

1. Reminder of Notation

$$\begin{aligned}
g_{\mu\nu} &= g^{\mu\nu} = \text{diag}(1, -1, -1, -1), \\
x_\mu &= g_{\mu\nu} x^\nu & x^\mu &= g^{\mu\nu} x_\nu, \\
\partial_\mu &= \frac{\partial}{\partial x^\mu} = \left(\frac{\partial}{\partial x^0}, \nabla \right), \\
\partial^\mu &= \frac{\partial}{\partial x_\mu} = \left(\frac{\partial}{\partial x_0}, -\nabla \right), \\
\frac{\partial}{\partial a_\mu} (a_\nu b^\nu) &= \delta_\nu^\mu b^\nu = b^\mu, \\
p &\equiv \gamma^0 p^0 - \gamma^i p^i.
\end{aligned}$$

2. Basic Principles

$$\begin{aligned}
\mathcal{H} &= \Pi \dot{\phi} - \mathcal{L} & \Pi &= \frac{\partial \mathcal{L}}{\partial \dot{\phi}}, \\
S &= \int_{t_1}^{t_2} dt \int d\mathbf{x} \mathcal{L}, \\
\text{E-L eq.:} & \quad \frac{\partial \mathcal{L}}{\partial \phi_a} - \partial_\mu \left[\frac{\partial \mathcal{L}}{\partial \partial_\mu \phi_a} \right] = 0, \\
\text{Causality:} & \quad [\phi_a(\mathbf{x}, t), \Pi_b(\mathbf{y}, t)] = i\delta^3(\mathbf{x} - \mathbf{y}), \\
& \quad (\mathbf{x} - \mathbf{y})^2 < 0,
\end{aligned}$$

3. Canonical Quantization

$$\begin{aligned}
[\hat{\phi}(\mathbf{x}, t), \hat{\Pi}(\mathbf{y}, t)] &= i\delta^3(\mathbf{x} - \mathbf{y}), \\
[\hat{\phi}(\mathbf{x}, t), \hat{\phi}(\mathbf{y}, t)] &= 0, \\
[\hat{\Pi}(\mathbf{x}, t), \hat{\Pi}(\mathbf{y}, t)] &= 0.
\end{aligned}$$

4. Noether's Theorem

Invariance under a continuous transformation of the form: $\phi_a \rightarrow \phi_a + \alpha \Delta \phi_a$ where $\mathcal{L} \rightarrow \mathcal{L} + \alpha \partial_\mu J^\mu$ gives rise to conserved current and charge:

$$\begin{aligned}
j^\mu &= \sum_a \frac{\partial \mathcal{L}}{\partial \partial_\mu \phi_a} \Delta \phi_a - J^\mu, \\
Q &= \int j^0(\mathbf{x}, t) d\mathbf{x},
\end{aligned}$$

for which it holds that:

$$\begin{aligned}
\text{(i)} \quad \partial_\mu j^\mu(\mathbf{x}, t) &= 0 \\
\text{(ii)} \quad \frac{d}{dt} Q &= 0
\end{aligned}$$

5. Scalar Fields

$$\begin{aligned}
\partial_\mu \partial^\mu \phi(\mathbf{x}, t) + m^2 \phi(\mathbf{x}, t) &= 0, \\
\mathcal{L} &= \frac{1}{2} (\partial_\mu \phi) (\partial^\mu \phi) - \frac{m^2}{2} \phi^2, \\
T^\mu{}_\nu &= \frac{\partial \mathcal{L}}{\partial \partial_\mu \phi} \partial_\nu \phi - \delta^\mu{}_\nu \mathcal{L}, \\
\hat{\phi}(\mathbf{x}, t) &= \int \frac{d\mathbf{p}}{(2\pi)^3} A_{\mathbf{p}} (a_{\mathbf{p}} e^{-ipx} + a_{\mathbf{p}}^\dagger e^{ipx}), \\
[a_{\mathbf{p}}, a_{\mathbf{p}'}^\dagger] &= (2\pi)^3 \delta^3(\mathbf{p} - \mathbf{p}'), \\
[a_{\mathbf{p}}, a_{\mathbf{p}'}] &= 0, \quad [a_{\mathbf{p}}^\dagger, a_{\mathbf{p}'}^\dagger] = 0, \\
\hat{\mathbf{p}} &= \int \frac{d\mathbf{p}}{(2\pi)^3} \mathbf{p} a_{\mathbf{p}}^\dagger a_{\mathbf{p}}, \\
\hat{H} &= \int \frac{d\mathbf{p}}{(2\pi)^3} E_{\mathbf{p}} \frac{1}{2} \left[a_{\mathbf{p}}^\dagger a_{\mathbf{p}} + \frac{1}{2} (2\pi)^3 \delta(0) \right], \\
\Delta_F(x - y) &= \int \frac{d^4 p}{(2\pi)^4} \frac{i e^{ip \cdot (x - y)}}{p^2 - m^2 + i\varepsilon}.
\end{aligned}$$

