1. Reminder of Notation

$$g_{\mu\nu} = g^{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)\,,$$
$$x_\mu = g_{\mu\nu}x^\nu \qquad x^\mu = g^{\mu\nu}x_\nu\,,$$
$$\partial_\mu = \frac{\partial}{\partial x^\mu} = \left(\frac{\partial}{\partial x^0},\nabla\right)\,,$$
$$\partial^\mu = \frac{\partial}{\partial x_\mu} = \left(\frac{\partial}{\partial x_0},-\nabla\right)\,.$$
$$\frac{\partial}{\partial a_\mu}(a_\nu b^\nu) = \delta^\mu_\nu b^\nu = b^\mu\,,$$
$$\not{p} \equiv \gamma^0 p^0 - \gamma^i p^i\,.$$

2. Basic Principles

$$\mathcal{H} = \Pi\dot{\phi} - \mathcal{L} \qquad \Pi = \frac{\partial\mathcal{L}}{\partial\dot{\phi}}\,,$$
$$S = \int_{t_1}^{t_2}\mathrm{d}t\int \mathrm{d}\boldsymbol{x}\,\mathcal{L}\,,$$

E-L eq.: $$\frac{\partial\mathcal{L}}{\partial\phi_a} - \partial_\mu\left[\frac{\partial\mathcal{L}}{\partial\partial_\mu\phi_a}\right] = 0\,.$$

Causality: $$[\phi_a(\boldsymbol{x},t),\Pi_b(\boldsymbol{y},t)] = i\delta^3(\boldsymbol{x}-\boldsymbol{y})\,,$$
$$(\boldsymbol{x}-\boldsymbol{y})^2 < 0\,,$$

3. Cannonical Quantization

$$[\hat{\phi}(\boldsymbol{x},t),\hat{\Pi}(\boldsymbol{y},t)] = i\delta^3(\boldsymbol{x}-\boldsymbol{y})\,,$$
$$[\hat{\phi}(\boldsymbol{x},t),\hat{\phi}(\boldsymbol{y},t)] = 0\,,$$
$$[\hat{\Pi}(\boldsymbol{x},t),\hat{\Pi}(\boldsymbol{y},t)] = 0\,.$$

4. Noether's Theorem

Invariance under a continuous transformation of the form: $\phi_a \to \phi_a + \alpha\Delta\phi_a$ where $\mathcal{L} \to \mathcal{L} + \alpha\partial_\mu J^\mu$ gives rise to conserved current and charge:

$$j^\mu = \sum_a \frac{\partial\mathcal{L}}{\partial\partial_\mu\phi_a}\Delta\phi_a - J^\mu\,,$$
$$Q = \int j^0(\boldsymbol{x},t)\,\mathrm{d}\boldsymbol{x}\,,$$

for which it holds that:

(i) $\partial_\mu j^\mu(\boldsymbol{x},t) = 0$

(ii) $\dfrac{\mathrm{d}}{\mathrm{d}t}Q = 0$

5. Scalar Fields

$$\partial_\mu\partial^\mu\phi(\boldsymbol{x},t) + m^2\phi(\boldsymbol{x},t) = 0\,,$$
$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \frac{m^2}{2}\phi^2\,,$$
$$T^\mu{}_\nu = \frac{\partial\mathcal{L}}{\partial\partial_\mu\phi}\partial_\nu\phi - \delta^\mu{}_\nu\mathcal{L}\,,$$
$$\hat{\phi}(\boldsymbol{x},t) = \int\frac{\mathrm{d}\boldsymbol{p}}{(2\pi)^3}A_{\boldsymbol{p}}\left(a_{\boldsymbol{p}}e^{-ipx} + a^\dagger_{\boldsymbol{p}}e^{ipx}\right)\,,$$
$$[a_{\boldsymbol{p}},a^\dagger_{\boldsymbol{p}'}] = (2\pi)^3\delta^3(\boldsymbol{p}-\boldsymbol{p}')\,,$$
$$[a_{\boldsymbol{p}},a_{\boldsymbol{p}'}] = 0\,,\quad [a^\dagger_{\boldsymbol{p}},a^\dagger_{\boldsymbol{p}'}] = 0\,,$$
$$\hat{\boldsymbol{p}} = \int\frac{\mathrm{d}\boldsymbol{p}}{(2\pi)^3}\boldsymbol{p}\,a^\dagger_{\boldsymbol{p}}a_{\boldsymbol{p}}\,,$$
$$\hat{H} = \int\frac{\mathrm{d}\boldsymbol{p}}{(2\pi)^3}E_{\boldsymbol{p}}\frac{1}{2}\left[a^\dagger_{\boldsymbol{p}}a_{\boldsymbol{p}} + \frac{1}{2}(2\pi)^3\delta(0)\right]\,,$$
$$\Delta_F(x-y) = \int\frac{\mathrm{d}^4p}{(2\pi)^4}\frac{ie^{ip\cdot(x-y)}}{p^2-m^2+i\varepsilon}\,.$$

