

Polja

• Klasična polja:

• Skalarna polja: $\varphi(\vec{x}, t)$, $\psi(\vec{x}, t)$

• Vektorska polja: $\vec{v}(\vec{x}, t)$, $\vec{B}(\vec{x}, t)$, $\vec{E}(\vec{x}, t)$

$u(\vec{x}, t)$

↑ polje odmikov (npr. pri zvočnem valovanju)

Predvsem razlika med njimi je ~~razlika~~ lahko se vedejo do rotacijah.

• Polja pri QFT:

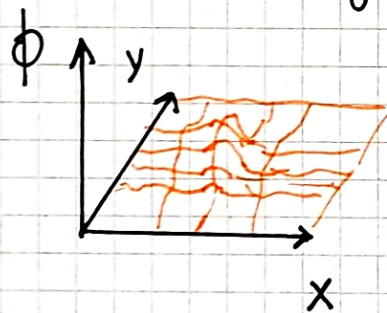
Polja bodo skalarna $\phi(\vec{x}, t)$; Vektorska $A^\mu(\vec{x}, t)$
 $V^\mu(\vec{x}, t)$; Spinorska $\psi(\vec{x}, t)$

$S=0$

$S=1$

$S=\frac{1}{2}$

Delci so ekscitacije teh polj.



V QFT so polja operatorji $\hat{\phi}(\vec{x}, t)$. Meri vrednost polja v $\vec{x}$ ob t .

Reminder: Števila vs. operatorji

$$AB=BA \quad \hat{A}\hat{B} \neq \hat{B}\hat{A}$$

V QM opišemo delec kot $a^\dagger|0\rangle$, v QFT pa je ista reprezentacija $\hat{\phi}^\dagger(\vec{x}, t)|0\rangle$.

Zakaj QFT?

- Osnovna teorija elementarnih delcev
 $\hookrightarrow$ Opisuje EM + močno + šibko interakcijo med njimi
- Osnovna teorija Narave (ker uspešno združuje QM + posebno teo. rel.)
- Skoraj enolično določa možne interakcije
 (za razliko od enodelčne QM, ki da popolno svobodo z izbiro potenciala)

Q. electrodynamics: dodamo še $U(1)_{EM}$ simetrijo

Standard model: dodamo še $SU(3)_c \times SU(2)_H \times U(1)_y$ simetrijo

Zakaj polja, ne delci (zakaj ne relativistična kvantna mehanika)?

1.) Enodelčna QM: $e^+e^- \rightarrow \gamma \rightarrow \mu^+\mu^-$

Lokalizirajmo delec na tako skalo, da zaradi nebločnosti letijo ven pari delec-antidelec:



$$\Delta p \approx \frac{\hbar}{L}$$

$$\Delta E \approx \Delta p c = \frac{\hbar c}{L} \approx 2mc^2$$

da odstranimo par

$$E \approx \sqrt{m^2 c^4 + p^2 c^2} \approx pc$$

$$L = \frac{\hbar c}{2mc^2} \Rightarrow \lambda_{comp} = \frac{h}{mc}$$

$$\lambda_{comp}^{el} \approx 0,02 \text{ \AA}$$

Torej odpade.

2.) Drugi razlog je **Kavzalnost**

$$\vec{0}, 0$$

$$\vec{x}, t$$

$$|\vec{x}| > ct$$

$$x^2 = (ct)^2 - \vec{x}^2 < 0$$

Meritve opazljivk ne smejo biti vzročno povezane. **Relativistična QM urši kavzalnost.** Poglejmo kar za QM:

$$U \sim \langle \vec{x}_t | e^{-i\hat{H}t} | \vec{x}_0 \rangle$$

$t \quad \uparrow \quad t=0$
 $\vec{x}_t \quad \text{II} \quad \vec{x}_0$

$$II = \int d\vec{p} |\vec{p}\rangle \langle \vec{p}|$$

imamo prosti delec.

$$\hat{H} = \frac{\hat{\vec{p}}^2}{2m}$$

$\rightarrow$

$$II \propto \int d\vec{p} e^{-i\frac{p^2}{2m}t} \langle \vec{x} | \vec{p} \rangle \langle \vec{p} | \vec{x}_0 \rangle$$

$$\propto \int d\vec{p} e^{-i\frac{p^2}{2m}t} e^{i\vec{p}(\vec{x}-\vec{x}_0)}$$

$$II_x \propto \int_{-\infty}^{\infty} dp_x e^{-i\frac{p_x^2}{2m}t + i p_x(x-x_0)} \propto \frac{1}{\sqrt{t}} e^{\frac{im(x-x_0)^2}{2t}}$$

↑
Uvedeš
popolni
kvadrat

$$\Rightarrow U \propto \frac{1}{t^{3/2}} e^{im\frac{(\vec{x}-\vec{x}_0)^2}{2t}}, \quad |U| \neq 0 \text{ za vsako kombinacijo } \vec{x}, \vec{x}_0, t$$

in to krši kausalnost.

Lega je opazljiva v QM in meritve so
Vzročno povezane.

Poglejmo še za relativistično QM:

$$U \propto \langle \vec{x} | e^{-i\hat{H}t} | \vec{x}_0 \rangle \propto \int d\vec{p} e^{-i\sqrt{p^2+m^2}\frac{t}{\hbar}} e^{i\vec{p}(\vec{x}-\vec{x}_0)} \propto$$

$$e^{-i\hat{H}t} | \vec{p} \rangle = e^{-i\sqrt{m^2+p^2}t} | \vec{p} \rangle$$

$$c=1 \quad \hbar=1$$

$$\propto \int_{-1}^1 d\cos\theta \int_0^{\infty} p^2 dp e^{-i\sqrt{p^2+m^2}t} e^{ip|\vec{x}-\vec{x}_0|\cos\theta} \propto$$

$$\propto \int_0^{\infty} \frac{e^{ip|\vec{x}-\vec{x}_0|} - e^{-ip|\vec{x}-\vec{x}_0|}}{p|\vec{x}-\vec{x}_0|} e^{-i\sqrt{p^2+m^2}t} p^2 dp$$

Za vsah $\vec{x}, \vec{x}_0, t$: $|U| > 0$ tudi za $|\vec{x}| > ct$ torej
relativistična QM. Krši kausalnost.

Klasická mechanika

Argument: t

Posplošeni koordinati: $x_i(t)$ oz. $q_i(t)$

$$L(q_i, \dot{q}_i)$$

Konjugirani moment:

$$p_i(t) = \frac{dL}{dq_i}$$

Kvantna mehanika

$\hat{x}_i(t), \hat{p}_i(t)$ promerjano v operatorja

$$[\hat{x}_i(t), \hat{p}_j(t)] = i\hbar \delta_{ij}$$

$$[\hat{x}_i, \hat{x}_j] = 0$$

$$[\hat{p}_i, \hat{p}_j] = 0$$

Klasická teorija polja

a) Diskretno

Argumenti: $t, \vec{x}_j$

Posplošene koordinati: $\phi_a(t, \vec{x}_j)$ za vsak $\vec{x}_j$ druga posplošena koordinata

$$\mathcal{L}(\phi_a(t, \vec{x}_j), \partial_\mu \phi_a(t, \vec{x}_j));$$

Konjugirani moment:

$$\pi_a(\vec{x}_j, t) = \frac{\partial \mathcal{L}}{\partial \dot{\phi}_a(\vec{x}_j, t)}$$

b) Kontinuum

$t, \vec{x}_j \rightarrow \phi_a(t, \vec{x}) \propto$ posp. coord.

$$\mathcal{L}(\phi_a(\vec{x}, t), \partial_\mu \phi_a(\vec{x}, t))$$

$$\pi_a(\vec{x}, t) = \frac{\partial \mathcal{L}}{\partial \dot{\phi}_a(\vec{x}, t)}$$

Kvantna teorija polja

a)

$\hat{\phi}_a(\vec{x}_j, t), \hat{\pi}_a(\vec{x}_j, t)$: promerjani v operatorje

$$[\hat{\phi}_a(\vec{x}_j, t), \hat{\pi}_b(\vec{x}_k, t)] = i\hbar \delta_{ab} \delta_{jk}$$

$$[\hat{\phi}_a(\vec{x}_j, t), \hat{\phi}_b(\vec{x}_k, t)] = 0$$

$$[\hat{\pi}_a(\vec{x}_j, t), \hat{\pi}_b(\vec{x}_k, t)] = 0$$

Recept
za QFT
Ta opis se
slada z
vsem kar
hočemo

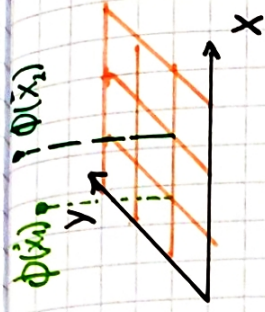
$$\hat{\phi}(\vec{x}, t), \hat{\pi}(\vec{x}, t)$$

$$[\hat{\phi}_a(\vec{x}, t), \hat{\pi}_b(\vec{y}, t)] = i\hbar \delta_{ab} \delta(\vec{x} - \vec{y})$$

$$[\hat{\phi}_a(\vec{x}, t), \hat{\phi}_b(\vec{y}, t)] = 0$$

$$[\hat{\pi}_a(\vec{x}, t), \hat{\pi}_b(\vec{y}, t)] = 0$$

(Kanonická kvantizacija)



$$\partial_\mu = \frac{\partial}{\partial x^\mu}$$

3 slike

1.) Schrodingerjeva

$$|n(t)\rangle = e^{-i\hat{H}t} |n(0)\rangle$$

$\hat{O}$ so neodvisni od časa

$$\underbrace{\hat{X}, \hat{P}}_{QM} ; \rightarrow \hat{\Phi}(\vec{x}), \hat{\Pi}(\vec{x})$$

2.) Heisenbergova

$|n\rangle$ neodvisne od časa

$$\hat{O}(t) = e^{i\hat{H}t} \hat{O}(0) e^{-i\hat{H}t}$$

$$\hookrightarrow \hat{\Phi}(\vec{x}, t), \hat{\Pi}(\vec{x}, t)$$

3.) Interakcijska (Diracova)

Notacije pri nas:

$$g^{\mu\nu} = (1, -1, -1, -1)$$

$$x^\mu = (x^0, x^i)$$

$$x_\mu = g_{\mu\nu} x^\nu = (x^0, -x^i)$$

$$\frac{\partial}{\partial a_\mu} (a_\nu b^\nu) = b^\mu \rightarrow \partial^\mu = \frac{\partial}{\partial x_\mu}$$

$$\partial^\mu = \frac{\partial}{\partial x_\mu} = \left(\frac{\partial}{\partial x^0}, -\nabla \right)$$

Osnove klasične teorije polja

$$\mathcal{L} = \mathcal{L}(\phi, \partial_\mu \phi)$$

Enačbe gibanja: Euler-Lagrange

$$S = \int_{t_1}^{t_2} dt \int d\vec{x} \mathcal{L} ; \quad \phi \rightarrow \phi + \delta\phi \quad \left. \begin{array}{l} \delta\phi|_{t_1, t_2} = 0 \\ \delta\phi|_{|\vec{x}| \rightarrow \infty} = 0 \end{array} \right\} : \delta S = 0$$

$$\delta S = 0 = \int dt \int d\vec{x} \left[\frac{\partial \mathcal{L}}{\partial \phi} \delta\phi + \frac{\partial \mathcal{L}}{\partial \partial_\mu \phi} \delta \partial_\mu \phi \right]$$

$$\partial_\mu \left[\frac{\partial \mathcal{L}}{\partial \partial_\mu \phi} \delta\phi \right] = \partial_\mu \left[\frac{\partial \mathcal{L}}{\partial \partial_\mu \phi} \right] \delta\phi + \frac{\partial \mathcal{L}}{\partial \partial_\mu \phi} \partial_\mu \delta\phi$$

$$\Rightarrow \delta S = 0 = \int dt \int d\vec{x} \left[\frac{\partial \mathcal{L}}{\partial \phi} \delta\phi - \partial_\mu \left[\frac{\partial \mathcal{L}}{\partial \partial_\mu \phi} \right] \delta\phi + \underbrace{\partial_\mu \left[\frac{\partial \mathcal{L}}{\partial \partial_\mu \phi} \delta\phi \right]}_{=0} \right]$$

Torej za poljuben $\delta\phi$ bo $\delta S = 0$ če bo:

$$\boxed{\frac{\partial \mathcal{L}}{\partial \phi_a} - \partial_\mu \left[\frac{\partial \mathcal{L}}{\partial \partial_\mu \phi_a} \right] = 0}$$

Euler-Lagrangeove enačbe !