6. Dirac Spinor Fields (Fermions)

$$\begin{aligned}
\left(i\gamma^\mu \frac{\partial}{\partial x^\mu} - m \right) \psi(x) &= 0, \\
\mathcal{L}(x) &= \bar{\psi}_a (i\gamma^\mu \partial_\mu - m)_{ab} \psi_b(x), \\
\bar{\psi} &\equiv \psi^\dagger \gamma^0, \\
\psi(x) &= \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\mathbf{p}}}} \sum_s (a_{\mathbf{p}}^s u^s(p) e^{-ip \cdot x} \\
&\quad + b_{\mathbf{p}}^{s\dagger} v^s(p) e^{ip \cdot x}), \\
\bar{\psi}(x) &= \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\mathbf{p}}}} \sum_s (b_{\mathbf{p}}^s \bar{v}^s(p) e^{-ip \cdot x} \\
&\quad + a_{\mathbf{p}}^s \bar{u}^s(p) e^{ip \cdot x}), \\
\{\psi_a(\mathbf{x}, t), \psi_b^\dagger(\mathbf{y}, t)\} &= \delta_{ab} \delta(\mathbf{x} - \mathbf{y}), \\
\{\psi_a(\mathbf{x}, t), \psi_b(\mathbf{y}, t)\} &= 0, \\
\{\psi_a^\dagger(\mathbf{x}, t), \psi_b^\dagger(\mathbf{y}, t)\} &= 0, \\
\{a_{\mathbf{p}}^r, a_{\mathbf{q}}^{s\dagger}\} &= \{b_{\mathbf{p}}^r, b_{\mathbf{q}}^{s\dagger}\} = (2\pi)^3 \delta^{rs} \delta(\mathbf{p} - \mathbf{q}), \\
u^s(\mathbf{p}) &= M_{\mathbf{p}} u^s(0) = \sqrt{\frac{E + m}{m}} \begin{pmatrix} \xi^s \\ \frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{E + m} \xi^s \end{pmatrix}; \\
\xi^1 &= \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \xi^2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \\
v^s(\mathbf{p}) &= M_{\mathbf{p}} v^s(0) = \sqrt{\frac{E + m}{m}} \begin{pmatrix} \frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{E + m} \eta^s \\ \eta^s \end{pmatrix}; \\
\eta^1 &= \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \eta^2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \\
\bar{u}^r(p) u^s(p) &= 2m \delta^{rs}, \\
\bar{v}^r(p) v^s(p) &= -2m \delta^{rs},
\end{aligned}$$