6. Dirac Spinor Fields (Fermions)

$$\left(i\gamma^\mu\frac{\partial}{\partial x^\mu} - m\,\mathbb{1}\right)\psi(x) = 0\,,$$
$$\mathcal{L}(x) = \bar{\psi}_a\,(i\gamma^\mu\partial_\mu - m)_{ab}\,\psi_b(x)\,,$$
$$\bar{\psi} \equiv \psi^\dagger\gamma^0\,,$$
$$\psi(x) = \int\frac{\mathrm{d}^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_{\boldsymbol{p}}}}\sum_s\big(a^s_{\boldsymbol{p}}u^s(p)e^{-ip\cdot x}$$
$$+b^{s\dagger}_{\boldsymbol{p}}v^s(p)e^{ip\cdot x}\big)\,,$$
$$\bar{\psi}(x) = \int\frac{\mathrm{d}^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_{\boldsymbol{p}}}}\sum_s\big(b^s_{\boldsymbol{p}}\bar{v}^s(p)e^{-ip\cdot x}$$
$$+a^s_{\boldsymbol{p}}\bar{u}^s(p)e^{ip\cdot x}\big)\,,$$
$$\{\psi_a(\boldsymbol{x},t),\,\psi^\dagger_b(\boldsymbol{y},t)\} = \delta_{ab}\delta(\boldsymbol{x}-\boldsymbol{y})\,,$$
$$\{\psi_a(\boldsymbol{x},t),\,\psi_b(\boldsymbol{y},t)\} = 0\,,$$
$$\{\psi^\dagger_a(\boldsymbol{x},t),\,\psi^\dagger_b(\boldsymbol{y},t)\} = 0\,,$$
$$\{a^r_{\boldsymbol{p}},\,a^{s\dagger}_{\boldsymbol{q}}\} = \{b^r_{\boldsymbol{p}},\,b^{s\dagger}_{\boldsymbol{q}}\} = (2\pi)^3\delta^{rs}\delta(\boldsymbol{p}-\boldsymbol{q})\,,$$
$$u^s(\boldsymbol{p}) = M_{\boldsymbol{p}}u^s(0) = \sqrt{\frac{E+m}{m}}\begin{pmatrix}\xi^s\\ \frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{E+m}\xi^s\end{pmatrix}\,;$$
$$\xi^1 = \begin{pmatrix}1\\0\end{pmatrix}\,,\ \xi^2 = \begin{pmatrix}0\\1\end{pmatrix}\,,$$
$$v^s(\boldsymbol{p}) = M_{\boldsymbol{p}}v^s(0) = \sqrt{\frac{E+m}{m}}\begin{pmatrix}\frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{E+m}\eta^s\\ \eta^s\end{pmatrix}\,;$$
$$\eta^1 = \begin{pmatrix}1\\0\end{pmatrix}\,,\ \eta^2 = \begin{pmatrix}0\\1\end{pmatrix}\,,$$
$$\bar{u}^r(p)u^s(p) = 2m\delta^{rs}\,,$$
$$\bar{v}^r(p)v^s(p) = -2m\delta^{rs}\,,$$