Ta člen se prevede na integral po robu ampak definirali smo, da je $\delta\phi = 0$ na robu.

Recimo če je polje elektromagnetno polje so EL Maxwellove enačbe.

Poglejmo si H (hamiltonian) in Π (konjugiran moment). Trenutno smo v klasični sliki.

Klasična
Mehanika : $p_i = \frac{\partial L}{\partial \dot{q}_i}$

$$H = \sum p_i \dot{q}_i - L$$

klasina
Teorija
Polja

$$L = \int d\vec{x} \mathcal{L} = \sum_j \Delta \vec{x} \mathcal{L}(\vec{x}_j) ; \mathcal{L} = \mathcal{L}(\phi(\vec{x}_j), \partial_\mu \phi(\vec{x}_j))$$

$$\rho(\vec{x}_i) = \frac{\partial L}{\partial \dot{\phi}(\vec{x}_i)} = \frac{\partial \sum_j \Delta \vec{x} \mathcal{L}(\phi(\vec{x}_j))}{\partial \dot{\phi}(\vec{x}_i)} = \Delta \vec{x} \frac{\partial \mathcal{L}}{\partial \dot{\phi}(\vec{x}_i)} \equiv \pi(\vec{x}_i)$$

$$H = \sum_i \Delta \vec{x} \pi(\vec{x}_i) \dot{\phi}(\vec{x}_i) - \sum_i \Delta \vec{x} \mathcal{L} = \sum_i \Delta \vec{x} (\pi(\vec{x}_i) \dot{\phi}(\vec{x}_i) - \mathcal{L}(\vec{x}_i))$$

$H(\vec{x}_i)$

Hamiltonova gostota

Realna skalarna polja

Ekscitacije tega polja predstavljajo ^{električno nevtralne} skalarne delce.

$$\text{Realno} \rightarrow \phi(\vec{x}, t) = \phi^*(\vec{x}, t)$$

Zapišimo enačbe gibanja. Želimo, da rešitve zadoščajo relativistični kinematiki in Lorentzovi invarianci.

$$\hookrightarrow E^2 = \vec{p}^2 + m^2$$

Lorentzova invarianca:

Če ϕ zadošča en. gib. potem tudi $\phi \xrightarrow[\text{trans.}]{\text{Lor.}} \phi'$ zadošča en. gib.

"Kupimo", da je ta enačba zadostna za te pogoje:

$$\underline{\partial_\mu \partial^\mu \phi(x, t) + m^2 \phi(x, t) = 0} \quad \text{Klein-Gordonova enačba}$$

Lahko pa sicer preverimo prvi pogoj. Rešitve so oblike: Ae^{ipx}

$$A(\pm ip)^2 e^{\pm ipx} + Am^2 e^{\pm ipx} = 0$$

$$-p^2 + m^2 = 0$$

$$\Rightarrow p = (E, \vec{p})$$

$$-E^2 + \vec{p}^2 + m^2 = 0$$

$$p^2 = p_\mu p^\mu \quad \checkmark$$

Lagrangovo gostoto ugotovimo in preverimo če dobimo K-G en.:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{m^2}{2}\phi^2$$

To damo v $E-L$, da preverimo: $\frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial \partial_\mu \phi} = 0$

$$\Rightarrow -m^2\phi - \partial_\mu(\partial^\mu \phi) = 0 \quad \text{To pa je K-G.}$$

Torej je Lagrangian:

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{m^2}{2}\phi^2 \\ &= \frac{1}{2}(\dot{\phi}^2 - (\nabla\phi)^2 - m^2\phi^2)\end{aligned}$$

Lagrangian
(brez interakcij)

$$\pi = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} \rightarrow \pi = \dot{\phi}$$

Poglejmo še hamiltonian:

$$H = \pi\dot{\phi} - \mathcal{L} = \dot{\phi}^2 - \frac{1}{2}(\dot{\phi}^2 - (\nabla\phi)^2 - m^2\phi^2)$$

[Vaja: Polje odmiklov v 1D kristalu z zvočnim valom]

$$I-1 \quad I \quad I+1$$

sonarbo sonarbo

$$\phi_{I-1} \quad \phi_I \quad \phi_{I+1}$$

o

1

$$0 = \begin{pmatrix} 0 & 0 \end{pmatrix} \frac{1}{2} - 0 = 0$$

2

$$\begin{aligned} \left(\frac{1}{2} - \left(\frac{1}{2} \right) \left(\frac{1}{2} \right) \right) \frac{1}{2} &= 0 \\ \left(\frac{1}{2} - \left(\frac{1}{2} \right) \right) \frac{1}{2} &= 0 \end{aligned}$$

3

$$0 = \frac{1}{2} - \frac{1}{2} = 0$$

1

$$\left(\frac{1}{2} - \left(\frac{1}{2} \right) \right) \frac{1}{2} = 0 = \frac{1}{2} - \frac{1}{2} = 0$$

Zveze med simetrijami in ohranitvenimi zakoni (Noether-jin teorem 1915)

Teorem:

Če so enačbe gibanja invariantne na $\phi_a \rightarrow \phi_a + \alpha \Delta \phi_a$ in če se ob tem $\mathcal{L} \rightarrow \mathcal{L} + \alpha \partial_\mu J^\mu$ potem obstajajo ohranjene:

$$i) \partial_\mu j^\mu(\vec{x}, t) = 0$$

$$j^\mu = \sum_a \frac{\partial \mathcal{L}}{\partial \partial_\mu \phi_a} \Delta \phi_a - J^\mu j^\mu \dots \text{ohranjen tok}$$

$$ii) \frac{d}{dt} Q = 0$$

$$Q = \int_{\text{cel prostor}} j^0(\vec{x}, t) d\vec{x} \quad Q \dots \text{ohranjen naboj}$$

• Transformacije so zvezne (torej ne inverzija prostora/naboja).

• $\alpha = \text{konst.} \Rightarrow \alpha \neq \alpha(x)$ torej so to globalne transformacije

• $\Delta \phi_a|_{\text{rob}}$ ni nujno 0 (ker transformiramo celem prostoru enako)

$$t \rightarrow t + a^0 : \frac{dH}{dt} = 0$$

$$x^i \rightarrow x^i + a^i : \frac{dp^i}{dt} = 0$$

Dokaz: ϕ_a zadošča E-L enačbam

$$\text{EL invariantne} \rightarrow \Delta S = 0 \rightarrow \begin{cases} \Delta \mathcal{L} = 0 \\ \Delta \mathcal{L} = \alpha \partial_\mu J^\mu \end{cases}$$

$\mathcal{L}(\phi_a, \partial_\mu \phi_a) \rightarrow$ klasično še (niso še operatorji)

$$\Delta \mathcal{L} = \alpha \partial_\mu j^\mu = \underbrace{\left(\sum_a \frac{\partial \mathcal{L}}{\partial \phi_a} \Delta \phi_a + \frac{\partial \mathcal{L}}{\partial \partial_\mu \phi_a} \partial_\mu \Delta \phi_a \right)}_{\text{sprememba v prvem redu}} \alpha$$
$$\text{①} \quad \partial_\mu \left[\frac{\partial \mathcal{L}}{\partial \partial_\mu \phi_a} \Delta \phi_a \right] - \partial_\mu \left[\frac{\partial \mathcal{L}}{\partial \partial_\mu \phi_a} \right] \Delta \phi_a \quad \text{②}$$

① + ② = 0, ker ϕ_a zadošča E-L enačbam

Ostane nam:

$$1 \quad \partial_\mu \left(\underbrace{\sum_a \frac{\partial \mathcal{L}}{\partial \partial_\mu \phi_a} \Delta \phi_a}_{j^\mu(\vec{x}, t)} - \mathcal{J}^\mu \right) = 0$$

$$j^\mu(\vec{x}, t) \Rightarrow \partial^\mu j_\mu = 0 = \frac{d j^0}{dt} + \nabla \cdot \vec{j}$$

$$\frac{d j^0}{dt} = -\nabla \cdot \vec{j}(\vec{x}, t)$$

$j \dots$ tdi Noetherjinega naboja.

