-1 \leq l \leq J+J'+1$$

Pri $3^+ \to 2^+$, fermi?

$$1 \leq l \leq 5 \ ; \quad \text{da se } P \text{ ohranja}$$

$$l = 2, 4 \Rightarrow F2, F4$$

GT? $0 \leq l \leq 6$

$$l = 0, 2, 4, 6$$

↑ Ta bo dominanten ⇒ GT0 je dominanten proces razpada.

Pri $3^+ \to 0^+$, fermi?

$$3 \leq l \leq 3 \quad // \text{ ne gre}$$

GT?

$$2 \leq l \leq 4 \ ; \quad l = 2, 4 \Rightarrow \text{dominanten je } GT2$$

Torej: → Le ocena

$$\frac{\tau(3^+ \to 2^+)}{\tau(3^+ \to 0^+)} \approx \frac{|V_{fi}(3^+ \to 0^+)|^2}{|V_{fi}(3^+ \to 2^+)|^2} = \frac{|G_F \int \psi_f^* \psi_i \frac{1}{2!}\left(\frac{i\vec{q}\cdot\vec{r}}{\hbar}\right)^2 d^3r|^2}{|G_F \int \psi_f^* \psi_i \, 1 \, d^3r|^2} :$$

$$\sim \frac{\left|\frac{1}{2!} \frac{q^2 R_j^2}{\hbar^2} \int \psi_f^* \psi_i \, d^3r\right|^2}{|1 \int -\parallel- \ |^2} = \frac{1}{4}\left(\frac{q R_j}{\hbar}\right)^4 \sim 10^{-6}$$

Eksperimentalno pa pride 4.5x več torej je kar okay ocena za tako grob približek.

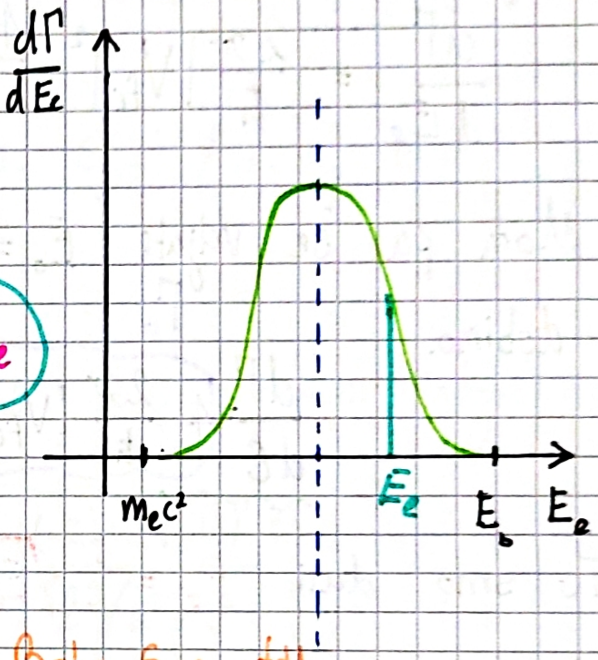
1.5.5 [Spekter β razpadov]

Zanimajo nas τ, $\langle E_{e^-} \rangle$, E_{max} (tam kjer je $\frac{d\Gamma}{dE_e}$ največji)

ne mala energija

Naloga pravi, da vzamemo aproksimacijo :

$$E_0 = E_e + E_\nu \gg m_e c^2$$
$$= m_e c^2 + Q$$



Gremo z Fermijevim zlatim pravilom:

$$d\Gamma = \frac{2\pi}{\hbar} \underbrace{|V_{fi}|}^{2} \boxed{\frac{d\rho_f(E_0)}{dE_e} dE_e}$$

Predpostavimo, da je to precej konstantno

Zanima nas prvo gostota stanj :

Po Bohr-Sommerfeldovem načelu

$$dN = 2V \frac{d^3 p_e}{h^3} \cdot 2V \frac{d^3 p_\nu}{h^3}$$

Elektron je fermion

Neutrino je fermion

$$d^3 p_e = 4\pi p_e dp_e$$

dp_e

$$E_e^2 = c^2 p_e^2 + m_e^2 c^4 \qquad / \cancel{\frac{dE}{dp_e}}$$

$$E_e dE = c^2 p_e dp_e$$

$$\Rightarrow dN = 4 \boxed{V^2} \frac{(4\pi)^2 p_e^2 dp_e \, dp_\nu \, p_\nu^2}{h^6}$$

Se pokrajša z volumnom v matričnem elementu

Lahko postavimo na 1

$$= 4 \frac{(4\pi)^2}{h^6} p_e \frac{E_e dE_e}{c^2} \cdot \frac{E_\nu^2 dE_\nu}{c^2}$$

Fiksiramo E_e v celotni energiji $E = E_e + E_\nu$ od tod sledi $dE = dE_\nu$

$$\Rightarrow \frac{dN}{dE} = \frac{dN}{dE_\nu}$$

$$\boxed{\frac{dN}{dE_\nu}} = \frac{4(4\pi)^2}{h^6}\,\frac{E_e p_e E_\nu^2}{c^5}\,dE_e$$

$d\rho_F$ ↓

$$d\rho_F = \frac{4(4\pi)^2}{h(hc)^5}\,p_e E_e E_\nu^2\,dE_e$$

Torej:

$$\frac{d\Gamma}{dE_e} = \frac{2\pi}{\hbar}|V_{fi}|^2\,\frac{4(4\pi)^2}{h(hc)^5}\,p_e E_e E_\nu{}^2\,dE_e$$

Mora pa še veljati $E_0 = E_e + E_\nu$. Vpeljemo še $\mathcal{E} = \frac{E_e}{m_e c^2}$ in dobimo.