$$u^{r\dagger}(p)u^s(p) = 2E_{\boldsymbol{p}}\delta^{rs}\,,$$
$$v^{r\dagger}(p)v^s(p) = +2E_{\boldsymbol{p}}\delta^{rs}\,,$$
$$\overline{u}^r(p)v^s(p) = \overline{v}^r(p)u^s(p) = 0\,,$$
$$u^{r\dagger}(\boldsymbol{p})v^s(-\boldsymbol{p}) = v^{r\dagger}(-\boldsymbol{p})u^s(\boldsymbol{p}) = 0\,,$$
$$\hat{H} = \int \frac{\mathrm{d}\boldsymbol{p}}{(2\pi)^3} E_{\boldsymbol{p}} \sum_s \left(a^{s\dagger}_{\boldsymbol{p}} a^s_{\boldsymbol{p}} + b^{s\dagger}_{\boldsymbol{p}} b^s_{\boldsymbol{p}}\right)\,,$$
$$\hat{\boldsymbol{p}} = \int \frac{\mathrm{d}\boldsymbol{p}}{(2\pi)^3} \boldsymbol{p} \sum_s \left(a^{s\dagger}_{\boldsymbol{p}} a^s{}_{\boldsymbol{p}} + b^{s\dagger}_{\boldsymbol{p}} b^s{}_{\boldsymbol{p}}\right)\,,$$
$$\hat{Q}_{\text{el.}} = e \int \frac{\mathrm{d}\boldsymbol{p}}{(2\pi)^3} \sum_s (a^{s\dagger}_{\boldsymbol{p}} a^s{}_{\boldsymbol{p}} - b^s{}_{\boldsymbol{p}} b^{s\dagger}_{\boldsymbol{p}})\,,$$
$$\hat{\boldsymbol{J}} = \hat{\boldsymbol{S}} + \hat{\boldsymbol{L}} =$$
$$= \int \mathrm{d}^3x \hat{\psi}^\dagger \left[\frac{1}{2}\Sigma + (\boldsymbol{x} \times (-i\nabla))\right]_z \hat{\psi}\,,$$
$$\Sigma^i = \frac{1}{2}\begin{bmatrix}\sigma^i & 0\\ 0 & \sigma^i\end{bmatrix}\,,$$
$$\Delta_F(x-y) \equiv \langle 0|\mathcal{T}\{\psi(x)\overline{\psi}(y)\}|0\rangle =$$
$$= \int \frac{\mathrm{d}^4p}{(2\pi)^4} \frac{i(\not{p}+m)}{p^2 - m^2 + i\varepsilon} e^{-ip\cdot(x-y)}\,,$$
$$\left(a_{\boldsymbol{p}} e^{-ip\cdot x} + a_{\boldsymbol{p}}{}^\dagger e^{ip\cdot x}\right) \xrightarrow{t=0} \left(a_{\boldsymbol{p}} + a_{\boldsymbol{p}}{}^\dagger\right) e^{ip\cdot x}\,,$$
$$|p\rangle = \sqrt{2E_{\boldsymbol{p}}} a^\dagger_{\boldsymbol{p}} |0\rangle\,,$$

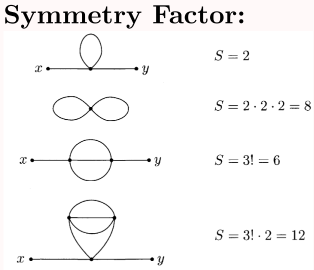
Majorana Fermions: $b^s_{\boldsymbol{p}} = a^s_{\boldsymbol{p}}$.