2 Dokažimo že ti)

$$\int \frac{d j^0(\vec{x}, t)}{dt} d\vec{x} = - \int \nabla \cdot \vec{j} d\vec{x}$$

cel prostor



$$2 \quad \frac{d}{dt} \int j^0(\vec{x}, t) d\vec{x} = - \vec{j} \Big|_{\text{rob}} = 0 \quad ; \quad \vec{j} \Big|_{|\vec{x}| \rightarrow \infty} = 0$$

1

[Še nekaj primerov]

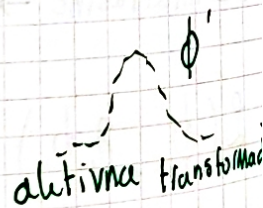
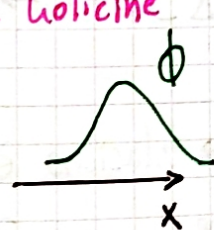
$$\vec{x} \rightarrow R\vec{x} : \frac{d \vec{j}}{dt} = 0$$

$$\phi \rightarrow e^{i\alpha} \phi : \frac{d Q_{\text{el}}}{dt} = 0$$

Primer: Izpeljava tenzorja energije in gibalne količine

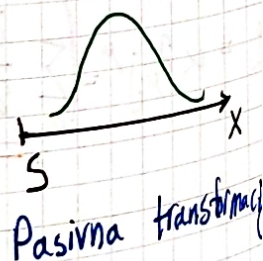
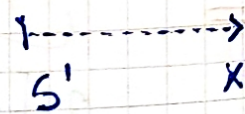
→ Za poljuben $\mathcal{L}(\phi, \partial_\mu \phi)$

Transformacije naj bodo oblike $x^\mu \rightarrow x^\mu + a^\mu$



$$S = \int \mathcal{L}(x) d^4x$$

$$\Rightarrow \int \mathcal{L}(x+a) d^4x = \int \mathcal{L}(x') dx' = S$$



$$\Delta \phi = ?$$

$$\phi(x) \rightarrow \phi(x+a) = \phi(x) + \underbrace{a^\nu}_{\alpha} \underbrace{\partial_\nu \phi}_{\Delta \mathcal{L}}$$

$$J^\mu = ? \quad \mathcal{L}(x) \rightarrow \mathcal{L}(x+a) = \mathcal{L}(x) + a^\nu \partial_\nu \mathcal{L}$$

$$\underbrace{\partial_\mu (\delta_\nu^\mu \mathcal{L})}_{J^\mu_\nu}$$

$$\underline{J^\mu_\nu = \frac{\partial \mathcal{L}}{\partial \partial_\mu \phi} \partial_\nu \phi - \delta_\nu^\mu \mathcal{L} = T^\mu_\nu}$$

$$\partial_\mu T^\mu_\nu(\vec{x}, t) = 0$$

Tenzor energije in gibalne količine

$$\nu=0: \quad Q = \int j^0_0 d\vec{x} = \int \left(\frac{\partial \mathcal{L}}{\partial \partial_0 \phi} \partial_0 \phi - \mathcal{L} \right) d\vec{x} =$$

$$= \int \underbrace{\left(\underbrace{\Pi(\vec{x}) \dot{\phi}(\vec{x})}_{H(\vec{x})} - \mathcal{L} \right) d\vec{x}}_{\text{Energija } E} \quad \frac{dE}{dt} = 0$$

$$\nu=i: \quad Q = \int j^0_i dx = \int \underbrace{\Pi(\vec{x}) \partial_i \phi(\vec{x})}_{\text{gostota gib. kol.}} d\vec{x} \quad \frac{dp^i}{dt} = 0$$

p^i ... gibalna količina, ki jo nosi polje

DN: $\vec{x} \rightarrow R\vec{x}$ $Q = ?$

$x \rightarrow x + \theta y$

$y \rightarrow y - \theta x$

$$J_z = \int d\vec{x} \Pi(\vec{x}) [\vec{x} \times (-i\nabla)]_z \phi(\vec{x})$$

[Vaja: $\mathcal{L} = \partial_\mu \phi^\dagger \partial^\mu \phi - \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 \phi = |\frac{\phi_1}{\phi_2}|^2]$

a) $\mathcal{L}$ = invariant na $\phi \rightarrow \phi + i\alpha^a \partial^a \phi$ $\alpha^1 = \alpha, \alpha^{2,3} = 0$

$\phi \rightarrow \phi + i\alpha \partial^1 \phi$ $\partial^1 = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}$

b) $j^\mu = ?$

$$\begin{aligned}
 a) \partial_\mu \phi^\dagger \partial^\mu \phi &= \partial_\mu (\phi^\dagger - i\alpha \phi^\dagger \partial^1 \phi) \partial^\mu (\phi + i\alpha \partial^1 \phi) \\
 &= \partial_\mu \phi^\dagger \partial^\mu \phi + i\alpha (-\partial_\mu \phi^\dagger) \partial^1 \partial^\mu \phi + \phi^\dagger \partial^1 \partial^\mu \phi + \mathcal{O}(\alpha^2) \\
 &= \partial_\mu \phi^\dagger \partial^\mu \phi \quad \text{torej je invariantno!} \quad \begin{matrix} \Delta \mathcal{L} = 0 \\ \delta^\mu = 0 \end{matrix}
 \end{aligned}$$

Enako se polaze za ostale člene.

$$\begin{aligned}
 b) \Delta \phi_1 &= i \phi_2 & \Delta \phi_1^* &= -i \phi_2^* \\
 \Delta \phi_2 &= i \phi_1 & \Delta \phi_2^* &= -i \phi_1^*
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{L}(\phi_1, \phi_1^*, \partial_\mu \phi_1, \partial_\mu \phi_1^*, \phi_2, \phi_2^*, \partial_\mu \phi_2, \partial_\mu \phi_2^*) &= \partial_\mu (\phi_1^* \phi_2) \partial^\mu (\phi_1) + f(\phi_1) \\
 &= \partial_\mu \phi_1^* \partial^\mu \phi_1 + \partial_\mu \phi_2^* \partial^\mu \phi_2 \\
 j^\mu &= \sum_{i=1,2} \frac{\partial \mathcal{L}}{\partial \partial_\mu \phi_i} \Delta \phi_i + \sum_{i=1,2} \frac{\partial \mathcal{L}}{\partial \partial_\mu \phi_i^*} \Delta \phi_i^* \\
 &= i(\partial^\mu \phi_1^*) \phi_2 - i(\partial^\mu \phi_1) \phi_2^* + (\text{doli zamenjuna indeksa}) = \\
 &= i(\partial^\mu \phi^\dagger \partial^1 \phi - \phi^\dagger \partial^1 \partial^\mu \phi) \quad \partial_\mu j^\mu = 0
 \end{aligned}$$

$$\begin{aligned}
 &= i(\partial^\mu \phi_1^*) \phi_2 - i(\partial^\mu \phi_1) \phi_2^* + (\text{doli zamenjuna indeksa}) = \\
 &= i(\partial^\mu \phi^\dagger \partial^1 \phi - \phi^\dagger \partial^1 \partial^\mu \phi) \quad \partial_\mu j^\mu = 0
 \end{aligned}$$

DN: $\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi$ je invariant na $\phi \rightarrow e^{i\alpha} \phi$

Kvantizacija realnih skalarnih polj $\phi(\vec{x}, t) \rightarrow \hat{\phi}(\vec{x}, t)$

Klasirno polje:

Morajo Zadoštkiti Klein-Gordonovi enačbi $\partial_\mu \phi \partial^\mu \phi + m^2 \phi^2 = 0$

$$\phi = A e^{\pm i p x}, \quad p^2 = m^2, \quad \mathcal{L} = \dot{\phi}^2 - \dots$$

$$\phi = \int \frac{d\vec{p}}{(2\pi)^3} A_{\vec{p}} (e^{-i p x} + e^{i p x}) = \phi^*(\vec{x}, t)$$

To zadošča Klein-Gordonovi enačbi.

$$\pi = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = \dot{\phi} =$$

QFT: $\hat{\phi}(\vec{x}, t), \hat{\pi}(\vec{x}, t)$

Postuliramo $[\hat{\phi}(\vec{x}, t), \hat{\pi}(\vec{y}, t)] = i\hbar \delta(\vec{x} - \vec{y})$ in zahtevamo naj

$\hat{\phi}(\vec{x}, t)$ zadošča klasični ~~en~~ enačbi gibanja (Klein-Gordonova)

Pričakovane m $\langle \hat{\phi}(\vec{x}_0, t) \rangle$ na se obnesejo v skladu s to teorijo.

$$\hat{\phi}(\vec{x}, t) = \int \frac{d\vec{p}}{(2\pi)^3} A_{\vec{p}} (a_{\vec{p}} e^{-i\vec{p}\vec{x}} + a_{\vec{p}}^\dagger e^{i\vec{p}\vec{x}})$$

$$a_{\vec{p}} |0\rangle = 0, [a_{\vec{p}}, a_{\vec{p}'}^\dagger] = (2\pi)^3 \delta(\vec{p} - \vec{p}') \rightarrow e^{-i(E_{\vec{p}}t - \vec{p}\vec{x})}$$

$$\text{KG enačba: } \partial_\mu \partial^\mu \hat{\phi}(\vec{x}, t) + m^2 \hat{\phi}(\vec{x}, t) = \int \frac{d\vec{p}}{(2\pi)^3} A_{\vec{p}} [a_{\vec{p}} \{(-i\vec{p})'(-i\vec{p})_\mu + m^2\} e^{-i\vec{p}\vec{x}}]$$

Hočemo pokazati še, da veljajo komutacijske relacije

$$\hat{\mathcal{L}} = \frac{1}{2} \partial_\mu \hat{\phi} \partial^\mu \hat{\phi} - \frac{m^2}{2} \hat{\phi}^2 = \frac{1}{2} \hat{\pi}^2 + \dots$$

$$\hat{\pi}(\vec{x}, t) = \frac{\partial \hat{\mathcal{L}}}{\partial \dot{\hat{\phi}}} = \dot{\hat{\phi}}(\vec{x}, t) = \int \frac{d\vec{p}}{(2\pi)^3} A_{\vec{p}} (-iE_{\vec{p}}) (a_{\vec{p}} e^{-i\vec{p}\vec{x}} - a_{\vec{p}}^\dagger e^{i\vec{p}\vec{x}})$$

Trdimo:

$$A_{\vec{p}} = \frac{1}{\sqrt{2E_{\vec{p}}}} \leftarrow [\hat{\phi}(\vec{x}, t), \hat{\pi}(\vec{y}, t)] = i\delta(\vec{x} - \vec{y})$$

Preverimo ob $t=0$ (ker mora itak za vsa t veljati):

$$t=0: \hat{\phi}(\vec{x}, 0) = \int \frac{d\vec{p}}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} (a_{\vec{p}} + a_{-\vec{p}}^\dagger) e^{i\vec{p}\vec{x}}$$

$$\hat{\pi}(\vec{y}, 0) = \int \frac{d\vec{p}'}{(2\pi)^3} \frac{(-iE_{\vec{p}'})}{\sqrt{2E_{\vec{p}'}}} (a_{\vec{p}'} - a_{-\vec{p}'}^\dagger) e^{i\vec{p}'\vec{y}}$$

$$[\hat{\phi}, \hat{\pi}] = \int \frac{d\vec{p}}{(2\pi)^3} \int \frac{d\vec{p}'}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} \frac{(-iE_{\vec{p}'})}{\sqrt{2E_{\vec{p}'}}} e^{i(\vec{p}\vec{x} + \vec{p}'\vec{y})} \cdot \underbrace{(-[a_{\vec{p}}, a_{\vec{p}'}^\dagger] + [a_{-\vec{p}}^\dagger, a_{-\vec{p}'}])}_{(2\pi)^3 \delta(\vec{p} + \vec{p}')} =$$

$$= \int \frac{d\vec{p}}{(2\pi)^3} \frac{2E_{\vec{p}} i}{2E_{\vec{p}}} e^{i\vec{p} \cdot (\vec{x} - \vec{y})} = i\delta(\vec{x} - \vec{y})$$