$$\frac{d\Gamma}{d\mathcal{E}} = \underbrace{\overbrace{4\,\frac{2\pi}{\hbar}|V_{fi}|^2\,\frac{(4\pi)^2}{(hc)^6}(m_e c^2)^5}^{\Gamma_0}\,\overbrace{\mathcal{E}(\mathcal{E}_0-\mathcal{E})^2\sqrt{\mathcal{E}^2-1}}^{f(\mathcal{E})}}$$

↑ smo dali $E_\nu = E_0 - E_e$

Tu smo dali $p_e = \frac{1}{c}\sqrt{E_e^2-(m_e c^2)^2}$

↓ Igrali se bomo s to funkcijo

Celotna širina:

$$\Gamma = \int_1^{\mathcal{E}_0 \gg 1} \frac{d\Gamma}{d\mathcal{E}}\,d\mathcal{E}$$

Torej:

$$\Gamma = \Gamma_0 \underbrace{\int_1^{\mathcal{E}_0} f(\mathcal{E})\,d\mathcal{E}}_{I(\mathcal{E}_0)}$$

$$I(\mathcal{E}_0) = \int_1^{\mathcal{E}_0} \mathcal{E}(\mathcal{E}_0-\mathcal{E})^2\sqrt{\mathcal{E}^2-1}\,d\mathcal{E}\ ;\ u = \frac{\mathcal{E}}{\mathcal{E}_0}$$

↗ Uvedemo brezdimenzijsko novo spr.

$$= \mathcal{E}_0^5 \int_{1/\mathcal{E}_0}^{1} u(1-u)^2\sqrt{u^2-\frac{1}{\mathcal{E}_0^2}}$$

V vodilnem redu ko je $\mathcal{E}_0 \gg 1$ lahko

$$I^{(0)}(\mathcal{E}_0) \approx \mathcal{E}_0^5 \int_0^1 du\, u^2(1-u)^2 = \frac{\mathcal{E}_0^5}{30}$$

Integral lahko aproksimiramo če ga razvijemo:

$$z = 1/\mathcal{E}_0 \qquad h(z) = \int_z^1 du\, u(1-u)^2 \sqrt{u^2-z^2}$$

$$F(z) = \int_z^1 f(u)\,du = G(1) - G(z) \Big/ \frac{d}{dz}$$

$$F'(z) = 0 - \frac{dG}{dz}$$

ampak pazimo še dodatno imamo z v mejah. Torej odvajamo integral z parametrom:

$$\frac{dh}{dz} = \left[-u(1-u)^2 \underbrace{\sqrt{u^2-z^2}}_{z^2-z^2}\right]\Bigg|_{u=z} + \int_z^1 du \frac{(-2)u(1-u)^2 z}{2\sqrt{u^2-z^2}}$$

$$\frac{dh}{dz}\bigg|_{z\to 0} = 0$$

$$\frac{dh^2}{dz^2}\bigg|_{z\to 0} = \underbrace{\frac{u(1-u)^2 z}{\sqrt{u^2-z^2}}}_{}\Bigg|_{\substack{z\to 0\\ u\to z}} - \int_z^1 du \frac{u(1-u)^2\left(\sqrt{u^2-z^2} - \frac{1}{2}\frac{-2z^2}{\sqrt{u^2-z^2}}\right)}{u^2-z^2}$$

Tole je malo strange a pravo $z \to 0$ in bo ok (in res tako pride) } Natančnejši račun nam da, da je to okay

$$\frac{dh^2}{dz^2}\bigg|_{z\to 0} = -\int_0^1 du \frac{u(1-u)^2 u}{u^2} = -\frac{1}{3}$$

Torej okoli točke 0 razvito:

$$h(z) \approx \frac{1}{30} \underset{\overset{\uparrow}{0\cdot z}}{} - \frac{1}{3}\frac{1}{2!} z^2$$

Torej: aproksimacija našega integrala:

$$I^{(1)}(\varepsilon_0) = 0$$

$$I^{(2)}(\varepsilon_0) = -\frac{1}{6}\frac{\varepsilon_0^5}{\varepsilon_0^2} - \frac{1}{6}\varepsilon_0^3$$

$$\Rightarrow \quad I(\varepsilon_0) \approx \frac{\varepsilon_0^5}{30}\left(1 - \frac{5}{\varepsilon_0^2}\right) \quad \Rightarrow \frac{1}{\tau} = \Gamma_0 I(\varepsilon_0)$$

Povprečna energija

$$\langle E_e \rangle = E_0 \langle \varepsilon \rangle$$

$$\langle \varepsilon \rangle = \frac{\int_1^{\varepsilon_0} f(\varepsilon)\varepsilon \, d\varepsilon}{\int_1^{\varepsilon_0} f(\varepsilon)\, d\varepsilon} = \frac{\varepsilon_0}{2}\left(1 + \frac{5}{2\varepsilon_0^2}\right)$$

Maksimum spektra

$$f(\varepsilon) = \varepsilon(\varepsilon_0 - \varepsilon)^2\sqrt{\varepsilon^2 - 1} \quad / \cdot \ln$$

$$\ln(f(\varepsilon)) = \ln\varepsilon + 2\ln(\varepsilon_0 - \varepsilon) + \frac{1}{2}\ln(\varepsilon^2 - 1) \quad / \cdot \frac{d}{d\varepsilon}$$

$$\frac{d}{d\varepsilon}\ln f(\varepsilon) = \frac{1}{\varepsilon} - \frac{2}{\varepsilon_0 - \varepsilon} + \frac{\varepsilon}{\varepsilon^2 - 1} = \boxed{0}$$

Zahtevamo, da je maksimum

$$\Rightarrow \frac{(\varepsilon_0 - \varepsilon)(\varepsilon^2 - 1) - 2\varepsilon(\varepsilon^2 - 1) + \varepsilon^2(\varepsilon_0 - \varepsilon)}{\varepsilon(\varepsilon_0 - \varepsilon)(\varepsilon^2 - 1)} = 0$$

nima ε_0 2. reda nižji

$$-4\varepsilon^3 + 2\varepsilon_0\varepsilon^2 + \cancel{3\varepsilon} - \cancel{\varepsilon_0} = 0$$