7. Complex Scalar Fields

$$\mathcal{L} = \partial_\mu\phi\partial^\mu\phi^\dagger - m^2\phi\phi^\dagger\,,$$
$$\hat{\phi}(\boldsymbol{x},t) = \int \frac{\mathrm{d}\boldsymbol{p}}{(2\pi)^3} \frac{1}{\sqrt{2E_{\boldsymbol{p}}}} \left(a_{\boldsymbol{p}} e^{-ip\cdot x} + b^\dagger_{\boldsymbol{p}} e^{ip\cdot x}\right)\,,$$
$$\hat{\phi}(\boldsymbol{x},t) \neq \hat{\phi}^\dagger(\boldsymbol{x},t)\,,$$
$$\left[a_{\boldsymbol{p}},\, a^\dagger_{\boldsymbol{q}}\right] = (2\pi)^3\delta(\boldsymbol{p}-\boldsymbol{q})\,,$$
$$\left[\hat{\phi}(\boldsymbol{x},t),\, \hat{\Pi}(\boldsymbol{y},t)\right] = i\delta(\boldsymbol{x}-\boldsymbol{y})\,.$$

8. Interactions

- **Scalar-Scalar:**
$$\mathcal{L}_I = -\frac{\lambda}{4!}\phi^4\,,$$
- **Fermion-Vector:**
$$\mathcal{L}_I = -A^\mu \sum_f e_f \overline{\psi}_f \gamma_\mu \psi_f\,,$$
- **Fermion-Scalar (Yukawa):**
$$\mathcal{L}_I = -g\phi\overline{\psi}\psi\,,$$
- **Fermion-Pseudoscalar:**
$$\mathcal{L}_I = -g\overline{\psi}i\gamma^5\phi\psi\,,$$
- **Scalar-Vector:**
$$\mathcal{L}_1 = \tilde{g}V_\mu\phi\partial^\mu\phi^\dagger\,,$$
$$\mathcal{L}_2 = \tilde{g}V_\mu\phi V^{\mu\dagger}\phi^\dagger\,,$$
- **Do remember that:**
$$\mathcal{H}_I = -\mathcal{L}_I\,,$$
- **Correlation Functions:**
$$\langle\Omega|\mathcal{T}\{\phi_1(x_1)\phi_2(x_2)\ldots\}|\Omega\rangle =$$
$$\lim_{T\to\infty(1-i\varepsilon)} \frac{\langle 0|\mathcal{T}\left\{\phi_I(x_1)\ldots\phi_I(x_n)\mathcal{X}\right\}0\rangle}{\langle 0|\mathcal{T}\left\{\exp\left[-i\int_{-T}^{T}\int \mathrm{d}^4z\mathcal{H}_{\text{int}}\right]\right\}|0\rangle}\,,$$
$$\mathcal{X} = \exp\left[-i\int_{-T}^{T}\int \mathrm{d}^4z\mathcal{H}_{\text{int}}(z)\right]\,,$$
- **Symmetry Factor:**



- **Probability Amplitude:**
$$\hat{S} = \mathbb{1} + i\hat{T}\,,$$
$$iT_{fi} = (2\pi)^4\delta^4(p_1 + p_2 - k_A - k_B)iM_{fi}\,,$$
$$\mathrm{d}\sigma = \mathrm{d}\Pi|\bar{M}|^2 \frac{1}{2E_A 2E_B|v_A - v_B|} \text{ v is velocity}\,,$$
$$\mathrm{d}\Pi = \frac{p_1}{16\pi^2 E_{\text{CMS}}}\mathrm{d}\Omega\,,$$

Nonpolarized scattering:

$$|\bar{M}|^2 = \frac{1}{N_s N_r}\sum_{s,r}\sum_{s',r'}|M(s,r\to s',r')|^2\,,$$

Fermions:

$$\sum s = 1, 2u^s(\boldsymbol{p})\overline{u}^s(\boldsymbol{p}) = \not{p} + m\,,$$
$$\sum s = 1, 2v^s(\boldsymbol{p})\overline{v}^s(\boldsymbol{p}) = \not{p} - m\,,$$

Photons:

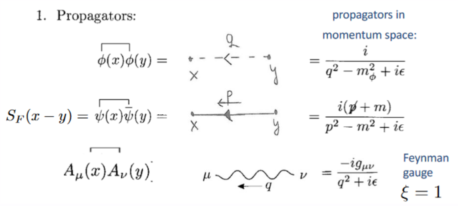
$$\sum_{r=1,2}\varepsilon^{r\,\dagger=*}_\mu\varepsilon^r_\nu = -g_{\mu\nu}$$