Po istem postopku bi bilo potrebno preveriti še:

$$[\hat{\phi}(\vec{x}, t), \hat{\phi}(\vec{y}, t)] = 0$$

$$[\hat{\pi}(\vec{x}, t), \hat{\pi}(\vec{y}, t)] = 0$$

Energija je kvantizirana,
ker so $A_{\vec{p}}$ točno določeni

Operatorji za opazljivke v QFT

$\hat{A}$: meri pričakovano vrednost opazljivke A

$$\hat{A}|\Psi_n\rangle = A_n|\Psi_n\rangle \quad \leftarrow \text{Lastne funkcije } \hat{A}$$

$$|\Psi\rangle = \sum |c_n| |\Psi_n\rangle$$

$$P(A_n) = |c_n|^2$$

Želimo:

$$\hat{A} = F(\hat{\phi}, \partial_\mu \hat{\phi}); \quad \hat{A} = \hat{A}^\dagger$$

$$\hat{A}: \hat{\phi}(\vec{x}, t), \hat{\pi}(\vec{x}, t), \hat{H}, \hat{P}, \dots \text{ ampak } \underline{\underline{ne}} \hat{X}(t)$$

Ni smisla operator
hrajn za polje, ki se
nahaja povsod

Operator $\hat{\vec{p}}$:

$$X^2 \rightarrow X^2 + \frac{\hbar^2}{4a^2}$$

$$1. \quad \hat{p}^i = \int \hat{\pi}(\vec{x}) \partial^i \hat{\phi}(\vec{x}) d\vec{x} \quad \partial^i \rightarrow -\nabla$$

$$1. \quad \hat{\vec{p}} = \int d\vec{x} \int \frac{d\vec{p}}{(2\pi)^3} \frac{-iE_{\vec{p}}}{\sqrt{2E_{\vec{p}}}} (a_{\vec{p}} - a_{-\vec{p}}^\dagger) \int (-i\vec{p}) \frac{d\vec{p}}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} (a_{\vec{p}} + a_{-\vec{p}}^\dagger)$$

$$1. \quad e^{i\vec{p}' \cdot \vec{x} + i\vec{p} \cdot \vec{x}}$$

od odvoda

$$\int d\vec{x} \int \frac{d\vec{p}'}{(2\pi)^3} e^{i(\vec{p}' + \vec{p}) \cdot \vec{x}} F(\vec{p}, \vec{p}') = F(\vec{p}, -\vec{p}')$$

$$(2\pi)^3 \delta(\vec{p}' + \vec{p}) \Rightarrow \vec{p}' = -\vec{p}$$

$$\hat{\vec{p}} = \int \frac{d\vec{p}}{(2\pi)^3} (-i\vec{p}) \left(-\frac{i}{2} \right) \left(\underbrace{a_{-\vec{p}} a_{\vec{p}}}_{=0} - \underbrace{a_{\vec{p}}^\dagger a_{-\vec{p}}}_{\text{lahko}} - \underbrace{a_{-\vec{p}}^\dagger a_{\vec{p}}}_{=0} + \underbrace{a_{\vec{p}} a_{-\vec{p}}^\dagger}_{\text{obrnemo}} \right)$$

$$\int d\vec{p} \cdot \vec{p} \delta(0) = 0$$

↑
lih integral

$$\hat{\vec{p}} = \int \frac{d\vec{p}}{(2\pi)^3} \vec{p} a_{\vec{p}}^\dagger a_{\vec{p}}$$

Operator gibalne
količine

Lastna stanja?

Poglejmo če so $a_{\vec{p}}^\dagger |0\rangle$ lastna;

$$\hat{\vec{p}} a_{\vec{p}}^\dagger |0\rangle = [\hat{\vec{p}}, a_{\vec{p}}^\dagger] |0\rangle + a_{\vec{p}}^\dagger \hat{\vec{p}} |0\rangle$$

lastna vrednost

$\vec{p} a_{\vec{p}}^\dagger |0\rangle$

0

$$[\hat{\vec{p}}, a_{\vec{p}}^\dagger] = \int \frac{d\vec{p}'}{(2\pi)^3} \vec{p}' [a_{\vec{p}'}^\dagger a_{\vec{p}'}^\dagger, a_{\vec{p}}^\dagger] = \vec{p} a_{\vec{p}}^\dagger$$

$a_{\vec{p}}^\dagger (2\pi)^3 \delta(\vec{p}' - \vec{p})$

Trdimo, da je tudi $a_{\vec{p}}^{\dagger n} |0\rangle$ lastno stanje

$$\hat{\vec{p}} a_{\vec{p}}^{\dagger n} |0\rangle = [a_{\vec{p}}^{\dagger n}, \hat{\vec{p}}] |0\rangle + a_{\vec{p}}^{\dagger n} \hat{\vec{p}} |0\rangle = n \vec{p} a_{\vec{p}}^{\dagger n} |0\rangle$$

↑
n kvantov v isti Fourierov način

→ Bose-Einsteinova statistika: V isti ~~Fourier~~ Fourier. način je lahko naloženih poljubno število eksitacij.

Operator $\hat{H}: \bar{E}_n, |n\rangle$

$$t \rightarrow t + \alpha^0$$

$$\hat{H} = \int \hat{H}(\vec{x}) d\vec{x} = \int [\hat{\pi}^2(\vec{x}) + (\nabla \hat{\phi})^2 + m^2 \hat{\phi}^2] d\vec{x} = \dots =$$

Zabavna
domaća
naloge

$$= \int \frac{d\vec{p}}{(2\pi)^3} E_{\vec{p}} \frac{1}{2} (a_{\vec{p}}^\dagger a_{\vec{p}} + a_{\vec{p}} a_{\vec{p}}^\dagger)$$

$$a_{\vec{p}}^\dagger a_{\vec{p}} + (2\pi)^3 \delta(0)$$

$$\hat{H} = \int \frac{d\vec{p}}{(2\pi)^3} E_{\vec{p}} \left(a_{\vec{p}}^\dagger a_{\vec{p}} + \frac{1}{2} (2\pi)^3 \delta(0) \right)$$

Hamiltonov
operator

Energija vakuma je neskončna!!!

$$\hookrightarrow \hat{H}|0\rangle = E_0|0\rangle \Rightarrow E_0 = \int \frac{d\vec{p}}{(2\pi)^3} E_{\vec{p}} \frac{1}{2} (2\pi)^3 \delta(0) = \infty$$

Poglejmo še za ostale $E - E_0 > 0$:

$$\hat{H} a_{\vec{p}}^\dagger |0\rangle = \underbrace{[a_{\vec{p}}^\dagger, \hat{H}] |0\rangle}_{E_{\vec{p}} a_{\vec{p}}^\dagger |0\rangle} + \underbrace{a_{\vec{p}}^\dagger \hat{H} |0\rangle}_{E_0 |0\rangle}$$

$$\hat{H} a_{\vec{p}}^\dagger |0\rangle = (E_{\vec{p}} + E_0) a_{\vec{p}}^\dagger |0\rangle \Rightarrow E - E_0 = E_{\vec{p}} = \sqrt{\vec{p}^2 + m^2}$$

$a_{\vec{p}}^\dagger |0\rangle$ z $\vec{p}$ in $E_{\vec{p}}$: Te eksitacije polja predstavljajo skalarni delci z gibalno količino

$$a_{\vec{p}_1}^{\dagger n_1} \dots a_{\vec{p}_n}^{\dagger n_n} |0\rangle: \vec{p} = n_1 \vec{p}_1 + \dots + n_n \vec{p}_n$$

$$E - E_0 = n_1 E_1 + \dots + n_n E_n$$

Energija valuvna

$$1) H = \int_{-L/2}^{L/2} d\vec{x} \mathcal{H}$$

$$(2\pi)^3 \delta(0) = \int_{-L/2}^{L/2} d\vec{x} \int d\vec{x}' e^{i(\vec{p}+\vec{p}')\vec{x}} \underbrace{\delta(\vec{p}+\vec{p}')}_{[a_{\vec{p}}, a_{-\vec{p}'}]} = V$$

$$1) \frac{E_0}{V} = \int \frac{d\vec{p}}{(2\pi)^3} \frac{1}{2} E_{\vec{p}} \left. \begin{array}{l} \text{Kot harmonski oscilator} \\ \text{[Pravzaprav kot 1 HO na Fourier na\u010digi]} \end{array} \right\} \frac{1}{2} \hbar \omega = \hbar \omega \left(n + \frac{1}{2} \right)$$

Ocenimo, do katere sega na\u0161a teorija:

$$\frac{E_0}{V} = \int p^2 dp \frac{1}{2} \sqrt{p^2 + m^2} \sim \frac{(1 \text{ TeV})^4}{4} = \frac{(10^{12} \text{ eV})^4}{4}$$

Za neskon\u010dnost, bi lahko bila odgovorna gravitacija, ki \u010duti energijo valuvna.

E:

4% : vidna snov

22% : temna snov

74% : temna energija $\Lambda_{\text{eff}} = (10^{-3} \text{ eV})^4$

Normalizacija stanj

$$|\vec{p}\rangle = B_{\vec{p}}^\dagger a_{\vec{p}}^\dagger |0\rangle$$

\u017delimo, da je $\langle \vec{q} | \vec{p} \rangle = B_{\vec{q}} B_{\vec{p}}^\dagger$.

$$(2\pi)^3 B_{\vec{p}}^2 \delta(\vec{q} - \vec{p})$$

$$\langle \vec{q}, \vec{p} \rangle = B_{\vec{q}}^\dagger B_{\vec{p}} \langle 0 | a_{\vec{q}} - a_{\vec{p}}^\dagger | 0 \rangle = (2\pi)^3 B_{\vec{p}}^2 \delta(\vec{q} - \vec{p})$$

Mal\u010do lep\u0161e napisano:

$$\langle \vec{q}, \vec{p} \rangle = B_{\vec{q}}^\dagger B_{\vec{p}} \langle 0 | a_{\vec{q}} a_{\vec{p}}^\dagger | 0 \rangle = |B_{\vec{p}}|^2 (2\pi)^3 \delta(\vec{p} - \vec{q})$$

Normalizacija iz pogoja $\langle \vec{q} | \vec{p} \rangle = \langle \vec{q}' | \vec{p}' \rangle$

← Skalarni produkt se ohrani za vse Lorentz t.

Potisk v z smeri:

$$p'_3 = \gamma(p_3 + \beta E) \quad p'_{1,2} = p_{1,2}$$

$$\delta(\vec{p}' - \vec{q}') = \delta(\vec{p} - \vec{q}) \cdot \frac{1}{\frac{dp'_3}{dp_3}} = \delta(\vec{p} - \vec{q}) \frac{2E}{2E'}$$

$$\delta(f(x) - f(x_0)) = \frac{\delta(x - x_0)}{\frac{df}{dx}(x_0)}$$

$$\Rightarrow \langle \vec{q} | \vec{p} \rangle = |B_p|^2 (2\pi)^3 \delta(\vec{p} - \vec{q})$$

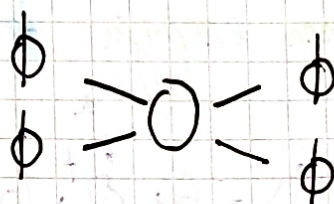
$$\langle \vec{q} | \vec{p}' \rangle = |B_p|^2 (2\pi)^3 \delta(\vec{p} - \vec{q}) \frac{2E}{2E'}$$

$$\frac{|B_{\vec{p}}|^2}{2E} = \frac{|B_{\vec{p}'}|^2}{2E'}; \quad B_{\vec{p}} = \sqrt{2E_{\vec{p}}}$$

$$\Rightarrow |\vec{p}\rangle = \sqrt{2E_{\vec{p}}} a_{\vec{p}}^\dagger |0\rangle$$

Valovni paket

$|\phi\rangle$: probabilistična int.