$$\begin{aligned}
u^{r\dagger}(p)u^s(p) &= 2E_{\mathbf{p}}\delta^{rs}, \\
v^{r\dagger}(p)v^s(p) &= +2E_{\mathbf{p}}\delta^{rs}, \\
\bar{u}^r(p)v^s(p) &= \bar{v}^r(p)u^s(p) = 0, \\
u^{r\dagger}(\mathbf{p})v^s(-\mathbf{p}) &= v^{r\dagger}(-\mathbf{p})u^s(\mathbf{p}) = 0, \\
\hat{H} &= \int \frac{d\mathbf{p}}{(2\pi)^3} E_{\mathbf{p}} \sum_s \left(a_{\mathbf{p}}^{s\dagger} a_{\mathbf{p}}^s + b_{\mathbf{p}}^{s\dagger} b_{\mathbf{p}}^s \right), \\
\hat{\mathbf{p}} &= \int \frac{d\mathbf{p}}{(2\pi)^3} \mathbf{p} \sum_s \left(a_{\mathbf{p}}^{s\dagger} a_{\mathbf{p}}^s + b_{\mathbf{p}}^{s\dagger} b_{\mathbf{p}}^s \right), \\
\hat{Q}_{\text{el.}} &= e \int \frac{d\mathbf{p}}{(2\pi)^3} \sum_s \left(a_{\mathbf{p}}^{s\dagger} a_{\mathbf{p}}^s - b_{\mathbf{p}}^s b_{\mathbf{p}}^{s\dagger} \right), \\
\hat{\mathbf{J}} &= \hat{\mathbf{S}} + \hat{\mathbf{L}} = \\
&= \int d^3x \hat{\psi}^\dagger \left[\frac{1}{2} \Sigma + (\mathbf{x} \times (-i\nabla)) \right]_z \hat{\psi}, \\
\Sigma^i &= \frac{1}{2} \begin{bmatrix} \sigma^i & 0 \\ 0 & \sigma^i \end{bmatrix}, \\
\Delta_F(x-y) &\equiv \langle 0 | \mathcal{T} \{ \psi(x) \bar{\psi}(y) \} | 0 \rangle = \\
&= \int \frac{d^4p}{(2\pi)^4} \frac{i(\not{p} + m)}{p^2 - m^2 + i\varepsilon} e^{-ip \cdot (x-y)}, \\
(a_{\mathbf{p}} e^{-ip \cdot x} + a_{\mathbf{p}}^\dagger e^{ip \cdot x}) &\xrightarrow{t=0} (a_{\mathbf{p}} + a_{\mathbf{p}}^\dagger) e^{ip \cdot x}, \\
|p\rangle &= \sqrt{2E_{\mathbf{p}}} a_{\mathbf{p}}^\dagger |0\rangle, \\
\text{Majorana Fermions: } &b_{\mathbf{p}}^s = a_{\mathbf{p}}^s.
\end{aligned}$$

7. Complex Scalar Fields

$$\begin{aligned}
\mathcal{L} &= \partial_\mu \phi \partial^\mu \phi^\dagger - m^2 \phi \phi^\dagger, \\
\hat{\phi}(\mathbf{x}, t) &= \int \frac{d\mathbf{p}}{(2\pi)^3} \frac{1}{\sqrt{2E_{\mathbf{p}}}} (a_{\mathbf{p}} e^{-ip \cdot x} + b_{\mathbf{p}}^\dagger e^{ip \cdot x}), \\
\hat{\phi}(\mathbf{x}, t) &\neq \hat{\phi}^\dagger(\mathbf{x}, t), \\
[a_{\mathbf{p}}, a_{\mathbf{q}}^\dagger] &= (2\pi)^3 \delta(\mathbf{p} - \mathbf{q}), \\
[\hat{\phi}(\mathbf{x}, t), \hat{\Pi}(\mathbf{y}, t)] &= i\delta(\mathbf{x} - \mathbf{y}).
\end{aligned}$$

8. Interactions

- **Scalar-Scalar:**

$$\mathcal{L}_I = -\frac{\lambda}{4!} \phi^4,$$
- **Fermion-Vector:**

$$\mathcal{L}_I = -A^\mu \sum_f e_f \bar{\psi}_f \gamma_\mu \psi_f,$$
- **Fermion-Scalar (Yukawa):**