Približek: če je $\varepsilon_0 \gg 1$ bo verjetno rešitev tudi reda ε_0. Zanemarimo dva reda.

$$-2\varepsilon = -\varepsilon_0 \Rightarrow \varepsilon = \frac{\varepsilon_0}{2}$$

To tudi lahko popravimo (oblike):

$$\varepsilon = \frac{\varepsilon_0}{2}(1+X)$$

$$\downarrow \quad \varepsilon^3 = \left(\frac{\varepsilon_0}{3}\right)^3(1+3X)$$

To vstavimo nazaj:

$$-4\left(\frac{\varepsilon_0}{2}\right)^3(1+3x)+2\varepsilon_0\left(\frac{\varepsilon_0}{2}\right)^2(1+2x)+3\left(\frac{\varepsilon_0}{2}\right)^{\swarrow (1-x)}-\varepsilon_0=0$$

$$\cancel{-4\left(\frac{\varepsilon_0}{2}\right)^3}-12\left(\frac{\varepsilon_0}{2}\right)^3x+\cancel{2\varepsilon_0\left(\frac{\varepsilon_0}{2}\right)^2}+4\varepsilon_0\left(\frac{\varepsilon_0}{2}\right)^2X+3\left(\frac{\varepsilon_0}{2}\right)+$$

$$+\boxed{3\left(\frac{\varepsilon_0}{2}\right)X}-\varepsilon_0=0$$

Zanemarljiv proti prejšnjem členu z X

$$-\frac{12}{9}\varepsilon_0^3X+\varepsilon_0^3X=-\frac{3\varepsilon_0}{2}+\varepsilon_0$$

$$X=\frac{-\frac{\varepsilon_0}{2}}{-1/2\,\varepsilon_0^3}=\boxed{\frac{1}{\varepsilon_0^2}}$$ Popravki so pod kontrolo

1.5.6

Nek koloktivij [Izotop Stroncija]

$${}^{90}_{\substack{49\\3}}\mathrm{Sr} \qquad q=0{,}54\,\frac{W}{g}$$

$$\ln 2\,\tau = t_{1/2}$$

$${}^{90}_{38}\mathrm{Sr} \rightarrow {}^{90}_{39}\mathrm{Y}\ e^-\ \bar{\nu}_e \qquad t_{1/2}=28{,}8\ \text{let}$$

$${}^{90}_{39}\mathrm{Y} \rightarrow {}^{90}_{40}\mathrm{Zr}\ e^-\ \bar{\nu}_e \qquad t'_{1/2} \ll t_{1/2}$$

Oceni masno razliko jeder $\Delta m = m_Y - m_{Zr}$

št razpadov na čas

energija ki se sprosti

$$q=\frac{P}{m} \qquad P=\frac{dN}{dt}\left(\langle E_e\rangle - m_e c^2\right)$$

↳ odšteta masa, ki se ne sprosti

$$A = \frac{N}{\tau} = \frac{N\ln 2}{t_{1/2}}$$

$m_y = m_{Zr} + E_0$

Torej:

$$q = \frac{P}{m} = \frac{P}{N m_y} = \frac{\frac{N\ln 2}{t_{1/2}}\left(\langle E_e\rangle - m_e c^2\right)}{N \boxed{m_y}}$$

aprohsimiramo kar 90u

$$\langle E_e \rangle = m_e c^2 \langle \mathcal{E} \rangle = m_e c^2 \left(\frac{\mathcal{E}_0}{2}\left(1 + \frac{5}{2\mathcal{E}_0^2}\right)\right)$$

Pot naprej bi bila:

$$q \to \langle E_e \rangle \to \mathcal{E}_0 \to \mathcal{E}_0 m_e c^2 = E_0 = \Delta m c^2$$

Pride $E_0 = -2{,}0$ MeV

1.5.7 [Razpad tricija in spekter]

$${}^3H \to {}^3He\ e^-\ \bar{\nu}_e \quad ; \quad m_\nu > 0$$

Ne smemo zanemariti!

Zanima nas tudi spekter pri $E_e \lesssim E_0$.

$$K(E_e) = \sqrt{\frac{1}{p_e E_e} \frac{d\Gamma}{dE_e}}$$

Zadnjič smo izpeljali

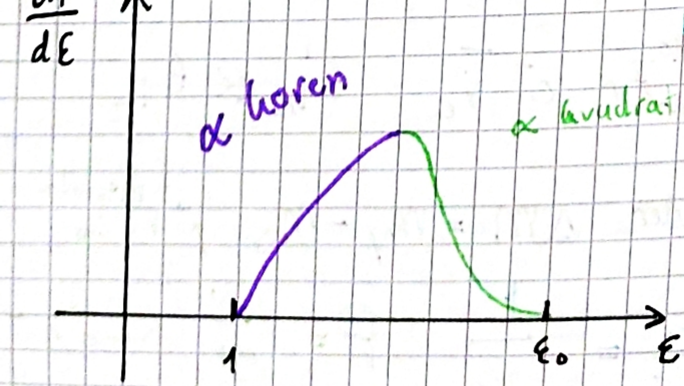
Dominantno za velike $\mathcal{E}$

$$\frac{d\Gamma}{d\mathcal{E}} = A\mathcal{E}\sqrt{\mathcal{E}^2 - 1}\,\boxed{\left(\mathcal{E}_0 - \mathcal{E}\right)^2}$$

$$\mathcal{E} = \frac{E_e}{m_e c^2}$$

$$\mathcal{E}_0 = \frac{E_0}{m_e c^2}$$

$$E_0 = E_e + E_\nu = Q + m_e c^2$$



Razvijemo za • ε: $\sqrt{\varepsilon^2-1} = \sqrt{(\varepsilon-1)(\varepsilon+1)} \sim \boxed{\sqrt{\varepsilon-1}}$