Pogoj: $\langle \phi | \phi \rangle = 1$

$$|\phi\rangle = \int \frac{d\vec{p}}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} f(\vec{p}) |\vec{p}\rangle$$

Za DN pa lahko dobaz, da je $\langle \phi | \phi \rangle = 1$, $\int \frac{d\vec{p}}{(2\pi)^3} |f(\vec{p})|^2 = 1$

Kompletnostne relacije

VQM:

$$I = \sum_n |n\rangle \langle n|$$

$$I|m\rangle = \sum_n |n\rangle \langle n|m\rangle = |m\rangle \quad \checkmark$$

VQFT:

$$I = \int$$

$$I|\vec{q}\rangle = \int \frac{d\vec{p}}{(2\pi)^3} \frac{1}{2E_{\vec{p}}} |\vec{p}\rangle \underbrace{\langle \vec{p} | \vec{q} \rangle}_{2E_{\vec{p}} \delta(\vec{p}-\vec{q})} = |\vec{q}\rangle$$

Zgled [nazaj k 1D verigi ionov]:

$N_{\text{atomov}} \sim N_A$ s periodičnim robnim pogojem:

$$\tilde{\phi}(x,t) = \sum_k A_k [e^{-i(\omega_k t - kx)} + e^{i(\omega_k t - kx)}]$$

poljubni in $\omega_k = v \cdot k$

↑ hitrost potovanja motnje (dobljeno iz valovne enačbe).

Upoštevamo periodični RP:

$$k \cdot L = j \cdot 2\pi \rightarrow k = j \frac{2\pi}{L}$$

Za določen k , je A_k poljuben $\Rightarrow E \propto |A_k|^2$ zvezno

Sedaj pa to za vajo kvantiziramo.

$$\hat{\phi}(x,t) = \sum_k A_k [a_k e^{-i(\omega_k t - kx)} + a_k^\dagger e^{i(\omega_k t - kx)}]$$

Pričakujemo, da nam to generira kvant $\Rightarrow$ Fonon z dano valovno dolžino k

Normalizacijo bomo določili iz zahtere po določenem komutatorju.

$$\Pi(x) = \frac{\partial \mathcal{L}}{\partial \dot{\tilde{\Phi}}(x)} = m \dot{\tilde{\Phi}}(x) =$$

polje gibalne količine

$$= m \sum_k A_k \left[-i\omega_k a_k e^{-i(\omega_k t - kx)} + i\omega_k a_k^\dagger e^{i(\omega_k t - kx)} \right]$$

$$\hat{\tilde{\Phi}}(x, 0) = \sum_k A_k [a_k + a_{-k}^\dagger] e^{ikx}$$

$$\hat{\Pi}(y, 0) = \sum_{k'} A_{k'} (-i\omega_{k'}) [a_{k'} - a_{-k'}^\dagger] e^{ik'y} m$$

Zahtevamo: $[\hat{\tilde{\Phi}}(x, t), \hat{\Pi}(y, t)] = i\hbar \delta_{xy}$

ob $t=0$

$$[\hat{\tilde{\Phi}}(x, 0), \hat{\Pi}(y, 0)] = \sum_k \sum_{k'} A_k A_{k'} (-i\omega_{k'}) e^{i(kx + k'y)} (-2[a_k, a_{k'}^\dagger]) m$$

$$= \sum_k A_k^2 m(i\omega_k) e^{ik(x-y)} \cdot 2 = \begin{cases} 1 & ; x=y \\ 0 & ; x \neq y \end{cases} = i\delta_{xy}$$

k smo
zanemali
...

$$\downarrow$$

$$\frac{1}{2\omega_k \cdot N \cdot m}$$

$$A_k = \frac{1}{\sqrt{2\omega_k N m}}$$

$\Downarrow$

Zdaj lahko zapišemo:

$$\hat{H} = \sum_x \frac{m}{2} \dot{\tilde{\Phi}}(x)^2 + \frac{\hbar^2 a^2}{2} (\partial_x \tilde{\Phi})^2 = \dots = \sum_k \omega_k \left(\frac{1}{2} + a_k^\dagger a_k \right)$$

V bistvu je to kar LHO!

Fourierovi način z razloženimi k so neodvisni

Poglejmo si lastna stanja:

$$|0\rangle \hat{H} |0\rangle = E_0 |0\rangle : E_0 = \sum_k \frac{1}{2} \omega_k$$

Vsota ničelnih
energij

$$a_h^\dagger |0\rangle : E = \omega_h \left(\frac{1}{2} + 1\right)$$

Kolektivne eksitacije v polju odvihov = **fonon**

$$a_h^{\dagger n} |0\rangle : E = \omega_h \left(\frac{1}{2} + n\right)$$

n kvantov energije v Fourierovem načinu
 $h = n$ **fononov**

Pri določenem načinu h : diskretne energije in Bose-Einsteinova statistika

Dualnost, valovna funkcija

$$\hat{\phi} = \dots$$

$e^{\pm i p x}$: so rešitve gibalne enačbe za polje (Klein-Gordon)

$|p\rangle \propto a_p^\dagger |0\rangle$: eksitacije polj

$$\hat{\phi}^\dagger(x,t) |0\rangle = \int \frac{d\vec{p}}{(2\pi)^3} \frac{1}{2E_p} e^{+i p x} |\vec{p}\rangle \xrightarrow{\text{non-rel. lim.}} \int \frac{d\vec{p}}{(2\pi)^3} \frac{1}{2m} e^{i E t} e^{-i \vec{p} \cdot \vec{x}} |\vec{p}\rangle$$

let povedati, ko nečuj kreiramo

interpretacija: skalarni delec pri $\vec{x}$
(operacije $\hat{x}$ ne uporabljamo v QFT)

$$\Psi_{\vec{q}}(\vec{x}, t) = \langle \vec{x} | \vec{q} \rangle = \langle 0 | \hat{\phi}(x, t) | \vec{q} \rangle =$$

Valovna funkcija je projekcija na stanje z dobro lego

$$= \int \frac{d\vec{p}}{(2\pi)^3} \frac{1}{2E_p} e^{-i p x} \langle \vec{p} | \vec{q} \rangle = e^{-i q x} \underbrace{\int \frac{d\vec{p}}{(2\pi)^3} \frac{1}{2E_p} \delta(\vec{p} - \vec{q})}_{\langle \vec{x} | \vec{q} \rangle} q^0 = E_{\vec{q}}$$

Člen $a_{\vec{q}}$ v $\hat{\phi}$ anihilira začetno stanje $|\vec{q}\rangle$ z valovno funkcijo $e^{-i q x}$, $q_0 > 0$.

$$\Psi_{\vec{q}}^\dagger(\vec{x}, t) = \langle \vec{q} | \vec{x} \rangle = \langle \vec{q} | \hat{\phi}^\dagger(\vec{x}, t) | 0 \rangle = e^{i q x}; \text{ to je complex conj. VF } \Psi_{\vec{q}}(\vec{x}, t) = e^{-i q x}$$

Člen $a_{\vec{q}}^\dagger$ v $\hat{\phi}^\dagger$ kreira končno stanje z valovno funkcijo

$$\Psi_{\vec{q}}(\vec{x}, t) = e^{-i q x} \quad q_0 > 0 \quad | \quad \text{Energije so pozitivne.}$$

Kavzalnost (= vzročna povezanost) in koreliranost

Meritvi $O_1(x)$ in $O_2(y)$ ne smeta vplivati ena na drugo če $(x-y)^2 < 0$. Torej:

$$\hat{O}_1 \hat{O}_2 |\Psi\rangle = \hat{O}_2 \hat{O}_1 |\Psi\rangle$$

$$\hat{O} = f(\hat{\phi}, \hat{\pi}, \dots)$$

$$[\hat{O}_1(x), \hat{O}_2(y)] = 0 \quad \text{za} \quad (x-y)^2 < 0$$

Imajmo dva dogodka kavičnega tipa $\Rightarrow$ obstaja oz. gremo v sistem, kjer je $x_0 = y_0 = t$ na različnih mestih in $|\vec{x} - \vec{y}| > 0$. Tahirat:

$$[\hat{\phi}(\vec{x}, t), \hat{\phi}(\vec{y}, t)] = 0$$

Vseeno $\vec{x} \neq \vec{y}$

$$[\hat{\pi}(x, t), \hat{\pi}(\vec{y}, t)] = 0$$

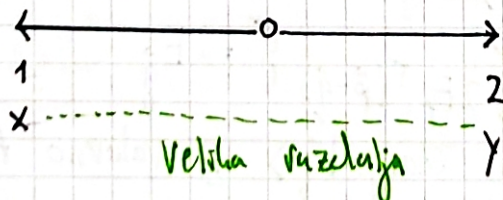
$$[\hat{\phi}(x, t), \hat{\pi}(\vec{y}, t)] = i\delta(\vec{x} - \vec{y}) \stackrel{\downarrow}{=} 0$$

Kavzalnost je trivialno zadoščena za $\hat{\phi} = \hat{\phi}^\dagger$. Pri kompl. skalar poljih in fermionih bo pa tu treba imeti kreator za antidelece, drugače ne bo zadoščena.

Prepletenost

Poglejmo primer iz kvantne mehanike:

$$|\Psi\rangle = (|\uparrow\rangle_1 |\uparrow\rangle_2 + |\downarrow\rangle_1 |\downarrow\rangle_2) \frac{1}{\sqrt{2}} = |s=1, s_z=0\rangle$$