$$\mathcal{L}_I = -g \phi \bar{\psi} \psi,$$
- **Fermion-Pseudoscalar:**

$$\mathcal{L}_I = -g \bar{\psi} i \gamma^5 \phi \psi,$$

- **Scalar-Vector:**

$$\mathcal{L}_1 = \tilde{g} V_\mu \phi \partial^\mu \phi^\dagger,$$

$$\mathcal{L}_2 = \tilde{g} V_\mu \phi V^{\mu\dagger} \phi^\dagger,$$

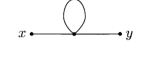
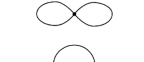
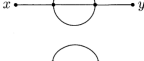
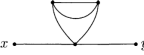
- **Do remember that:**

$$\mathcal{H}_I = -\mathcal{L}_I,$$

- **Correlation Functions:**

$$\begin{aligned}
\langle \Omega | \mathcal{T} \{ \phi_1(x_1) \phi_2(x_2) \dots \} | \Omega \rangle &= \\
\lim_{T \rightarrow \infty (1-i\varepsilon)} \frac{\langle 0 | \mathcal{T} \{ \phi_I(x_1) \dots \phi_I(x_n) \mathcal{X} \} | 0 \rangle}{\langle 0 | \mathcal{T} \{ \exp[-i \int_{-T}^T \int d^4z \mathcal{H}_{\text{int}}(z)] \} | 0 \rangle}, \\
\mathcal{X} &= \exp \left[-i \int_{-T}^T \int d^4z \mathcal{H}_{\text{int}}(z) \right],
\end{aligned}$$

- **Symmetry Factor:**

	$S = 2$
	$S = 2 \cdot 2 \cdot 2 = 8$
	$S = 3! = 6$
	$S = 3! \cdot 2 = 12$

- **Probability Amplitude:**

$$\hat{S} = 1 + i\hat{T},$$

$$iT_{fi} = (2\pi)^4 \delta^4(p_1 + p_2 - k_A - k_B) iM_{fi},$$

$$d\sigma = d\Pi |\bar{M}|^2 \frac{1}{2E_A 2E_B |v_A - v_B|} \text{ v is velocity},$$

$$d\Pi = \frac{p_1}{16\pi^2 E_{\text{CMS}}} d\Omega,$$

Nonpolarized scattering:

$$|\bar{M}|^2 = \frac{1}{N_s N_r} \sum_{s,r} \sum_{s',r'} |M(s, r \rightarrow s', r')|^2,$$

Fermions:

$$\sum_s s = 1, 2 u^s(\mathbf{p}) \bar{u}^s(\mathbf{p}) = \not{p} + m,$$

$$\sum_s s = 1, 2 v^s(\mathbf{p}) \bar{v}^s(\mathbf{p}) = \not{p} - m,$$

Photons:

$$\sum_{r=1,2} \varepsilon_\mu^{r\dagger} \varepsilon_\nu^r = -g_{\mu\nu}$$

9. Feynman Rules

When transforming from momentum to position space, remember:

$$\int d^4x \exp[-ix(\sum p_i)] = (2\pi)^4 \delta^4(\sum p_i)$$

Rules in momentum space:

1. Propagators:

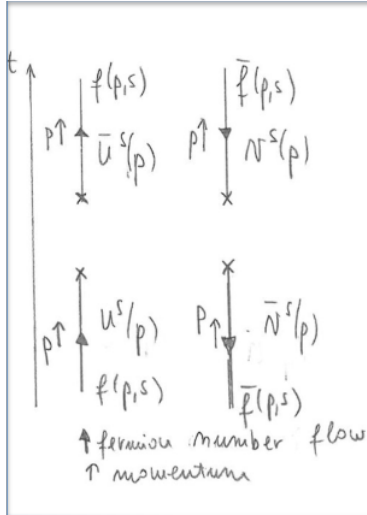
$$\begin{aligned} \overline{\phi(x)}\phi(y) &= \text{diagram} = \frac{i}{q^2 - m_\phi^2 + i\epsilon} \\ S_F(x-y) = \overline{\psi(x)}\psi(y) &= \text{diagram} = \frac{i(\not{p} + m)}{p^2 - m^2 + i\epsilon} \\ A_\mu(x)A_\nu(y) &= \text{diagram} = \frac{-ig_{\mu\nu}}{q^2 + i\epsilon} \quad \text{Feynman gauge } \xi = 1 \end{aligned}$$