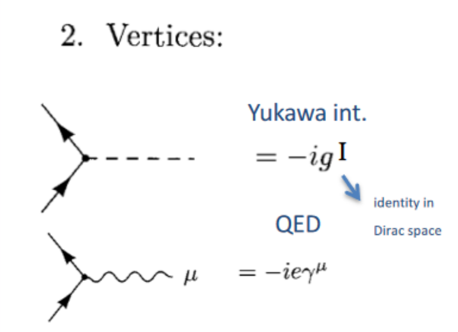
9. Feynman Rules

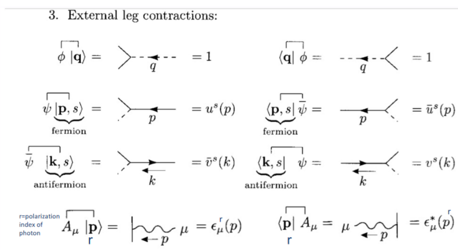
When transforming from momentum to position space, remember:

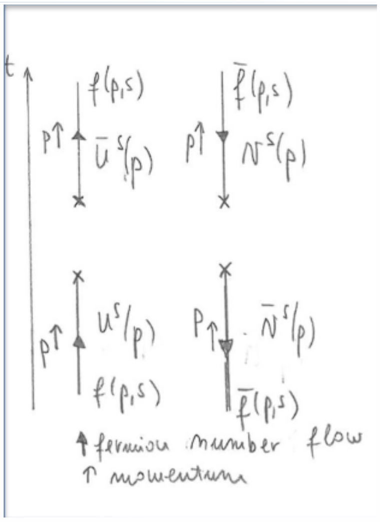
$$\int d^4x \exp\left[-ix\left(\sum p_i\right)\right] = (2\pi)^4\delta^4\left(\sum p_i\right)$$

Rules in momentum space:









$$\langle\phi|\partial_\mu\phi = (-ip_\mu)e^{-ipx}$$

$$\partial_\mu\phi|\phi\rangle = (ip_\mu)e^{ipx}$$

Some additional relations:

$$\begin{aligned}
\mathrm{tr}\,\mathbb{1} &= 4\,,\\
\mathrm{tr}\,(\text{odd number of }\gamma) &= 0\,,\\
\mathrm{tr}\,\gamma^\mu\gamma^\nu &= 4g^{\mu\nu}\,,\\
\mathrm{tr}\,(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) &= 4(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho})\,,\\
\mathrm{tr}\,(\gamma^5) &= 0\,,\\
\mathrm{tr}\,(\gamma^\mu\gamma^\nu\gamma^5) &= 0\,,\\
\mathrm{tr}\,(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^5) &= -4i\varepsilon^{\mu\nu\rho\sigma}\,,\\
\varepsilon^{\alpha\beta\gamma\delta}\varepsilon_{\alpha\beta\gamma\delta} &= -24\,,\\
\varepsilon^{\alpha\beta\gamma\nu}\varepsilon_{\alpha\beta\gamma\delta} &= -6\delta^\nu{}_\delta\,,\\
\varepsilon^{\alpha\beta\gamma\mu\nu}\varepsilon_{\alpha\beta\gamma\rho\sigma} &= -2\left(\delta^\mu{}_\rho\delta^\nu{}_\sigma - \delta^\mu{}_\sigma\delta^\nu{}_\rho\right)\,,\\
\gamma^\mu\gamma^\nu\gamma_\mu &= -2\gamma^\nu\,,\\
\gamma^\mu\gamma^\nu\gamma^\rho\gamma_\mu &= 4g^{\nu\rho}\,,\\
\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_\mu &= -2\gamma^\sigma\gamma^\rho\gamma^\nu\,,\\
\gamma^{\mu\dagger} &= \gamma^0\gamma^\mu\gamma^0\,,\\
\mathrm{tr}\,(\gamma^\mu\not{a}\gamma^\mu\not{b}) &= 4(a^\mu b^\nu - a^\nu b^\mu - g^{\mu\nu}a\cdot b)\,,
\end{aligned}$$