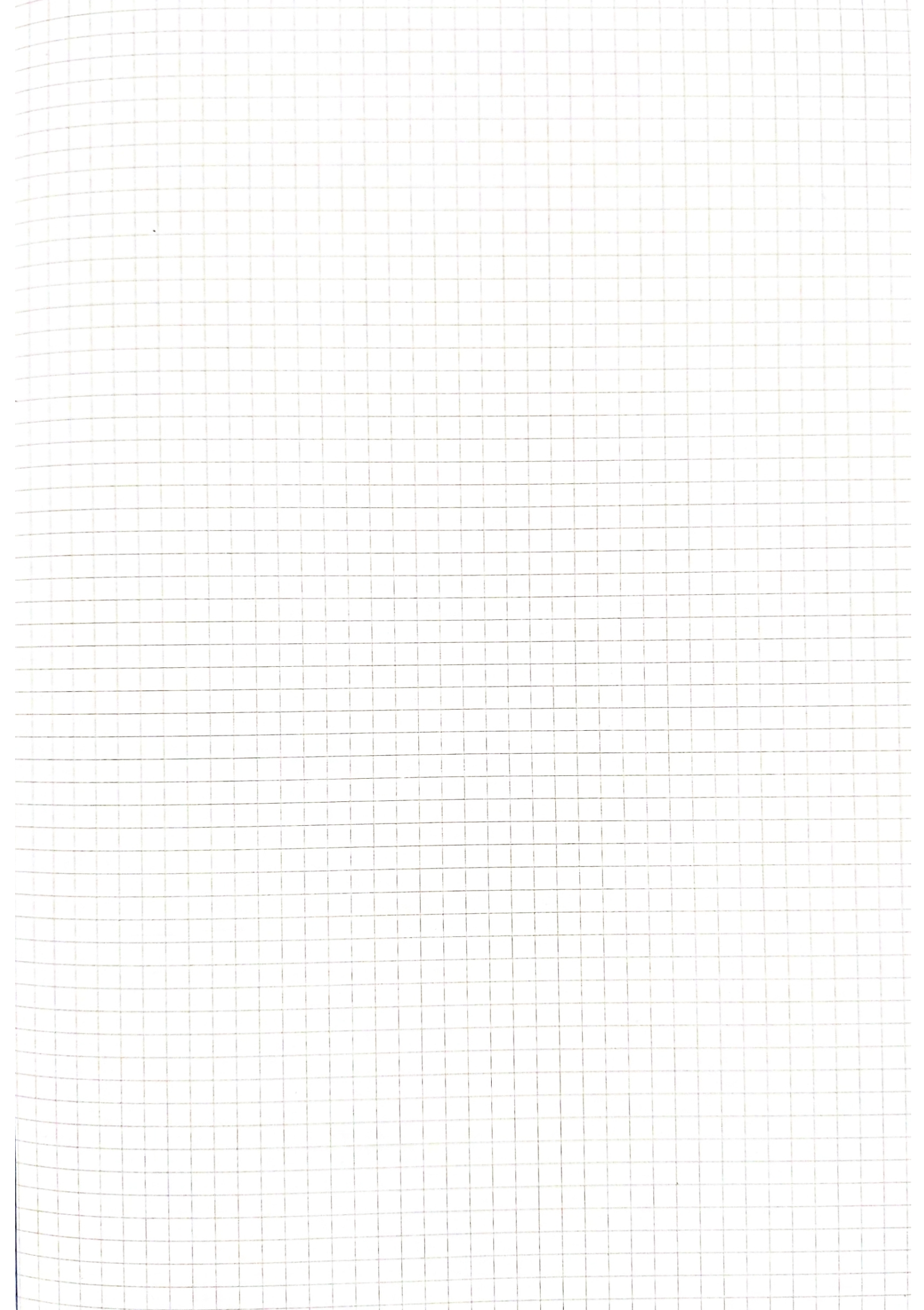


Pomerita spin hkrati

Če preveriš kavzalnost vidimo, da velja. Isto je če prvo pomei 1 in potem 2 ali pa obratno. Zapisano:

$$\hat{S}_{z1} \hat{S}_{z2} |\Psi\rangle = \hat{S}_{z2} \hat{S}_{z1} |\Psi\rangle$$

Meritvi sta korelirani!



$$X' \rightarrow \Lambda X$$

$$\phi' \rightarrow \phi(\Lambda^{-1}x) = \phi(t, x + \theta y, y - \theta x, z)$$

$$= \phi + \frac{\partial \phi}{\partial x} (\theta y) + \frac{\partial \phi}{\partial y} (-\theta x) =$$

$$= [1 - i\theta (\vec{x} \times (-i\nabla))_z] \phi = [1 - i\theta \underbrace{i(xJ^y - yJ^x)}_{\omega_{12} \hat{J}^{12}}] \phi$$

$$\hat{J}^{\mu\nu} = i(x^\mu J^\nu - x^\nu J^\mu) \dots \text{generator rotacij}$$

Podobno je za potislice (boosts)

$$M = I - i\omega_{03} \hat{J}^{03}(s)$$

$$\Lambda = \begin{vmatrix} \gamma & 1 & \beta\gamma & 0 \\ 0 & 1 & 0 & 0 \\ \beta\gamma & 0 & \gamma & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \xrightarrow[\beta]{\text{majhen}} \begin{vmatrix} 1 & 0 & \beta & 0 \\ 0 & 1 & 0 & 0 \\ \beta & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

$$\begin{aligned} \phi'(x) &\rightarrow \phi(\Lambda^{-1}x) = \phi(t - \beta z, x, y, z - \beta t) = \\ &= \phi + \frac{\partial \phi}{\partial t} (-\beta z) + \frac{\partial \phi}{\partial z} (-\beta t) = \\ &= [1 - i\beta i(tJ^z - zJ^t)] \end{aligned}$$

Lorentzove transformacije

$$M = \exp \left[\sum_{\mu\nu} -\frac{i}{2} \omega_{\mu\nu} \hat{J}^{\mu\nu}(s) \right]$$

Želimo, da $\hat{J}^{\mu\nu}_{(s)}$ zadostijo istim komutacijskim relacijam. V 3D je to:

$$[\hat{J}^i, \hat{J}^j] = i\epsilon^{ijk} \hat{J}^k$$

Vaja

[1.] $M = ?$ za potiski vektorja v smeri z

$$(J^{03})^\alpha_\beta = i (g^{0\alpha} g^3_\beta - g^0_\beta g^{3\alpha}) =$$

Vrstica
→ stolpec

$$= i \begin{vmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{vmatrix}$$

$$g^{\mu\nu} = \begin{vmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{vmatrix}$$

$$g^\mu_\nu = \delta_{\mu\nu}$$

MatrixExp[...]

$$M = \exp \left[-\frac{i}{2} (\omega_{03} J^{03} + \omega_{30} J^{30}) \right] = \exp \left[\begin{vmatrix} 0 & 0 & 0 & \omega_{03} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \omega_{03} & 0 & 0 & 0 \end{vmatrix} \right] =$$

ista
Zlena

$$= \begin{bmatrix} \cosh \omega_{03} & & \sinh \omega_{03} & \\ & 1 & & \\ & & 1 & \\ \sinh \omega_{03} & & \cosh \omega_{03} & \end{bmatrix} = \begin{bmatrix} \gamma & & \beta\gamma & \\ & 1 & & \\ & & 1 & \\ \beta\gamma & & \gamma & \end{bmatrix}$$

interpretiramo

kot

$$\Rightarrow \frac{\sinh \omega_{03}}{\cosh \omega_{03}} = \tanh \omega_{03} = \beta = \frac{v}{c} \quad \omega_{03} = \operatorname{arctanh} \beta \equiv \eta$$

"rapidity"

[2.] $M = 2$ za rot. vekt. okoli z za Θ . DN

$$J^{12}_{(1)} = i \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\rightarrow M = \begin{bmatrix} 1 & & & \\ & \cos \theta & -\sin \theta & \\ & \sin \theta & \cos \theta & \\ & & & 1 \end{bmatrix} = \Lambda$$

$$\omega_{12} = \Theta$$

torej je to čisto
normalna rotacija

[3. $M = ?$ za $s = 1/2$ rotacija okoli z]

$$J_{(1/2)}^{12} = \frac{1}{2} \epsilon_{12k} \begin{pmatrix} b^3 & \\ & b^3 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & & \\ & -1 & \\ & & 1 & \\ & & & -1 \end{pmatrix}$$

$$M = \exp[-i\theta J_{(1/2)}^{12}] = \begin{bmatrix} e^{-i\theta/2} & & & \\ & e^{i\theta/2} & & \\ & & e^{-i\theta/2} & \\ & & & e^{i\theta/2} \end{bmatrix}$$

Rotacija okoli smeri θ :

$$M = \begin{bmatrix} \exp(-\frac{i}{2} \vec{\theta} \cdot \vec{\sigma}) & 0 \\ 0 & \exp(-\frac{i}{2} \vec{\theta} \cdot \vec{\sigma}) \end{bmatrix} =$$

↑ vektor paulijevih matrik

$$= \begin{bmatrix} \cos \frac{\theta}{2} - i \frac{\hat{\theta}}{\theta} \cdot \vec{\sigma} \sin \frac{\theta}{2} & 0 \\ 0 & \cos \frac{\theta}{2} - i \frac{\hat{\theta}}{\theta} \cdot \vec{\sigma} \sin \frac{\theta}{2} \end{bmatrix}$$

Pri $M(\theta=2\pi) = -I \Rightarrow \psi \xrightarrow{\theta=2\pi} -\psi$. K sreči je verjetnost v QM $|\psi|^2$,
v QFT pa je $\sigma: \psi^* \dots \psi$ invariantno.

[4. $M = ?$ za $s = 1/2$ potisk v z]

$$J_{(1/2)}^{03} = -\frac{i}{2} \begin{pmatrix} b^3 & \\ & -b^3 \end{pmatrix} = -\frac{i}{2} \begin{bmatrix} 1 & & \\ & -1 & \\ & & -1 & \\ & & & 1 \end{bmatrix}$$

$$M = \exp[-i\omega_{03} J_{(1/2)}^{03}]$$

Za potisk v smeri i^s

$$\omega_{03} = \eta = \operatorname{arctgh} \beta \quad M = \operatorname{ch}\left(\frac{\eta}{2}\right) I - \operatorname{sh}\frac{\eta}{2} \begin{bmatrix} b^i & 0 \\ 0 & -b^i \end{bmatrix}$$

V kiralni upodobitvi (Diracovih matrik) so

$$M = \begin{bmatrix} \# & 0 \\ 0 & \# \end{bmatrix}$$

bločno diagonalne. Lorentzove transformacije $\psi_L \xrightarrow{x} \psi_R$; $\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$

$$\psi_L = P_L \psi \Rightarrow P_L = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \frac{1}{2}(1 - \gamma_5) \quad \gamma_5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3$$

$$P_L \begin{bmatrix} \psi_L \\ \psi_R \end{bmatrix} = \begin{bmatrix} \psi_L \\ 0 \end{bmatrix}$$

$$P_R = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \frac{1}{2}(1 + \gamma_5)$$

[Pokaži, da je $\bar{\psi}\psi$ inv. na LT; $\bar{\psi} = \psi^\dagger \gamma^0$; $\psi^\dagger \psi$ ni invar.]

$$\psi \rightarrow M\psi$$

$$\bar{\psi} \rightarrow (M\psi)^\dagger \gamma^0 = \psi^\dagger M^\dagger \gamma^0 = \bar{\psi} \overbrace{\gamma^0 M^\dagger \gamma^0}^{M^{-1}}$$

$$\bar{\psi}\psi \rightarrow \bar{\psi} M^{-1} M\psi$$

$$\bar{\psi} \rightarrow \bar{\psi} M^{-1} \text{ za vse LT}$$

Potisk: $M = \exp\left[-\frac{1}{2} \begin{bmatrix} \not{\partial}^3 & 0 \\ 0 & -\not{\partial}^3 \end{bmatrix}\right]$

$$\gamma^0 M^\dagger \gamma^0 = \begin{vmatrix} 0 & I \\ I & 0 \end{vmatrix} \exp\left[+\frac{1}{2} \begin{bmatrix} \not{\partial}^3 & 0 \\ 0 & -\not{\partial}^3 \end{bmatrix}\right] \begin{vmatrix} 0 & I \\ I & 0 \end{vmatrix} = M^{-1}$$