2. Vertices:

$$\begin{aligned} \text{Yukawa int.} &= -ig \mathbf{I} \\ \text{QED} &= -ie\gamma^\mu \end{aligned}$$

3. External leg contractions:

$$\begin{aligned} \overline{\phi}[\mathbf{q}] &= \text{diagram} = 1 & \phi[\mathbf{q}] &= \text{diagram} = 1 \\ \overline{\psi}[\mathbf{p}, s] &= \text{diagram} = u^s(p) & \psi[\mathbf{p}, s] &= \text{diagram} = \bar{u}^s(p) \\ \overline{\psi}[\mathbf{k}, s] &= \text{diagram} = \bar{v}^s(k) & \psi[\mathbf{k}, s] &= \text{diagram} = v^s(k) \\ \overline{A}_\mu[\mathbf{p}] &= \text{diagram} = \epsilon_\mu^*(p) & A_\mu[\mathbf{p}] &= \text{diagram} = \epsilon_\mu(p) \end{aligned}$$



$$\langle \phi | \partial_\mu \phi = (-ip_\mu) e^{-ipx}$$

$$\partial_\mu \phi | \phi \rangle = (ip_\mu) e^{ipx}$$

Some additional relations:

$$\begin{aligned} \text{tr } \mathbb{1} &= 4, \\ \text{tr}(\text{odd number of } \gamma) &= 0, \\ \text{tr } \gamma^\mu \gamma^\nu &= 4g^{\mu\nu}, \\ \text{tr}(\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma) &= 4(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho}), \\ \text{tr}(\gamma^5) &= 0, \\ \text{tr}(\gamma^\mu \gamma^\nu \gamma^5) &= 0, \\ \text{tr}(\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma^5) &= -4i\epsilon^{\mu\nu\rho\sigma}, \\ \epsilon^{\alpha\beta\gamma\delta} \epsilon_{\alpha\beta\gamma\delta} &= -24, \\ \epsilon^{\alpha\beta\gamma\delta} \epsilon_{\alpha\beta\gamma\delta} &= -6\delta^\nu_\delta, \\ \epsilon^{\alpha\beta\gamma\mu\nu} \epsilon_{\alpha\beta\gamma\rho\sigma} &= -2(\delta^\mu_\rho \delta^\nu_\sigma - \delta^\mu_\sigma \delta^\nu_\rho), \\ \gamma^\mu \gamma^\nu \gamma_\mu &= -2\gamma^\nu, \\ \gamma^\mu \gamma^\nu \gamma^\rho \gamma_\mu &= 4g^{\nu\rho}, \\ \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma_\mu &= -2\gamma^\sigma \gamma^\rho \gamma^\nu, \\ \gamma^{\mu\dagger} &= \gamma^0 \gamma^\mu \gamma^0, \\ \text{tr}(\gamma^\mu \not{a} \gamma^\mu \not{b}) &= 4(a^\mu b^\nu - a^\nu b^\mu - g^{\mu\nu} a \cdot b), \end{aligned}$$

10. Quantum Electrodynamics

$$\hat{A}_\mu(x) = \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{1}{\sqrt{2E_{\mathbf{k}}}} \sum_r (\epsilon_\mu^r(\mathbf{k}) e^{-i\mathbf{k}\cdot x} a_{\mathbf{k}}^r + \epsilon_\mu^{r*}(\mathbf{k}) e^{i\mathbf{k}\cdot x} a_{\mathbf{k}}^{r\dagger}),$$

$$\Delta_{\mu\nu}(x-y) = \int \frac{d^4k}{(2\pi)^4} \frac{ig_{\mu\nu}}{k^2 + i\epsilon} e^{-ik\cdot(x-y)},$$