10. Quantum Electrodynamics

$$\hat{A}_\mu(x) = \int \frac{d\boldsymbol{k}}{(2\pi)^3}\frac{1}{\sqrt{2E_{\boldsymbol{k}}}}\sum_r^3\left(\varepsilon^r_\mu(\boldsymbol{k})e^{-i\boldsymbol{k}\cdot x}a^{\boldsymbol{k}} + \right.$$

$$\left. + \varepsilon^{r*}_\mu(\boldsymbol{k})e^{i\boldsymbol{k}\cdot x}a^\dagger_{\boldsymbol{k}}\right)\,,$$

$$\Delta_{\mu\nu}(x-y) = \int\frac{d^4k}{(2\pi)^4}\frac{ig_{\mu\nu}}{k^2 + u\varepsilon}e^{-ik\cdot(x-y)}\,,$$

Ward Identity:

$$k_\mu M^\mu = 0\,,$$

Polarizations:

$$\begin{aligned}
\varepsilon^{r=0=s}_\mu &= (1,0,0,0)\,,\\
\varepsilon^{r=1=T}_\mu &= (0,1,0,0)\,,\\
\varepsilon^{r=2=T}_\mu &= (0,0,1,0)\,,\\
\varepsilon^{r=3=L}_\mu &= (0,0,0,1)\,,
\end{aligned}$$

11. Regularization

Feynmann parametrization:

$$\frac{1}{AB} = \int_0^1 \mathrm{d}x \frac{1}{[x(A-B)+B]^2} \,,$$
$$\frac{1}{A_1 \ldots A_n} = \int_0^1 \frac{\mathrm{d}x_1 \ldots \mathrm{d}x_n \delta(\sum_{x_i} -1)(n-1)!}{[x_1 A_1 + x_2 A_2 + \ldots + x_n A_n]^n} \,,$$
$$A = (k+q)^2 - m^2 + i\varepsilon \,, B = k^2 - m^2 + i\varepsilon \,,$$
$$\Rightarrow x(A-B) + B = l^2 - \Delta + i\varepsilon =$$
$$= (k^2 + xq)^2 - x^2 q^2 + xq^2 - m^2 + i\varepsilon \,,$$
$$A = (p-k)^2 + i\varepsilon \,, B = k^2 - m^2 + i\varepsilon \,,$$
$$\Rightarrow x(A-B) + B = l^2 - \Delta + i\varepsilon =$$
$$= (k - px)^2 - p^2 x^2 + xp^2 - m^2 + xm^2 + i\varepsilon \,.$$

Wick Rotation:

$$l^0 = i l_E^0 \,,$$
$$\int \frac{\mathrm{d}^d l_E}{(2\pi)^d} \frac{1}{(l_E^2 + \Delta)^n} =$$
$$= \frac{1}{(4\pi)^{(d/2)}} \frac{\Gamma_E(n - \frac{d}{2})}{\Gamma_E(n)} \left(\frac{1}{\Delta}\right)^{n-d/2} \,,$$
$$\int \frac{\mathrm{d}^d l_E}{(2\pi)^d} \frac{l_E^2}{(l_E^2 + \Delta)^n} =$$
$$= \frac{1}{(4\pi)^{(d/2)}} \frac{d}{2} \frac{\Gamma(n - \frac{d}{2} - 1)}{\Gamma(n)} \left(\frac{1}{\Delta}\right)^{n-\frac{d}{2}-1} \,,$$
$$\frac{d}{2}\Gamma(1 - \frac{d}{2}) = (-2)\Gamma(2 - \frac{d}{2}) \,,$$
$$\left(\frac{1}{\Delta}\right)^{1-\frac{d}{2}} = \left(\frac{1}{\Delta}\right)^{2-\frac{d}{2}} \Delta \,,$$
$$l^\mu l^\nu \to \frac{1}{d} l^2 g^{\mu\nu} \,,$$
$$\lim_{d\to 4} \frac{M^{4-d}\Gamma(2 - \frac{d}{2})}{(4\pi)^{d/2}\Delta^{(2-d/2)}} =$$
$$= \lim_{\eta \to 0} \frac{1}{(4\pi)^2} \left[\frac{2}{\eta} - \ln\frac{\Delta}{M^2} - \gamma + \ln(4\pi)\right] \,.$$