Lorentzove transformacije operatorjev polj

Skalarna polja, delci:

$$\Lambda: |p\rangle \rightarrow |\Lambda p\rangle$$

$$U(\Lambda)|p\rangle = |\Lambda p\rangle, a_p$$

$$\hat{U}(\Lambda) \sqrt{2E_p} a_p^\dagger |0\rangle = \sqrt{2E_{p'}} a_{p'}^\dagger |0\rangle$$

$$\mathbb{I} = U^{-1}(\Lambda) U(\Lambda)$$

$$\sqrt{2E_p} \hat{U} a_p^\dagger \underbrace{U^{-1} U}_{|0\rangle} |0\rangle = \sqrt{2E_{p'}} a_{p'}^\dagger |0\rangle$$

Vakuum je invariantna

$$\hat{U}(\Lambda) a_p^\dagger \underbrace{U^{-1}(\Lambda)}_{LT} = a_{\Lambda p}^\dagger \sqrt{\frac{2E_{\Lambda p}}{2E_p}}$$

$\hat{U}$ deluje na operatorje in stanje. Na stanju:

$$\hat{U} |n\rangle = |n'\rangle$$

in na operatorju

$$\hat{U} \hat{\phi} \hat{U}^{-1} = \hat{\phi}^{-1}$$

$$\hat{\phi}(x) \rightarrow \hat{U}(\lambda) \hat{\phi}(\lambda)^{-1}$$

Enačbe gibanja $\mathcal{L}, H, \dots$

Velikostna polja: $A^\mu(x)$: en. gib. $\partial_\mu F^{\mu\nu} = 0$
 lokalna $\rightarrow \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$

Spinorska (Diracova) polja: Ψ_a ; $a=1,2,3,4$

enake gibanja:

- i) inverzno na Lorentzove tr.
- ii) rešitve zadoščajo $p^2 = m^2$

iii) prvi odvod po času
 (ni odvisnosti na začetku)

$$\rightarrow \Psi(\vec{x}, t=0)$$

~~$$\frac{d}{dt} \Psi(\vec{x}, t=0)$$~~

Diracova enačba

(1928)

$$(i\gamma^\mu \frac{\partial}{\partial x^\mu} - mI) \Psi(x) = 0$$

$$\square \square = 1$$

Pokažimo, da veljajo zahtevi: LT polja

Dirac Equation

1.) Če Ψ zadošča DE, $\Psi'(x)$ zadošča DE.

$$(i\gamma^\mu \frac{\partial}{\partial x^\mu} - mI) M \Psi(\Lambda^{-1} x) \stackrel{?}{=} 0$$

$$M_{\frac{1}{2}} = \Lambda_{\frac{1}{2}}$$

$$MM^{-1} (i\gamma^\mu (\Lambda^{-1})^\nu{}_\mu \frac{\partial}{\partial \tilde{x}^\nu} - m) M \Psi(\tilde{x}) \stackrel{?}{=} 0$$

$$M (i\Lambda^\alpha{}_\mu \gamma^\mu (\Lambda^{-1})^\nu{}_\mu \frac{\partial}{\partial \tilde{x}^\nu} - m) \Psi(\tilde{x}) \stackrel{?}{=} 0$$

$$M (i\gamma^\nu \frac{\partial}{\partial \tilde{x}^\nu} - m) \Psi(\tilde{x}) = 0 \Rightarrow \text{kar pomeni, da tudi } \Psi' \text{ zadošča DE}$$

$$2.) (i\gamma^\mu \partial_\mu - m) \Psi(x) = 0$$

$$(i\gamma^\mu \partial_\mu - m)(i\gamma^\nu \partial_\nu - m) = (\partial_\mu \partial^\mu + m^2) \Psi = 0 ; \Psi_a \propto e^{\pm i p x}$$

Klein-Gordonova enačba

$$-(p)^2 + m^2 \Rightarrow p^2 = m^2 \Rightarrow p = (E_p, \vec{p})$$

3.) Očitno iz tega, da pogledas enačbo,

Rešitve diracove enačbe za proste fermione

$$\Psi_a(x) = u_a(\vec{p}) e^{-i p x}$$

$$\Psi_a(x) = v_a(\vec{p}) e^{i p x}$$

$$u(p) = \exp\left[-\frac{1}{2} \eta \begin{pmatrix} \gamma^3 & 0 \\ 0 & -\gamma^3 \end{pmatrix}\right] \sqrt{m} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$v(p) = \left[\cosh\left(\frac{1}{2} \eta\right) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \sinh\left(\frac{1}{2} \eta\right) \begin{pmatrix} \gamma^3 & 0 \\ 0 & -\gamma^3 \end{pmatrix} \right] \sqrt{m} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$(i \gamma^\mu \frac{\partial}{\partial x^\mu} - m) \begin{pmatrix} \psi \\ \chi \end{pmatrix} = e^{-i m t + i \vec{p} \cdot \vec{x}} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$= (i \gamma^\mu (-i m) - m) \begin{pmatrix} \psi \\ \chi \end{pmatrix}$$

$$= m \begin{pmatrix} -I & I \\ I & -I \end{pmatrix} \begin{pmatrix} \psi \\ \chi \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad u(\vec{0}) = \begin{pmatrix} \psi \\ \chi \end{pmatrix}$$

$$\sum_{s=1/2} u^s(p) \bar{u}^s(p) = \gamma p + m$$

Zapišimo $\mathcal{L}, \mathcal{H}, \Pi$ za klasično Diracovo polje

$$\mathcal{L}(\Psi_a, \partial_\mu \Psi_a) = \bar{\Psi}_a (i \gamma^\mu \dots$$

$$\mathcal{L}(\Psi_a, \partial_\mu \Psi_a) = \bar{\Psi}_a (i \gamma^\mu \frac{\partial}{\partial x^\mu} - m I)_{ab} \Psi_b(x)$$

E-L equm.

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial \partial_\mu \Psi_a} \right) - \frac{\partial \mathcal{L}}{\partial \Psi_a} \Rightarrow (i \gamma^\mu \frac{\partial}{\partial x^\mu} - m)_{ab} \Psi_b = 0$$

To pa je Diracova enačba, iz katere smo začeli.

Poglejmo še konjugirani moment:

$$\Pi_b(\vec{x}) = \frac{\partial \mathcal{L}}{\partial \dot{\Psi}_b(\vec{x})} = \bar{\Psi}_a i \gamma_a{}^0{}_{ba} = i (\bar{\Psi} \gamma_a)_5 = i \Psi_b^\dagger(\vec{x}).$$

$$H(\vec{x}) = \partial_a \Psi_a i \Psi_a^\dagger - \bar{\Psi} (i \gamma^0 \partial_0 \Psi + \vec{\gamma} \cdot \vec{\nabla} \Psi - m \Psi) ; \quad q p^i = \mathcal{L}$$

$$\mathcal{H}(\vec{x}) = \Psi^\dagger \partial_0 \Psi - \bar{\Psi} (i \vec{\gamma} \cdot \vec{\nabla} \Psi +$$

$$= \Psi^\dagger(\vec{x}) \gamma^0 \vec{\gamma} \cdot (-i \vec{\nabla}) + \gamma^0 m \Psi(\vec{x}) = \Psi^\dagger \omega | \Psi \rangle$$

$$= \gamma^\dagger(\vec{r}) = \gamma^0 \gamma^0 [\dots]$$

$$\hookrightarrow = \gamma^\dagger(\vec{x}) [\gamma^0 \vec{\gamma} \cdot (-\vec{\nabla}) + \gamma^0 m] \psi = \gamma^\dagger(x) [\gamma^0 \vec{\gamma} \cdot (-i \vec{\nabla}) + \gamma^0 m] \psi$$

$\vec{p} \quad \vec{d} = \vec{p}$

$\psi(x) = \psi(x) \gamma^0 \psi(x)$

Protiv kvantizaciji Dir. polja

$\psi(x)$ za DE.

Klasično

$$\psi(x) = \int \frac{d\vec{p}}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_{s=1,2} (A_{\vec{p}}^s u^s(p) e^{-ipx} + B_{\vec{p}}^s v^s(p) e^{ipx})$$

polje

poljubne

Operator fermionskih polj:

$$\psi(x) = \int \frac{d\vec{p}}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_{s=1,2} (a_{\vec{p}}^s u^s(p) e^{-ipx} + b_{\vec{p}}^s v^s(p) e^{ipx})$$

. 5x4x4

$$A_{\vec{p}}^s = B_{\vec{p}}^s = 1$$

- da bo zadoščena LNNV. relacija
- kg zgleda glede na razne

~~Tukaj~~ kje zidano

Če namesto $b_{\vec{p}}^{s+1}$ bi bil
java / miks / miks

wins / lins, python ne na dglu
(glej in path).

$$\mathcal{L}, \mathcal{H}, \mathcal{H} \quad \mathcal{L}(\psi_a, \partial_\mu \psi_a) = \bar{\psi}_a \gamma^\mu \partial_\mu \psi_a$$

Kanonische transformirajo lot:

$$[\hat{\psi}_a, \hat{\pi}_b] = i\hbar \delta_{ab}$$

$$[\psi(\vec{x}, t), \psi(\vec{y}, t)] = 0$$

$$[\pi(\vec{x}, t), \pi(\vec{y}, t)] = 0$$

$$[a_p^s, a_q^t] = \frac{(2\pi)^3}{c^3} \delta(\vec{p} - \vec{q}) \delta_{st}$$

Primeri $[\hat{p}_r^2(\vec{x}, 0), \pi_b(\vec{y}, 0)]$

$$= \int \frac{d\vec{p}}{(2\pi)^3} \frac{n}{\sqrt{2E_p}} i \sqrt{\frac{2E_p}{c}} \dots$$

$$\sum_{s,r} = (L a_{\vec{p}}^s a_{\vec{q}}^{st}) + M_s(\vec{p}) - n p^{\text{klas}}(10) +$$

$$+ [b_{-L}^{st}, b_{\vec{q}}^{-st}] \sqrt{2}(-\vec{p}) \sqrt{2}(-\vec{q})$$

$\sqrt{5\pi^5} \frac{20\pi k}{y^5}$

$$= i \int \frac{d\vec{p}}{(2\pi)^3} \frac{2E_p}{\sqrt{2E_p}} e^{i\vec{p}(\vec{x}-\vec{y})} \sum_s \underbrace{u(\vec{p})^s u(\vec{p})^s} + \sqrt{2}(-\vec{p}) \sqrt{2}(-\vec{q})$$

$y^\mu z - y^0 + \dots$

$$\Rightarrow i \int \frac{d\vec{p}}{d\vec{x} \frac{d\vec{p}}{d\vec{x}}} \frac{1}{2E_p} \frac{1}{2E_p} \dots$$

$$= i \int \frac{d\vec{p}}{(2\pi)^3} \frac{1}{2E_p} e^{i\vec{p}(\vec{x}-\vec{y})} 2E_p \underbrace{I_{ab}}$$

I_{ab}
 δ_{ab}

$\hat{\sigma} = \hat{H}; \hat{\vec{p}}, \hat{Q}_{el}, \hat{J}$ lastna stanja fermionov, lastne vrednosti:

$\hat{\sigma}$	H	$\vec{p}$	Q_{el}	$\hat{J}_z(\vec{p}=0)$
λ_{σ}	$+E_{\vec{p}}$	$\vec{p}$	e	$\begin{cases} +\frac{1}{2} & s=1 \\ -\frac{1}{2} & s=2 \end{cases}$
$\bar{\lambda}_{\sigma}$	$+E_{\vec{p}}$	$\vec{p}$	$-e$	$\begin{cases} -\frac{1}{2} & s=1 \\ +\frac{1}{2} & s=2 \end{cases}$

$$\hat{\sigma} a_{\vec{p}}^{st} |0\rangle = \lambda_{\sigma} a_{\vec{p}}^{st} |0\rangle$$

$$\hat{\sigma} b_{\vec{p}}^{st} |0\rangle = \bar{\lambda}_{\sigma} b_{\vec{p}}^{st} |0\rangle$$

$$E_p = +\sqrt{\vec{p}^2 + m^2}$$

$$|f(\vec{p}, s)\rangle = \sqrt{2E_p} a_{\vec{p}}^{st} |0\rangle$$

$$|\bar{f}(\vec{p}, s)\rangle = \sqrt{2E_p} b_{\vec{p}}^{st} |0\rangle$$

$$\hat{\Psi}(x, 0) = \int \frac{d\vec{p}}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} \sum_{s=1,2} \left(a_{\vec{p}}^s u^s(\vec{p}) + b_{-\vec{p}}^{st} u^s(-\vec{p}) \right) e^{i\vec{p}\vec{x}}$$

$\hat{H}$ operator energije

$$\hat{H} = \int d\vec{x} \underbrace{\hat{\Psi}^\dagger(\vec{x}) h_D \hat{\Psi}(\vec{x})}_{H(\vec{x})}$$

$$h_D = \vec{\alpha} \cdot (-i\vec{\nabla}) + \beta m$$

$$h_D \Psi = i \frac{d}{dt} \Psi \quad (\text{Diracova enačba})$$

$$\rightarrow h_D u(\vec{p}) e^{-ipx} = i(-iE_{\vec{p}}) u(\vec{p}) e^{-ipx}$$

$$\rightarrow h_D v(\vec{p}) e^{ipx} = -E_p v(\vec{p}) e^{ipx}$$

$$\hat{H} = \int d\vec{x} \int \frac{d\vec{p}}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} \sum_s \left(a_{\vec{p}}^{st} u^{st}(\vec{p}) + b_{-\vec{p}}^{st} v^{st}(\vec{p}) \right) e^{-i\vec{p}\vec{x}}$$

$$\int \frac{d\vec{q}}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{q}}}} \sum_r \left(+E_{\vec{q}} a_{\vec{q}}^r u^r(\vec{q}) + (-E_{\vec{q}}) b_{-\vec{q}}^{rt} v^r(\vec{q}) \right) e^{i\vec{q}\vec{x}}$$

$$= \int \frac{d\vec{p}}{(2\pi)^3} \sum \left(a_{\vec{p}}^{st} a_{\vec{p}}^s E_{\vec{p}} - \underbrace{b_{\vec{p}}^s b_{\vec{p}}^{st} E_{\vec{p}}}_{\text{antikomutira}} \right)$$

antikomutira

$$-b_{\vec{p}}^{st} b_{\vec{p}}^s + (2\pi)^3 \delta(0)$$

$$u^{st}(\vec{p}) u^r(\vec{p}) = 2E_{\vec{p}} \delta_{sr}$$

$$v^{st}(\vec{p}) v^r(\vec{p}) = 2E_{\vec{p}} \delta_{sr}$$

$$= \int \frac{d\vec{p}}{(2\pi)^3} E_{\vec{p}} \sum_s (a_{\vec{p}}^{s\dagger} a_{\vec{p}}^s + b_{\vec{p}}^{s\dagger} b_{\vec{p}}^s - \underbrace{(2\pi)^3 \mathcal{J}(0)})$$

implizira, da ima vakuum $-\infty$ energijo

za enodelčno stanje anti-delca potem:

$$\hat{H} b_{\vec{p}}^{s\dagger} |0\rangle = b_{\vec{p}}^{s\dagger} \underbrace{\hat{H} |0\rangle}_{E_0} + \underbrace{[\hat{H}, b_{\vec{p}}^{s\dagger}] |0\rangle}_{E_p b_{\vec{p}}^{s\dagger}}$$

$\Rightarrow E = E_0 + E_p$ } Energija je sicer dobro določena glede na vakuum.
 ↑
 Energija vakuma

Kaj pa večdelčno stanje 2 neinteragirajočih fermionov:

$$a_{\vec{p}_1}^{s_1\dagger} a_{\vec{p}_2}^{s_2\dagger} |0\rangle, \quad E = E_0 + E_{\vec{p}_1} + E_{\vec{p}_2}$$

$$a_{\vec{p}}^{s_1\dagger} a_{\vec{p}}^{s_2\dagger} |0\rangle = -a_{\vec{p}}^{s_2\dagger} a_{\vec{p}}^{s_1\dagger} |0\rangle = 0$$

↓
antikomutira

Kaj pa to?

$$a_{\vec{p}}^{s_1\dagger} b_{\vec{p}}^{s_2\dagger} |0\rangle = -b_{\vec{p}}^{s_2\dagger} a_{\vec{p}}^{s_1\dagger} |0\rangle \neq 0$$

Podoben račun za skalarno polje

$\hat{\vec{P}}$ Operator gibalne količine (ohranjen naboj pri translaciji v prostoru)

DN

Rezultat je pa:
$$\hat{\vec{P}} = \int \frac{d\vec{p}}{(2\pi)^3} \vec{p} \sum_s (a_{\vec{p}}^{s\dagger} a_{\vec{p}}^s + b_{\vec{p}}^{s\dagger} b_{\vec{p}}^s)$$

Q_{el} Operator električnega naboja (invarianca na transformacijo faze)

$$\psi \rightarrow e^{i\alpha} \psi; \alpha = \text{const.}$$

$$j^\mu = \frac{\partial \mathcal{L}}{\partial \partial_\mu \psi} \Delta \psi + \frac{\partial \mathcal{L}}{\partial \partial_\mu \bar{\psi}} \Delta \bar{\psi} - \mathcal{J}^\mu$$

$$\mathcal{L} = \bar{\psi}_a (i \gamma^\mu \partial_\mu - m)_{ab} \psi_b \Rightarrow \Delta \bar{\psi} = -i \bar{\psi}$$

$$\psi \rightarrow \psi' = (1 + i\alpha) \psi$$

$$\mathcal{J}^\mu = 0 \text{ ker } \mathcal{L} = \text{invariant.}$$

$$\frac{\partial \mathcal{L}}{\partial (\partial_a \psi_b)} = \bar{\psi}_b i \gamma^\mu_{ab}$$

$$\Rightarrow j^\mu = \bar{\psi}_a i \gamma^\mu_{ab} (i \psi_b)$$

$$j^\mu_{EM} = e \bar{\psi} \gamma^\mu \psi, \quad \partial_\mu j^\mu_{EM} = 0$$

$$j^\mu_{EM} = (j^0_{EM}, \vec{j}_{EM})$$

Kontinuitetna
enačba za
naboj

Naboj lahko dobimo iz 0-te komponente:

$$Q_{el} = \int d^3x e \bar{\psi} \gamma^0 \psi = \int d^3x e \psi^\dagger \psi = \int d\vec{x} e (\dots a^\dagger \dots + \dots b) (\dots a \dots + \dots b^\dagger) =$$

$$= e \int \frac{d\vec{p}}{(2\pi)^3} \sum_s \left(\underbrace{a_{\vec{p}}^{s\dagger} a_{\vec{p}}^s}_{\text{delci}} + \underbrace{b_{\vec{p}}^s b_{\vec{p}}^{s\dagger}}_{\text{anti delci}} \right)$$

$\uparrow$
delci

$\uparrow$
anti delci

$$! \left(-b_{\vec{p}}^{s\dagger} b_{\vec{p}}^s + (2\pi)^3 \delta(0) \right)$$

implicitna
neskončen naboj
voluma

$$\hat{Q}_{el} a_{\vec{p}}^\dagger |0\rangle = [\hat{Q}_{el}, a_{\vec{p}}^\dagger] |0\rangle = e a_{\vec{p}}^\dagger |0\rangle$$

$$\hat{Q}_{el} b_{\vec{p}}^\dagger |0\rangle = [\hat{Q}_{el}, b_{\vec{p}}^\dagger] |0\rangle = -e b_{\vec{p}}^\dagger |0\rangle$$

$\propto |\vec{p}\rangle$

ignorirali naboj voluma

Za elektronsko polje je $e = -e_0 \rightarrow e^-, e^+$
 $= \frac{2}{3} e_0 \rightarrow u, c, t$
 $= -\frac{1}{3} e_0 \rightarrow d, s, b$

Kaj pa če bi imeli?

$$\hat{\Psi}_\mu(\vec{x}, t) = \int \dots (a_p^s u e^{-ipx} + a_p^{st} v e^{ipx})$$

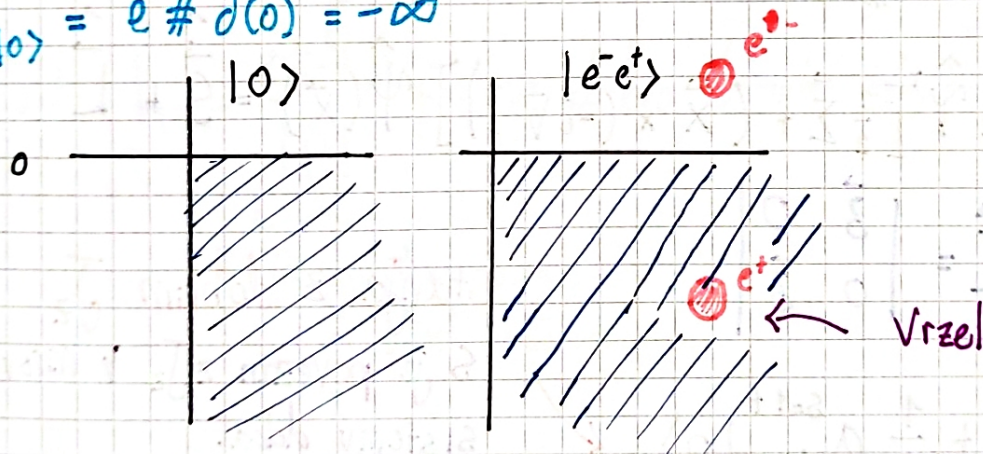
↑ Majorana

$$\hat{Q}_{el} = e \int \hat{\Psi}_\mu^\dagger \hat{\Psi}_\mu = 0$$

0 $\rightarrow$ V prejšnjem računu daj a namesto b in se boš odštela

Naboj vakuma:

$$e_{|0\rangle} = e \# \delta(0) = -\infty$$



↑ Vrtikalna količina (invarianca na rotacijo)

Najbolj nas bo zanimala $\hat{J}_z$. Načrt dela: - Rotacija okoli z za θ
 $\Delta v, f^\mu, Q = \hat{J}_z$

$$- \hat{J}_z : a, b$$

$$- \hat{J}_z a_0^{st} = ?$$