

$$\Psi \rightarrow \Psi' = M_{1/2} \Psi(\Lambda^{-1}x) = (1 - i\theta \overset{\omega_{12}}{J_{(1/2)}}) \Psi(x + \theta_y, y - \theta_x, z) =$$

$$\overset{12}{J_{(1/2)}} = \frac{1}{2} \underbrace{\begin{vmatrix} \vec{\sigma}^3 & 0 \\ 0 & \vec{\sigma}^3 \end{vmatrix}}_{\Sigma^3}, \quad \Lambda^{-1}x = \begin{vmatrix} 1 & \theta & 0 \\ -\theta & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} x \\ y \\ z \end{vmatrix} = \begin{vmatrix} x + \theta_y \\ y - \theta_x \\ z \end{vmatrix}$$

Uporabili Taylorjev razvoj

$$= \Psi + \theta \underbrace{\left[-\frac{1}{2} \Sigma^3 \Psi + \partial_x \Psi \cdot (y) - \partial_y \Psi \cdot (x) \right]}_{\Delta \Psi} = \Psi + \Delta \Psi \theta$$

$$j^0 = \frac{\partial \mathcal{L}}{\partial \partial_0 \Psi} \Delta \Psi = \bar{\Psi} i \gamma^0 \Delta \Psi; \quad Q = \int d\vec{x} \underset{\text{vektorsko}}{j^0}(\vec{x}) = \int d\vec{x} j^0(\vec{x}) =$$

$$= \int d\vec{x} \Psi^\dagger(\vec{x}) \left[\frac{1}{2} \Sigma^3 + (\vec{x} \times (-i\nabla))^3 \right] \Psi(x)$$

$$\Rightarrow \hat{J}_z = \int d\vec{x} \hat{\Psi}^\dagger(\vec{x}) \left[\frac{1}{2} \Sigma^3 + (\vec{x} \times (-i\nabla))^3 \right] \hat{\Psi}(x)$$

Za poljubno vrtilno količino pa:

$$\hat{\vec{J}} = \int d\vec{x} \hat{\Psi}^\dagger \left[\frac{1}{2} \vec{\Sigma} + (\vec{x} \times (-i\nabla)) \right] \hat{\Psi}(\vec{x}) = \hat{\vec{S}} + \hat{\vec{L}}$$

$$\vec{\Sigma} = \begin{vmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{vmatrix}$$

Stanja z dobrim J_z
 S je povezan z J_z v mirovnem sistemu deka.

$$\hat{J}_z a_{\vec{0}}^{s=1+} |0\rangle = +\frac{1}{2} a_{\vec{0}}^{s=1+} |0\rangle$$

$$\hat{J}_z a_{\vec{0}}^{s=2} |0\rangle = -\frac{1}{2} \dots$$

$$\hat{J}_z b_{\vec{0}}^{s=1+} |0\rangle = -\frac{1}{2} \dots$$

$$\hat{J}_z b_{\vec{0}}^{s=2+} |0\rangle = +\frac{1}{2} \dots$$

$$\hat{\Psi}(\vec{x}, t) = \int \frac{d\vec{p}}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_s (a_{\vec{p}}^s u_s(\vec{p}) e^{-ipx} + b_{\vec{p}}^{s\dagger} v^s(\vec{p}) e^{ipx})$$

Spin

Komponenta spina (S_z) in sučnost ($h = \text{helicity}$) za delce z $\vec{p} \neq 0$

Kaj če bi imeli?

$$\sum_s u^{s=1}(p_x) \neq u^{s=-1}(p_x)$$

$$\hat{S}_z a_{p_x}^{s=1\dagger} |0\rangle \neq a_{p_x}^{s=1\dagger} |0\rangle \quad \left. \vphantom{\hat{S}_z a_{p_x}^{s=1\dagger} |0\rangle} \right\} \text{ni lastno stanje operatorja spina } \hat{S}_z$$

S je povezan z komponento spina v mirovnem sistemu delca.