12. Renormalization

Conditions:

$$\Gamma^{(2)}_{e,R}(\not{p} = m_R) = 0 \,,$$
$$\frac{\partial \Gamma^{(2)}_{e,R}}{\partial \not{p}}(\not{p} = m_R) = 1 \,,$$
$$\Gamma^{(2)}_{\gamma,R}(q^2 = 0) = 0 \,,$$
$$\frac{\partial \Gamma^{(2)}_{\gamma,R}}{\partial q^2}(q^2 = 0) = 1 \,,$$
$$\Gamma^{(3),\mu}_R(q = 0) = -i e_R \gamma^\mu \,,$$

For the electron:

$$\mathrm{Prop}_e(p) = \text{propagator} + 1 \text{ particle irr.} + \cdots =$$
$$= \frac{i}{\not{p} - m} \sum (\frac{-i\Sigma_2}{\not{p} - m})^n = \frac{i}{\not{p} - m - \Sigma_2} =$$
$$= \frac{iZ_2}{\not{p} - m_R} = \frac{i}{\Gamma^{(2)}_{e,R}} \,,$$
$$Z_2 = \frac{1}{1 - \frac{\partial \Sigma_2}{\partial \not{p}}\Big|_{\not{p} = m_R}} \,.$$

For the photon:

$$\mathrm{Prop}_\gamma(q) = \text{propagator} + 1 \text{ particle irr.} + \cdots =$$
$$= \frac{-i g_{\mu\nu}}{q^2(1 - \Pi(q^2))} =$$
$$= \frac{-i g_{\mu\nu}}{q^2(1 - (\Pi_2(q^2) - \Pi_2(0)))} Z_2 =$$
$$= \frac{i g_\mu \nu}{\Gamma^{(2)}_{\gamma,R}(q^2)} >,$$
$$Z_2 = \frac{1}{1 - \Pi_2(0)} \,.$$

For the vertex:

$$\bar{u}(p')\Gamma^{(3),\mu}(q) \,,$$
$$\Gamma^{(3),\mu}(q) =$$
$$= -ie\left[\gamma^\mu(1 + \delta F_1(q^2) + i\frac{\sigma^{\mu\nu} q_\nu}{2m} F_2 q^2)\right] \,.$$

13. With Counterterms

Conditions:

$$\Gamma^{(2)}_{e,R}(p) = Z_2 \Gamma^{(2)}_e(p) \to \Gamma_R = Z_2^{-1/2}\Gamma\,,$$
$$\Gamma^{(2)}_{\gamma,R}(q) = Z_3 \Gamma^{(2)}_\gamma(q) \to A^\mu_R = Z_3^{-1/2}A^\mu\,,$$
$$\Gamma^{(3),\mu}_R(q=0) = Z_2\sqrt{Z_3}\gamma^\mu(1+\delta F_1(0)) =$$
$$= -ie\gamma^\mu \to e_R = \sqrt{Z_3}e\,,$$

Counterterms:

$$\begin{aligned} Z_2 &= 1+\delta_2\,,\\ Z_3 &= 1+\delta_2\,,\\ Z_1 &= 1+\delta_1\,,\\ Z_2 m &= m_R+\delta_m\,,\\ e &= e_R M^{\varepsilon/2}\,, \end{aligned}$$

Photon Propagator:

$$-i(g^{\mu\nu}q^2 - q^\mu q^\nu)\delta_3\,,$$
$$\text{propagator} = \frac{-ig_{\mu\nu}}{q^2(1-\Pi(q^2)+\delta_3)} =$$
$$= \frac{-ig_{\mu\nu}}{\Gamma^{(2)}_{\gamma,R}(q)}\,,$$
$$\delta_3 = \Pi_2(0)\,.$$

Electron propagator:

$$i(\not p\delta_2 - \delta_m)\,,$$
$$\text{propagator} = \frac{i}{\not p - m_R - \Sigma_2(p,m) + (\not p\delta_2 - \delta_m)}$$
$$= \frac{i}{\Gamma^{(2)}_{e,R}(p)}\,.$$