Ward Identity:

$$k_\mu M^\mu = 0,$$

Polarizations:

$$\epsilon_\mu^{r=0=s} = (1, 0, 0, 0),$$

$$\epsilon_\mu^{r=1=T} = (0, 1, 0, 0),$$

$$\epsilon_\mu^{r=2=T} = (0, 0, 1, 0),$$

$$\epsilon_\mu^{r=3=L} = (0, 0, 0, 1),$$

11. Regularization

Feynmann parametrization:

$$\begin{aligned}
\frac{1}{AB} &= \int_0^1 dx \frac{1}{[x(A-B) + B]^2}, \\
\frac{1}{A_1 \dots A_n} &= \int_0^1 \frac{dx_1 \dots dx_n \delta(\sum x_i - 1)(n-1)!}{[x_1 A_1 + x_2 A_2 + \dots + x_n A_n]^n}, \\
A &= (k+q)^2 - m^2 + i\varepsilon, B = k^2 - m^2 + i\varepsilon, \\
&\Rightarrow x(A-B) + B = l^2 - \Delta + i\varepsilon = \\
&= (k^2 + xq)^2 - x^2 q^2 + xq^2 - m^2 + i\varepsilon, \\
A &= (p-k)^2 + i\varepsilon, B = k^2 - m^2 + i\varepsilon, \\
&\Rightarrow x(A-B) + B = l^2 - \Delta + i\varepsilon = \\
&= (k - px)^2 - p^2 x^2 + xp^2 - m^2 + xm^2 + i\varepsilon.
\end{aligned}$$

Wick Rotation:

$$\begin{aligned}
l^0 &= il_E^0, \\
\int \frac{d^d l_E}{(2\pi)^d} \frac{1}{(l_E^2 + \Delta)^n} &= \\
&= \frac{1}{(4\pi)^{(d/2)}} \frac{\Gamma_E(n - \frac{d}{2})}{\Gamma_E(n)} \left(\frac{1}{\Delta}\right)^{n-d/2}, \\
\int \frac{d^d l_E}{(2\pi)^d} \frac{l_E^2}{(l_E^2 + \Delta)^n} &= \\
&= \frac{1}{(4\pi)^{(d/2)}} \frac{d}{2} \frac{\Gamma(n - \frac{d}{2} - 1)}{\Gamma(n)} \left(\frac{1}{\Delta}\right)^{n-\frac{d}{2}-1}, \\
\frac{d}{2} \Gamma(1 - \frac{d}{2}) &= (-2) \Gamma(2 - \frac{d}{2}), \\
\left(\frac{1}{\Delta}\right)^{1-\frac{d}{2}} &= \left(\frac{1}{\Delta}\right)^{2-\frac{d}{2}} \Delta, \\
l^\mu l^\nu &\rightarrow \frac{1}{d} l^2 g^{\mu\nu}, \\
\lim_{d \rightarrow 4} \frac{M^{4-d} \Gamma(2 - \frac{d}{2})}{(4\pi)^{d/2} \Delta^{(2-d/2)}} &= \\
&= \lim_{\eta \rightarrow 0} \frac{1}{(4\pi)^2} \left[\frac{2}{\eta} - \ln \frac{\Delta}{M^2} - \gamma + \ln(4\pi) \right].
\end{aligned}$$

12. Renormalization

Conditions:

$$\begin{aligned}
\Gamma_{e,R}^{(2)}(\not{p} = m_R) &= 0, \\
\frac{\partial \Gamma_{e,R}^{(2)}}{\partial \not{p}}(\not{p} = m_R) &= 1, \\
\Gamma_{\gamma,R}^{(2)}(q^2 = 0) &= 0, \\
\frac{\partial \Gamma_{\gamma,R}^{(2)}}{\partial q^2}(q^2 = 0) &= 1, \\
\Gamma_R^{(3),\mu}(q = 0) &= -ie_R \gamma^\mu,
\end{aligned}$$

For the electron:

$$\begin{aligned}
\text{Prop}_e(p) &= \text{propagator} + 1 \text{ particle irr.} + \dots = \\
&= \frac{i}{\not{p} - m} \sum \left(\frac{-i\Sigma_2}{\not{p} - m} \right)^n = \frac{i}{\not{p} - m - \Sigma_2} = \\
&= \frac{iZ_2}{\not{p} - m_R} = \frac{i}{\Gamma_{e,R}^{(2)}}, \\
Z_2 &= \frac{1}{1 - \left. \frac{\partial \Sigma_2}{\partial \not{p}} \right|_{\not{p}=m_R}}.
\end{aligned}$$