Za male ε

Zanima nas kako [illegible] [illegible] se spremeni spekter, ko dodamo še maso ν. Zadnjič smo delali

$$dN = 4\frac{(4\pi)^2}{h^6} p_e^2 dp_e \, p_\nu^2 dp_\nu$$

$$E_e^2 = p_e^2c^2 + m_e^2c^4 \to c^2 E_e dE_e = dp_e$$

Torej:

$$p_e^2 dp_e = \frac{p_e}{c^2} E_e dE_e$$

$$p_\nu^2 dp_\nu = \frac{p_\nu}{c^2} E_\nu dE_\nu$$

Pospravimo konstante v B in izračunamo $d\rho = \frac{dN}{dE_\nu}$

$$d\rho = B p_e E_e p_\nu E_\nu dE_e$$

$$p_\nu = \frac{1}{c}\sqrt{E_\nu^2 - m_\nu^2 c^4}$$

$E_e \to \varepsilon$

$$E_\nu = E_0 - E_e$$

$$\Rightarrow \varepsilon_\nu = \varepsilon_0 - \varepsilon$$

$$= \frac{1}{c}\sqrt{(\varepsilon_0-\varepsilon)^2 - \tilde{m}_\nu^2} \; ; \; \tilde{m}_\nu = \frac{m_\nu}{m_e}$$

"razenoteno"

$$\frac{d\Gamma}{d\varepsilon} = A\varepsilon\sqrt{\varepsilon^2-1}\,(\varepsilon_0-\varepsilon)\sqrt{(\varepsilon_0-\varepsilon)^2-\tilde{m}_\nu^2}$$

Zdaj je tudi ε_0 različen, ker velja $E_0 = E_e + E_\nu = Q + m_ec^2 + m_\nu c^2$.

Najdimo maksimalno energijo (kje je končna točka)

$$(\varepsilon_0-\varepsilon)^2 - \tilde{m}_\nu^2 = 0$$

$$\varepsilon_0 - \varepsilon_{max} = \tilde{m}_\nu$$

$$\varepsilon_{max} = \varepsilon_0 - \tilde{m}_\nu$$

Pogledamo odvisnost za okoli maksimalne energije

$$\left(\frac{d\Gamma}{d\varepsilon}\right)_{\varepsilon\to\varepsilon_{max}} = A\varepsilon_{max}\sqrt{\varepsilon_{max}^2-1}\left(\varepsilon_0-\varepsilon_{max}+x\right)\sqrt{(\varepsilon_0-\varepsilon_{max}+x)^2-\tilde{m}_\nu^2}$$

$$= A\varepsilon_{max}\sqrt{\varepsilon_{max}^2-1}\,(\tilde{m}_\nu+x)\sqrt{(\tilde{m}_\nu+x)^2-\tilde{m}_\nu^2}$$

Naredimo razvoj po X:

$$\frac{d\Gamma}{d\varepsilon} = A\,\varepsilon_{max}\sqrt{\varepsilon_{max}^2-1}\,\underbrace{(\tilde{m}_\nu+x)\sqrt{2m_\nu x+\cancel{x^2}}}_{\text{Vodilni red je }\sqrt{x}} \quad \text{Zanemarimo}$$

Poglejmo še:

Curie plot

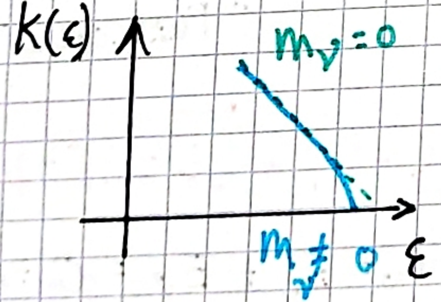
$$\boxed{K(\varepsilon)} = \sqrt{\frac{1}{\varepsilon\sqrt{\varepsilon^2-1}}\frac{d\Gamma}{d\varepsilon}} = A^{1/2}(\underbrace{\varepsilon_0-\varepsilon}_{\tilde{m}_\nu+x})^{1/2}(\underbrace{(\varepsilon_0-\varepsilon)^2-\tilde{m}_\nu^2}_{\tilde{m}_\nu+x})^{1/4}$$

a) $\tilde{m}_\nu = 0$

$$K(\varepsilon) = A^{1/2}\,x$$

b) $m_\nu \neq 0$

$$K(\varepsilon) = \sqrt{A\tilde{m}_\nu}\,(2\tilde{m}_\nu\,x)^{1/4}$$



Razpad γ -

1.6.1 [Razmerje razpadnih širin]

$$X^{**} \longrightarrow X^{*}\,\gamma$$

$$X^{**} \longrightarrow X\,\gamma$$

$$\Gamma_{E1} = \frac{E_\gamma^3}{3\pi \varepsilon_0 \hbar^4 c^3} \left| \int d^3r\, \psi_f^* e\vec{r}\, \psi_i \right|^2$$ Razpadna širina za E_1

$$\Gamma_{M1} = \frac{\mu_0 E_\gamma^{'3}}{3\pi \hbar^4 c^3} \left| \int d^3r\, \psi_f^* \frac{e}{2m_N} \left(\vec{l}^{\,\omega} + g^{\omega}\vec{s} \right) \psi_i \right|^2$$ Razpadna širina za M_1

$\uparrow$ le za proton

Povprečen radij jedra

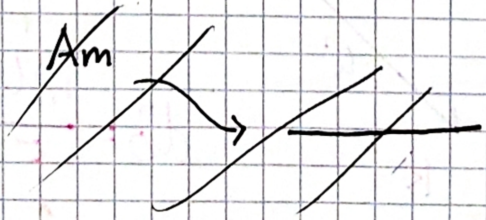
$$\frac{\Gamma_{E1}}{\Gamma_{M1}} \simeq \left(\frac{E_\gamma}{E_\gamma'}\right) \frac{1}{\varepsilon_0 \mu_0} \frac{e^2 \boxed{R^2}}{\left(\frac{e^2}{2^2 m_N^2} \hbar^2\right)} =$$