$$\hat{\vec{J}} = \hat{\vec{S}} + \hat{\vec{L}}; \quad [\hat{\vec{S}}, \hat{H}] \neq 0 \neq [\hat{\vec{L}}, \hat{H}]$$

Samo po sebi ni dobro kvantno število. Skupaj

$$\text{Oč.} \quad [\hat{\vec{J}}, \hat{H}] = 0$$

Sučnost $\hat{h} = \frac{\hat{\vec{p}} \cdot \hat{\vec{S}}}{|\hat{\vec{p}}|}$ pa je dobro kvantno število $[\hat{h}, \hat{H}] = 0$

projekcija spina na
gibalno količino

Karžalnost

Za dva dogodila krajinega tipa morata,

x y operatorja komutirati:

$$[\hat{\mathcal{O}}_1(x), \hat{\mathcal{O}}_2(y)] = 0 \quad \text{za } (x-y)^2 < 0$$

Ker imajo operatorji po par polj: $\hat{\mathcal{O}}_1(x) = \psi_a^\dagger(x) \Gamma_{ab}^{\frac{1}{2}} \psi_b(x)$

Vaja:

$$\vec{x} \neq \vec{y}: [\hat{\sigma}_1(\vec{x}, 0), \hat{\sigma}_2(\vec{y}, 0)] =$$

$$= \psi_a^\dagger(x) \Gamma_{ab}^{-1} \psi_b(x) \psi_c^\dagger(y) \Gamma_{cd}^{-2} \psi_d(y) - \psi_c^\dagger \Gamma_{cd}^{-2} \psi_d \psi_a^\dagger \Gamma_{ab}^{-1} \psi_b =$$

$$= \psi_c^\dagger(y) \psi_a^\dagger(x) \Gamma_{ab}^{-1} \psi_b(x) \Gamma_{cd}^{-2} \psi_d(y) - 2. \text{ člen} =$$

$$= \psi_c^\dagger \psi_d \psi_a^\dagger \Gamma_{ab}^{-1} \psi_b \Gamma_{cd}^{-2} - 2. \text{ člen} = 0$$

Fermionski propagator

$$S_F(x-y)_{ab} = \langle 0 | T \{ \psi_a(x) \bar{\psi}_b(y) \} | 0 \rangle = \begin{cases} \langle 0 | \psi_a(x) \bar{\psi}_b(y) | 0 \rangle; x^0 > y^0 \\ -\langle 0 | \bar{\psi}_b(y) \psi_a(x) | 0 \rangle; y^0 > x^0 \end{cases}$$

↑
časovni produkt

$$= \int \frac{d^4 p}{(2\pi)^4} \frac{i(\not{p} + m)_{ab}}{p^2 - m^2 + i\epsilon} e^{-ip(x-y)}$$

↑
iε odgovoren za

časovno vezanje

$$\not{p} = p^\mu \gamma_\mu = p^0 \gamma_0 - \vec{p} \vec{\gamma}$$

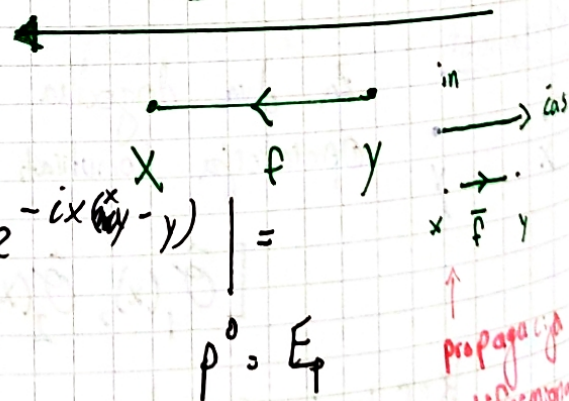
Dokazimo ekvivalenco:

$$x^0 > y^0:$$

$$\text{LHS: } S_{Fab}(x-y) = \int \dots \int_{s,r} \langle 0 | a_{\vec{p}}^s u(\vec{p}) e^{-ipx} \bar{u}(\vec{q}) e^{iqy} | 0 \rangle$$