For the photon:

$$\begin{aligned}
\text{Prop}_\gamma(q) &= \text{propagator} + 1 \text{ particle irr.} + \dots = \\
&= \frac{-ig_{\mu\nu}}{q^2(1 - \Pi(q^2))} = \\
&= \frac{-ig_{\mu\nu}}{q^2(1 - (\Pi_2(q^2) - \Pi_2(0)))} Z_2 = \\
&= \frac{ig_{\mu\nu}}{\Gamma_{\gamma,R}^{(2)}(q^2)} >, \\
Z_2 &= \frac{1}{1 - \Pi_2(0)}.
\end{aligned}$$

For the vertex:

$$\begin{aligned}
&\bar{u}(p') \Gamma^{(3),\mu}(q), \\
&\Gamma^{(3),\mu}(q) = \\
&= -ie \left[\gamma^\mu (1 + \delta F_1(q^2) + i \frac{\sigma^{\mu\nu} q_\nu}{2m} F_2(q^2)) \right].
\end{aligned}$$

13. With Counterterms

Conditions:

$$\begin{aligned}\Gamma_{e,R}^{(2)}(p) &= Z_2 \Gamma_e^{(2)}(p) \rightarrow \Gamma_R = Z_2^{-1/2} \Gamma, \\ \Gamma_{\gamma,R}^{(2)}(q) &= Z_3 \Gamma_\gamma^{(2)}(q) \rightarrow A_R^\mu = Z_3^{-1/2} A^\mu, \\ \Gamma_R^{(3),\mu}(q=0) &= Z_2 \sqrt{Z_3} \gamma^\mu (1 + \delta F_1(0)) = \\ &= -ie \gamma^\mu \rightarrow e_R = \sqrt{Z_3} e,\end{aligned}$$

Counterterms:

$$\begin{aligned}Z_2 &= 1 + \delta_2, \\ Z_3 &= 1 + \delta_2, \\ Z_1 &= 1 + \delta_1, \\ Z_2 m &= m_R + \delta_m, \\ e &= e_R M^{\varepsilon/2},\end{aligned}$$

Photon Propagator:

$$\begin{aligned}&-i(g^{\mu\nu} q^2 - q^\mu q^\nu) \delta_3, \\ \text{propagator} &= \frac{-ig_{\mu\nu}}{q^2(1 - \Pi(q^2) + \delta_3)} = \\ &= \frac{-ig_{\mu\nu}}{\Gamma_{\gamma,R}^{(2)}(q)}, \\ \delta_3 &= \Pi_2(0).\end{aligned}$$

Electron propagator:

$$\begin{aligned}&i(\not{p}\delta_2 - \delta_m), \\ \text{propagator} &= \frac{i}{\not{p} - m_R - \Sigma_2(p, m) + (\not{p}\delta_2 - \delta_m)} \\ &= \frac{i}{\Gamma_{e,R}^{(2)}(p)}.\end{aligned}$$

Vertex:

$$\begin{aligned}&-ie_R M^{\varepsilon/2} \gamma^\mu \delta_1, \\ \text{vertex} &\Rightarrow \delta_1 = -\delta F_1(0).\end{aligned}$$

14. Bonus

Pauli Matrices:

$$\begin{aligned}\sigma_1 &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \\ \sigma_2 &= \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \\ \sigma_3 &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},\end{aligned}$$

Gamma Matrices in Dirac Represen-

tation:

$$\begin{aligned}\gamma^0 &= \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix}, \\ \gamma^i &= \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \\ \gamma^5 &= \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix},\end{aligned}$$

Gamma Matrices in Weyl/Chiral Representation:

$$\begin{aligned}\gamma^0 &= \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}, \\ \gamma^i &= \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \\ \gamma^5 &= \begin{pmatrix} -\mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix},\end{aligned}$$