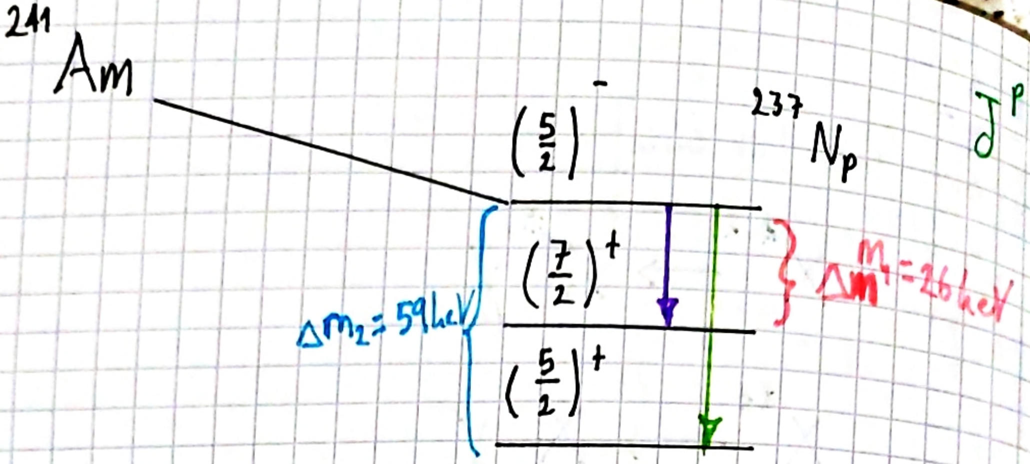
$$= \left(\frac{E_\gamma}{E_\gamma'}\right)^3 \left(\frac{2R m_N c^2}{\hbar c}\right)^2 = \left(\frac{E_\gamma}{E_\gamma'}\right)^3 \left(\frac{\text{fm} - 2 \cdot 100\,\text{GeV}}{0{,}2\,\text{fm}\ \ \text{GeV}}\right) =$$

$$\simeq 100 \left(\frac{E_\gamma}{E_\gamma'}\right)^3$$

1.6.2 [Razpad Americija]

$$^{241}\text{Am} \longrightarrow \alpha + {}^{237}\text{Np}$$





Zanima nas razmerje:

$$\frac{\boxed{\Gamma((\frac{5}{2})^- \to (\frac{5}{2})^+)}}{\boxed{\Gamma((\frac{5}{2})^- \to (\frac{7}{2})^+)}} = (*)$$

$$(\frac{5}{2})^- \to (\frac{5}{2})^+ \quad \Delta J = 0$$

$$|J - J'| \le l \le J + J'$$

$$0 \le l \le 5$$

Oba ~~[illegible]~~ $\underline{\underline{E_1}}$ $\Leftarrow$ $\underline{\underline{l=1}}$

Parnosti:

$E : \Delta P = (-1)^l$

$M : \Delta P = -(-1)^l$

$$(*) = \frac{E_\gamma^3}{E_\gamma'^3} = \left(\frac{\Delta m_2}{\Delta m_1}\right)^3 = \underline{\underline{11,7}}$$

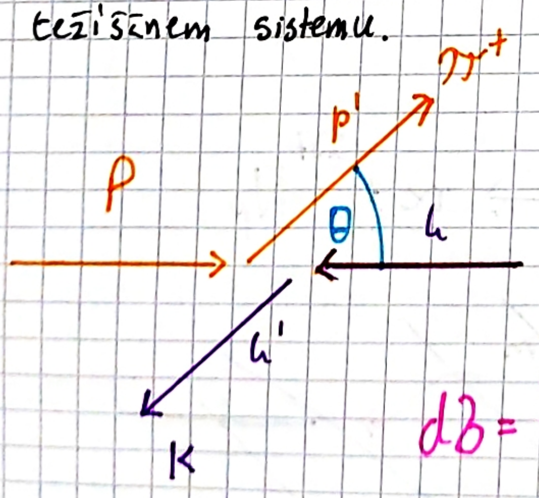
22.4 - 2.2.7 Elektromagnetno sipanje delcev brez spinov

$c = \hbar = 1$

$$\pi^+(p)\, K^+(k) \to \pi^+(p')\, K^+(k')$$

Obravnavaj trk v težiščnem sistemu.

$\vec{p} + \vec{k} = 0$



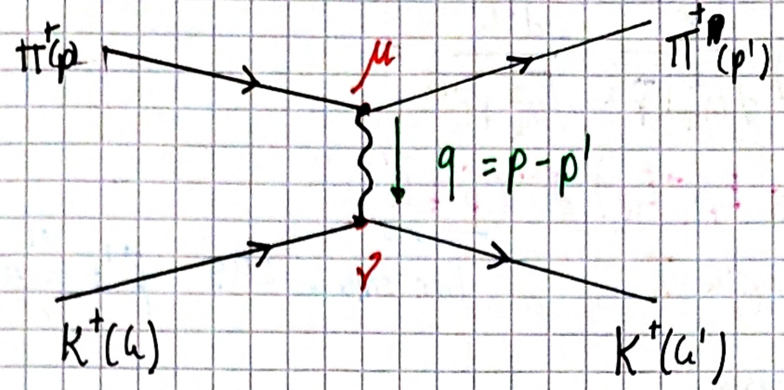
reducirana invariantna amplituda

$$dB = \frac{|M|^2}{F} dQ$$

fazni prostor

fluks

Feynmanov diagram predstavlja vse kar rabimo za $\mathcal{M}$



$$T_{fi} = i \int j^{fi}_\mu (x) A^\mu d^4x$$

$$j^{fi}_\mu (x) = eN^2 (p+p')_\mu e^{-i(p-p')x} = j^{Ni\pi}_\mu$$

Skalarna kvantna elektrodinamika

$$T_{fi} = -i \int j^\pi_\mu (x) \frac{-1}{q^2} j^{k\mu}(x) d^4x =$$

$$= \underbrace{(2\pi)^4 \delta^{(4)}(k+p-k'-p')}_{} \underbrace{(ie)(k+k') \frac{-ig^{\mu\nu}}{q^2} (ie)(p+p')_\nu}_{\mathcal{M}}$$