čas

$$= \int \frac{1}{(2\pi)^3} \frac{1}{2E_p} \langle 0 | 0 \rangle \sum_s u^s(\vec{p}) \bar{u}^s(\vec{q}) e^{-ix(\vec{p}-\vec{q})} =$$



Na plosojnicah žb

$$(p + m)_{ab}$$

$$\Rightarrow S_{Fab} = \int \frac{d^4 p}{(2\pi)^4} \frac{i(\not{p} + m)_{ab}}{p^2 - m^2 + i\epsilon} e^{-ip(x-y)} \Big|_{p^0 = E_p} \cdot \frac{-2\pi i}{2\pi}$$

↑
propagator
anti-Fermiona

↑
Nevem kaj to je

$$S_F(x-y)_{ab} = \int \frac{d^4 p}{(2\pi)^4} \frac{2\pi i}{2\pi} \frac{i(\not{p} + m)_{ab}}{p^0 - (E_p - i\epsilon)} e^{-ip(x-y)} \quad \left| \quad p^0 = -(E_p - i\epsilon) \right.$$

$S_F(x-y)$ je Greenova funkcija za Diracovo enačbo

$$(i\gamma^\mu \frac{\partial}{\partial x^\mu} - m)_{ab} S_F(x-y)_{bc} \propto \delta(x-y) \delta_{ac}$$

Preverimo, če to res velja.

$$\int \frac{d^4 p}{(2\pi)^4} \frac{i}{p^0 - m^2 + i\epsilon} \underbrace{(i\gamma^\mu (-i p_\mu) - m)_{ab} i(\not{p} + m)_{bc}}_{-\delta_{ac} \cdot 1} e^{-ip(x-y)} =$$

$$= \frac{(p-m)(p+m)}{p^0 - m^2 + i\epsilon} = \frac{p^0 p^0 - m^2 = (p^2 - m^2)}{p^0 - m^2 + i\epsilon}$$

$$\text{res} \propto \delta(x-y) \delta_{ac}$$

Majoranini in Diracovi fermioni

Rekli smo, da je $e_f = -e_{\bar{f}}$. Diracov fermion je tak, ki ni sam sebi anti delce. Majoranini pa tak, ki je, kar je možno le za delce z $e=0$.

$$\text{Dirac: } f \neq \bar{f} \quad a \neq b$$

$$\text{Majorana: } f = \bar{f} \quad a = b$$

Operator za Majoranine fermione je:

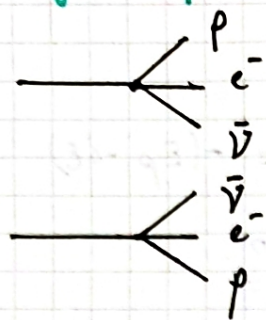
→ Diracovi bi pa imeli b_p^{st}

$$\hat{\psi}_M = \int \dots \sum_s (a_p^s u e^{-ipx} + \underbrace{a_p^{st}}_{b_p^{st}} v e^{-ipx})$$

Ekscitacije majoraninih polj so nevtralni delci.

ψ : ni znano ali so $\psi = \bar{\psi}$ ali $\psi \neq \bar{\psi}$

• dvojni razpad β

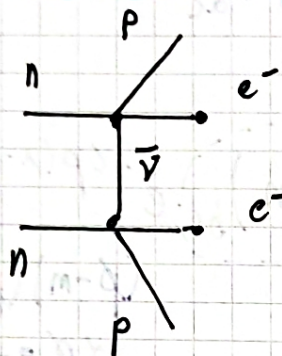


$$n \rightarrow p + e^- + \bar{\nu}_e$$

$\Leftarrow$ Možno za Diracove ali
Majorana

• brez-neutrinski dvojni β r.