Gauge Transformation:

$$A_\mu \rightarrow A_\mu - \frac{1}{e} \partial_\mu \alpha.$$

Levi-Civita Tensor:

$$\begin{aligned}&+1 \text{ if } ijk = 123, 231, 312, \\ &-1 \text{ if } ijk = 321, 213, 132, \\ &0 \text{ if any index is repeated.}\end{aligned}$$

15. Lorentz Transformations

Properties:

$$\begin{aligned}\Lambda_{\nu\alpha} \Lambda^\nu{}_\beta &= g_{\alpha\beta}, \\ \Lambda_\nu{}^\alpha &= (\Lambda^{-1})^\nu{}_\alpha,\end{aligned}$$

Scalars:

$$\phi(x) \rightarrow \phi(\Lambda^{-1}x),$$

Spinor:

$$\psi(x) \rightarrow M_{1/2} \psi(\Lambda^{-1}x),$$

Vector:

$$A^\mu(x) \rightarrow M^\mu{}_{\nu(1)} A^\nu(\Lambda^{-1}x),$$

Representation:

$$M_{(s)} = \exp\left(\sum_k -i\theta^k J_{(s)}^k\right),$$

$$M(\Lambda) = \exp\left[-\frac{i}{2} \omega_{\mu\nu} J_{(s)}^{\mu\nu}\right],$$

$$[J_{(s)}^l, J_{(s)}^m] = i\varepsilon^{lmk} J_{(s)}^k,$$

$$S_{1/2}^{0i} = \frac{i}{4} [\gamma^0, \gamma^i] = -\frac{i}{2} \begin{bmatrix} \sigma^i & 0 \\ 0 & -\sigma^i \end{bmatrix},$$

$$S_{1/2}^{ij} = \frac{i}{4} [\gamma^i, \gamma^j] = \frac{1}{2} \epsilon^{ijk} \begin{bmatrix} \sigma^k & 0 \\ 0 & \sigma^k \end{bmatrix}.$$

For Fermions:

Boost in z:

$$M_{1/2}^\beta = \cosh(\omega_{03}) \mathbb{1} - \sinh(\omega_{03}) \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix},$$

$$\omega_{03} = \operatorname{arctanh} \beta,$$

Rotation in z:

$$M_{1/2}^{12} = \cos\left(\frac{\theta}{2}\right) \mathbb{1} - i \sin\left(\frac{\theta}{2}\right) \begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix},$$

For Vectors:

Boost in z:

$$M_{(1)}^3 = \begin{pmatrix} \cosh(\omega_{03}) & 0 & 0 & \sinh(\omega_{03}) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \sinh(\omega_{03}) & 0 & 0 & \cosh(\omega_{03}) \end{pmatrix} =$$

$$= \begin{pmatrix} \gamma & 0 & 0 & \gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \gamma\beta & 0 & 0 & \gamma \end{pmatrix},$$

$$\omega_{03} = \operatorname{arctanh} \beta,$$

Rotation in z:

$$M_{(1)}^2 =$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta & 0 \\ 0 & \sin \theta & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

STOP DOING PHYSICS

- PARTICLES WERE NOT SUPPOSED TO BE LOOKED AT
- YEARS OF BONKERS THEORIES yet NO REAL-WORLD USE FOUND for looking beyond the FIRST ORDER DERIVATIVES
- Wanted to build a weirder model for a laugh? We had a tool for that: It was called "EXPERIMENTAL CONJECTURES"
- "Nice throw Jimmy, the ball really went $H(t) \psi(t) = i\hbar \frac{d}{dt} \psi(t)$! I'm feeling very $H = \sum_{n=1}^{\infty} \frac{p_n^2}{2m_n} + V(x_1, x_2, \dots, x_N)$ today" - Statements dreamed up by the utterly Deranged

LOOK at what Physicists have been demanding your Respect for all this time, with all the labs & computer programs we built for them
(This is REAL Physics, done by REAL Physicists):