Še vedno pojavi in zato damo v fazni prostor dQ

Torej:

$$\mathcal{M} = ie(p+p')^\mu \cdot ie(k+k')^\nu \frac{-ig_{\mu\nu}}{q^2}$$

$$\Rightarrow \mathcal{M} = \frac{ie^2}{q^2}(p+p')\cdot(k+k')$$

Poglejmo še dQ fazni prostor:

$$dQ = (2\pi)^4 \delta^{(4)}(p+k-p'-k') \frac{d^3p'}{(2\pi)^3 \underset{\sqrt{m_\pi + \vec{p}'^2}}{\underset{\|}{2E'_p}}} \, \frac{d^3k'}{(2\pi)^3 2E'_k}$$

Torej:

$$dQ = \frac{1}{4(2\pi)^2}\underbrace{\delta^{(4)}(\dots)}\frac{d^3p'}{\sqrt{m_\pi^2+\vec{p}'^2}}\frac{d^3k'}{\sqrt{m_K^2+\vec{k}'^2}} =$$

$$\delta(\boxed{E_p+E_k}-E_{p'}-E_{k'})\,\delta^{(3)}(\underbrace{\vec{p}'+\vec{k}'})$$

Težiščna energija $\sqrt{s}$

ker smo v CMS $\vec{p}'+\vec{k}'=0$

Napaka

$$= \frac{1}{4(2\pi)^2}\cancel{\delta(\sqrt{s}-E_{p'}-E_{k'})}$$

$$\delta\left(\sqrt{s}-\sqrt{m_\pi^2+\vec{p}'^2}-\sqrt{m_k^2+\vec{p}'^2}\right)\frac{d^3p'}{\sqrt{m_\pi^2+\vec{p}'^2}}\cdot\frac{1}{\sqrt{m_K^2+\vec{p}'^2}} =$$

$$d^3p' = |\vec{p}'|^2\,d|\vec{p}'|\,d\Omega = |\vec{p}'|E_p\,dE_p\,d\Omega$$

$$= \frac{1}{4(2\pi)^2}\delta\Big(\sqrt{s}-E_{p'}-\underbrace{\boxed{\sqrt{m_k+E_{p'}^2-m_\pi^2}}}_{E_{k'}}\Big)\,\frac{|\vec{p}'|E_{p'}}{E_p\,E_{k'}}\,dE_{p'}\,d\Omega$$

$$\int\delta(f(x))\,dx = \int\delta(f)\frac{df}{f'} = \frac{1}{|f'(x)|}\int\delta(f)\,df = \frac{1}{|f'(x_0)|}$$

$$\Rightarrow \delta(f(x)) = \sum_i\frac{1}{|f'(x_i)|}\delta(x-x_i)$$

Skratka moramo ugotoviti, kje je ničla zato, da lahko izračunamo $\delta(\dots)$.

$$\sqrt{s}-E_{p'}-\sqrt{m_k^2-m_\pi^2+E_{p'}^2} = 0$$

$$s-2\sqrt{s}E_{p'}+E_{p'}^2 = m_k^2-m_\pi^2+E_{p'}^2$$

$$E_{p'} = \frac{s+m_\pi^2-m_K^2}{2\sqrt{s}}$$

Potrebujemo pa še odvod (po $E_{p'}$):

$$-1-\frac{2E_{p'}}{2E_{k'}} = \frac{E_{k'}+E_{p'}}{E_{k'}}$$

Vstavimo $\left|\frac{1}{F}\right|$:

$$dQ = \frac{1}{4(2\pi)^2}\frac{E_k'}{\sqrt{s}}\frac{|\vec{p}'|}{E_k'}\,d\Omega$$

$$\Rightarrow \quad dQ = \frac{1}{4(2\pi)^2}\frac{|\vec{p}_f|}{\sqrt{s}}\,d\Omega \Bigg\}$$ Dvodelčni fazni prostor v težiščnem sistemu

in še Fluks

$$F = |v_\pi - v_k|\, 2E_k 2E_p =$$

$$= \sqrt{(p\cdot k)^2 - m_\pi^2 m_k^2}\cdot 4$$

$$p^2 = p\cdot p = (E_p, \vec{p})\cdot(E_p, \vec{p})$$
$$= E_p^2 - \vec{p}^{\,2} = \vec{p}^{\,2} + m_\pi^2 - \vec{p}^{\,2}$$
$$= m_\pi^2$$

$$\Rightarrow F = 4\sqrt{\left(\frac{(p+k)^2 - p^2 - k^2}{2}\right)^2 - m_\pi^2 m_k^2} =$$

$$= 4\sqrt{\left(s - m_\pi^2 - m_k^2 - 4m_\pi^2 m_k^2\right)\frac{1}{4}} =$$

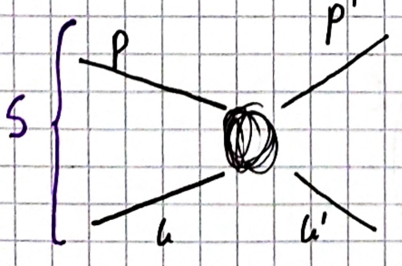
$$= 2\sqrt{(s + m_\pi^2 + m_k^2)^2 - 4(m_\pi^2 m_k + s m_\pi^2 + s m^2_{\cancel{k}\,k})}$$

$$= 2\cdot 2\sqrt{s}\,|\vec{p}_i| = 4\sqrt{s}\,|\vec{p}_i|$$

Tako je diferencialni sipalni presek:

$$\frac{d\sigma}{d\Omega} = \frac{|\mathcal{M}|^2}{64\pi^2 s}\frac{p_f}{p_i} \Bigg\}$$ 2 na 2 v CMS