Vertex:

$$-ie_R M^{\varepsilon/2}\gamma^\mu\delta_1\,,$$
$$\text{vertex} \Rightarrow \delta_1 = -\delta F_1(0)\,.$$

14. Bonus

Pauli Matrices:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\,,$$
$$\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\,,$$
$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\,,$$

Gamma Matrices in Dirac Representation:

$$\gamma^0 = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix}\,,$$
$$\gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}\,,$$
$$\gamma^5 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}\,,$$

Gamma Matrices in Weyl/Chiral Representation:

$$\gamma^0 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}\,,$$
$$\gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}\,,$$
$$\gamma^5 = \begin{pmatrix} -\mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix}\,,$$

Gauge Transformation:

$$A_\mu \to A_\mu - \frac{1}{e}\partial_\mu\alpha\,.$$

Levi-Civita Tensor:

$$\begin{aligned} &+1 \text{ if } ijk = 123, 231, 312\,,\\ &-1 \text{ if } ijk = 321, 213, 132\,,\\ &0 \text{ if any index is repeated}\,. \end{aligned}$$

15. Lorentz Transformations

Properties:

$$\Lambda_{\nu\alpha}\Lambda^\nu{}_\beta = g_{\alpha\beta}\,,$$
$$\Lambda_\nu{}^\alpha = (\Lambda^{-1})^\nu{}_\alpha\,,$$

Scalars:

$$\phi(x) \to \phi(\Lambda^{-1}x)\,,$$

Spinor:

$$\psi(x) \to M_{1/2}\psi(\Lambda^{-1}x)\,,$$

Vector:

$$A^\mu(x) \to M^\mu{}_{\nu(1)}A^\mu(\Lambda^{-1}x)\,,$$

Representation:

$$M_{(s)} = \exp(\sum_k -i\theta^k J^k_{(s)})\,,$$
$$M(\Lambda) = \exp\left[-\frac{i}{2}\,w_{\mu\nu}J^{\mu\nu}_{(s)}\right]\,,$$

$$[J^l_{(s)},\, J^m_{(s)}] = i\varepsilon^{lmk} J^k_{(s)}\,,$$

$$S^{0i}_{1/2} = \frac{i}{4}[\gamma^0,\, \gamma^i] = -\frac{i}{2}\begin{bmatrix}\sigma^i & 0\\ 0 & -\sigma^i\end{bmatrix},$$
$$S^{ij}_{1/2} = \frac{i}{4}[\gamma^i,\, \gamma^j] = \frac{1}{2}\epsilon^{ijk}\begin{bmatrix}\sigma^k & 0\\ 0 & \sigma^k\end{bmatrix}.$$

For Fermions:

Boost in z:

$$M^{\beta}_{1/2} = \cosh(\omega_{03})\mathbb{1} - \sinh(\omega_{03})\begin{pmatrix}\sigma_3 & 0\\ 0 & -\sigma_3\end{pmatrix},$$

$$\omega_{03} = \operatorname{arctanh}\beta\,,$$

Rotation in z:

$$M^{12}_{1/2} = \cos(\frac{\theta}{2})\mathbb{1} - i\sin(\frac{\theta}{2})\begin{pmatrix}\sigma_3 & 0\\ 0 & \sigma_3\end{pmatrix},$$

For Vectors:

Boost in z:

$$M^3_{(1)} = \begin{pmatrix}\cosh(\omega_{03}) & 0 & 0 & \sinh(\omega_{03})\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ \sinh(\omega_{03}) & 0 & 0 & \cosh(\omega_{03})\end{pmatrix} =$$
$$= \begin{pmatrix}\gamma & 0 & 0 & \gamma\beta\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ \gamma\beta & 0 & 0 & \gamma\end{pmatrix},$$

$$\omega_{03} = \operatorname{arctanh}\beta\,,$$

Rotation in z:

$$M^2_{(1)} =$$
$$\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & \cos\theta & -\sin\theta & 0\\ 0 & \sin\theta & \cos\theta & 0\\ 0 & 0 & 0 & 1\end{pmatrix}$$