Uvedimo parametrizacijo z Mandelstamove spremenljivkami



$$s = (p+k)^2$$
$$t = (p-p')^2$$
$$u = (p-k')^2$$
$$q^2 = t$$
$$s + t + u = \sum_i m_i^2$$

$$\mathcal{M} = \frac{ie^2}{q^2}(p+p')\cdot(k+k')$$

To moramo izraziti z s, t, u:

$$p+k = p'+k'$$

$$p\cdot k = \frac{s - m_\pi^2 - m_k^2}{2}$$

$$p\cdot k' = p'\cdot k = \frac{-u + m_\pi^2 + m_k^2}{2}$$

$$p'\cdot k' = p\cdot k$$

$$\mathcal{M} = \frac{ie^2}{t}(p\cdot k + p\cdot k' + p'\cdot k + p'\cdot k') = \frac{ie^2}{t}(s-u)$$

Sedaj pa izpelimo kotno porazdelitev v limiti $s \gg m_k^2$ $(> m_\pi^2)$, torej ultrarelativistično.

$$u = (p-k')^2 \qquad p^\mu = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\hat{e}_z\right)$$

pol energije ima

$$k^\mu = \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\hat{e}_z\right)$$

$$p'^\mu = \frac{\sqrt{s}}{2}(1, \hat{p})$$

$$k'^\mu = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\hat{p}\right)$$

So:

$$u = (p-k')^2 = -2\, p\cdot k' = -2\frac{s}{4}\left(1 + \overset{\cos\theta}{\boxed{\hat{p}\cdot\hat{e}_z}}\right) = -\frac{s}{2}(1+\cos\theta)$$

$$t = -u - s = -\frac{s}{2}(1-\cos\theta) \Rightarrow \mathcal{M} = -ie^2\frac{3+\cos\theta}{1-\cos\theta}$$

in je presek tako:

$$\frac{d\sigma}{d\Omega} = \frac{e^4}{64\pi^2 s}\left(\frac{3+\cos\theta}{1-\cos\theta}\right)^2$$

$$\mathcal{M} = \frac{ie^2}{q^2}(p+p')\cdot(k+k')$$

To moramo izraziti z s, t, u:

$$p+k = p'+k'$$

$$p\cdot k = \frac{s-m_\pi^2-m_k^2}{2}$$

$$p\cdot k' = p'\cdot k = \frac{-u+m_\pi^2+m_k^2}{2}$$

$$p'\cdot k' = p\cdot k$$

$$\mathcal{M} = \frac{ie^2}{t}(p\cdot k + p\cdot k' + p'\cdot k + p'\cdot k') = \frac{ie^2}{t}(s-u)$$

Sedaj pa izpelimo kotno porazdelitev v limiti $s \gg m_k^2$ $(> m_\pi^2)$, torej ultrarelativistično.

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$$k^\mu = \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\hat{e}_z\right)$$

$$p'^\mu = \frac{\sqrt{s}}{2}(1, \hat{p})$$

$$k'^\mu = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\hat{p}\right)$$

So:

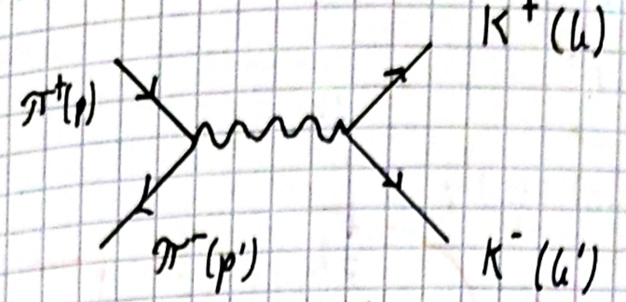
$$u=(p-k')^2 = -2\,p\cdot k' = -2\frac{s}{4}\left(1+\overset{\cos\theta}{\boxed{\hat{p}\cdot\hat{e}_z}}\right) = -\frac{s}{2}(1+\cos\theta)$$

$$t = -u-s = -\frac{s}{2}(1-\cos\theta) \Rightarrow \mathcal{M} = -ie^2\,\frac{3+\cos\theta}{1-\cos\theta}$$

in je presek tako:

$$\frac{d\sigma}{d\Omega} = \frac{e^4}{64\pi^2 s}\left(\frac{3+\cos\theta}{1-\cos\theta}\right)^2$$

$\pi^+\pi^- \to K^-K^+$



$$\pi^+(p)\pi^-(p') \to K^+(k)K^-(k')$$

rata antidelc

crossing

$$\pi^+(p)K^+(-k') \to K^+(k)\pi^+(-p')$$ } To pa že imamo

Od zadnjič: $s' = (p-k')^2$
$t' = (p+p')^2$
$u' = (p-k)^2$

Splošno : $u = (k'-p)^2$
$t = (k-p)^2$
$s = (p+p')^2$

Torej: $s' = u$
$u' = t$
$t' = s$

$$\Rightarrow |\mathcal{M}|^2 = e^4\left(\frac{s'-u'}{t'}\right)^2 = e^4\left(\frac{u-t}{s}\right)^2$$

θ

θ

Sipalni presek za tako anihilacijo bi potem bil

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\overset{1}{\boxed{\frac{p_f}{p_i}}}|\mathcal{M}|^2 = \frac{\alpha^2}{4s}\cos^2\theta$$

2.1.1. [Slepa močna interakcija za n in p]

Obravnavamo: $\underbrace{{}^{31}_{15}P \quad {}^{31}_{16}S}_{\text{Zrcalni jedri}}$

$$\tilde{W}_b(P) = W_b(p) + \boxed{W_2 \frac{\overset{15}{Z^2}}{\underset{31}{A^{1/3}}}}$$

Da pokrajšamo elektrostatski del in da ostane le močna interakcija

$$\frac{\tilde{W}_b(P)}{A} = 8{,}48\ MeV$$

+ še isti postopek za S

$$\frac{\tilde{W}_b(P) - \tilde{W}_b(S)}{A} = -0{,}027\ MeV$$

Izospinska simetrija

Nukleon

$$N = \begin{pmatrix} p \\ n \end{pmatrix}$$

$$N \longmapsto UN$$

2.1.2 [Grupa SU(2)]

$$SU(2) = \left\{ M \in \mathbb{C}^{2\times2};\ \overset{\text{Unitarna}}{\overbrace{M^\dagger M = I}},\ \det M = 1 \right\}$$

$$N^\dagger N = |p|^2 + |n|^2$$

$$(UN)^\dagger UN = N^\dagger U^\dagger U N = N^\dagger N$$

i) $M_1 M_2 = M$

$$M^\dagger M = M_2^\dagger M_1^\dagger M_1 M_2 = M_2^\dagger M_2 = I$$

ii) $e = I_{2\times2}$

iii) $$M^{-1} = M^{\dagger}$$

iv) Matrično množenje je asociativno

$\Rightarrow$ Res tvorijo grupo.

$$M = e^{iH} = 1 + iH + \dots$$

$$M^{\dagger} = 1 - iH - \dots = e^{-iH}$$

Take grupe so Liejeve grupe.

$$H = \cancel{\alpha_0 I} + \alpha_1 \frac{\sigma_1}{2} + \alpha_2 \frac{\sigma_2}{2} + \alpha_3 \frac{\sigma_3}{2}$$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Ker mora za SU(2) veljati $\det M = 1$ je $\alpha_0 = 0$.

$$\det e^{iH} = \det\left(I + i\left[\alpha_0 I + \alpha_1 \frac{\sigma_1}{2} + \alpha_2 \frac{\sigma_2}{2} + \alpha_3 \frac{\sigma_3}{2}\right]\right) =$$

$$= \det \begin{bmatrix} 1 + i(\alpha_0 + \frac{\alpha_3}{2}) & i\frac{\alpha_1}{2} + \frac{\alpha_2}{2} \\ i\frac{\alpha_1}{2} - \frac{\alpha_2}{2} & 1 + i(\alpha_0 - \frac{\alpha_3}{2} \end{bmatrix} =$$

$$= 1 + 2i\alpha_0 \; ; \; \alpha = 0$$ in to velja

Torej:

generatorji

$$M = e^{iH} = e^{i\alpha_i \boxed{\frac{\sigma_i}{2}}}$$ } Lie-jeve grupe !

št. generatorjev $SU(N) = N^2 - 1$

$$\frac{\sigma_i}{2} = L_i \qquad [L_i, L_j] = i\,\varepsilon_{ijk} I_k$$ } Kot za vrtilne količine

2 Torej: $|p\rangle = |\overset{\uparrow}{\underset{I}{\tfrac{1}{2}}}\ \overset{\uparrow}{\underset{I_3}{\tfrac{1}{2}}}\rangle_I$

(

$$I_\pm = I_1 \pm i I_2$$

$$|n\rangle = |\tfrac{1}{2}\ -\tfrac{1}{2}\rangle_I$$

$$I_\pm = I_1 \pm i I_2 = \left[\ \begin{array}{l} \frac{1}{2}\begin{pmatrix} 0 & 1+i(-i) \\ 1+ii & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = I_+ \\ \quad \dots \qquad\qquad\qquad\quad = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = I_- \end{array}\right.$$

$$|p\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad I_-|p\rangle = |n\rangle$$

$$|n\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad I_+|n\rangle = |p\rangle$$

2.1.1.3. [Ohranjena količina = Hamiltonian komutira]

$$[H, I_i] = 0 \Rightarrow m_p = m_n$$

Masa je povprečna energij
če minje

$$m_p = \langle p|H|p\rangle$$

$$m_n = \langle n|H|n\rangle$$

$$|p\rangle = I_+|n\rangle \,\Big|\, ^\dagger$$

$$\langle p| = \langle n|I_-$$

$$\hat{I}|I\,I_3\rangle = I(I+1)|I\,I_3\rangle$$

$$\hat{I}_3|\ \rangle = I_3|\ \rangle$$

Torej: $m_p = \langle p|H|p\rangle = \langle n|\underbrace{I_- H I_+}_{I_- I_+ H}|n\rangle =$

$$I_- I_+ = I_1^2 + I_2^2 - i I_2 I_1 + i I_1 I_2 =$$

$$= I^2 - I_3^2 + i[I_1, I_2] =$$

$$= I^2 - I_3^2 - I_3$$

$$= \langle n|H(I^2 - I_3^2 - I_3)|n\rangle = \langle n|H(\underbrace{\tfrac{1}{2}\cdot\tfrac{3}{2} - (-\tfrac{1}{2})^2 + \tfrac{1}{2}}_{=1})|n\rangle =$$

$$= \langle n|H|n\rangle$$

2.1.4. [Dva nukleona]

$I = 1$ triplet

$I = 0$ singlet

$$I_- |p\rangle|p\rangle = |n\rangle|p\rangle + |p\rangle|n\rangle$$

$$I_\pm |I\, I_3\rangle = \sqrt{(I+1)I - I_3(I_3 \pm 1)}\, |I, I_3 \pm 1\rangle$$

Triplet
$$\begin{cases} |1\,1\rangle = |p\rangle|p\rangle \\ |1\,0\rangle = \left(|p\rangle|n\rangle + |n\rangle|p\rangle\right)\frac{1}{\sqrt{2}} \\ |1\,{-1}\rangle = |n\rangle|n\rangle \end{cases}$$

$$|0\,0\rangle = \frac{1}{\sqrt{2}}\left(|p\rangle|n\rangle - |n\rangle|p\rangle\right)$$