$$\nu_n \rightarrow p + e^-$$



$\Leftarrow$ To pa je možno le za $\bar{\nu} = \nu$

Leptonsko število

$$L = L_e + L_\mu + L_\tau$$

$$\underline{L_e = 1}$$

$$\underline{L_{\bar{e}} = -1}$$

Vedno, da se L_e recimo sam po sebi ne dranja,
(npr. nevtrinske oscilacije)

Dirac

$$\Delta L = 0$$

$$2n \rightarrow 2p + 2e^- + 2\bar{\nu}_e$$

Majorana

$$\Delta L \neq 0$$

$$0 \rightarrow 0 + 2e^-$$

Masa nevtrinov (1998).

$\nu_\mu \rightarrow \nu_\tau$: nevtrinske oscilacije, $m_\nu \neq 0$, so pa zelo
male

Levo-ročni delci

$$\psi_L = \frac{1}{2} (1 - \gamma_5) \psi$$

Desno-ročni delci

$$\psi_R = \frac{1}{2} (1 + \gamma_5) \psi$$

Torej če so $\bar{\nu}$ Majoranini lahko
masni člen Zapišemo z ψ_L

Če imajo nevtrini maso morajo
imeti leve in desne.

Desno ročne ne moremo dobiti neposred
no, vsi so levi, a da se transpo
hirat na $\psi_{R,u} = C \bar{\psi}_L^T$

$\Leftarrow$ To se da če so Majoranini

Kompleksna skalarna polja

$\hat{\phi} \neq \hat{\phi}^\dagger$, eksitacije teh polj ustrezajo nabitemu skalarnemu delcu.

Npr. π^+ : $u\bar{d}$, π^- : $\bar{u}d$ + vsi drugi mezeni s $S=0$

$$\hat{\phi}(\vec{x}, t) = \int \frac{d\vec{p}}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} (a_{\vec{p}} e^{-i p x} + b_{\vec{p}}^\dagger e^{i p x}) \neq \hat{\phi}^\dagger$$

$\uparrow$ $\uparrow$
 anihilacija π^+ Lz kreacija antikvarkov π^-

$$[a_{\vec{p}}, a_{\vec{p}'}^\dagger] = (2\pi)^3 \delta(\vec{p} - \vec{p}')$$

$$\mathcal{L} = \partial_\mu \phi \partial^\mu \phi^\dagger - m^2 \phi \phi^\dagger$$

$$[\hat{\phi}(\vec{x}, t), \hat{\pi}(\vec{y}, t)] = i \delta(\vec{x} - \vec{y})$$

Iz tega sledi $\hat{\pi}, \hat{H}, \hat{P}, Q, \dots$

Karžalnost:

$$[\hat{\sigma}_1(\vec{x}, 0), \hat{\sigma}_2(\vec{y}, 0)] = 0; \vec{x} \neq \vec{y}$$

$$\hat{\sigma} = f(\hat{\phi}, \hat{\pi}, \hat{\phi}^\dagger)$$

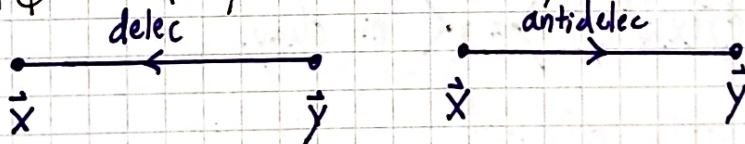
$$[\hat{\phi}(\vec{x}, 0), \hat{\pi}(\vec{y}, 0)] = 0 \leftarrow \text{zaradi komutacijskih relacij}$$

$$[\hat{\phi}(\vec{x}, 0), \hat{\phi}^\dagger(\vec{y}, 0)] = ? = \iint \dots \left(\underbrace{[a_{\vec{p}}, a_{-\vec{p}'}^\dagger]}_{() \cdot \delta(\vec{p} - \vec{p}')} + \underbrace{[b_{-\vec{p}}^\dagger, b_{-\vec{p}'}]}_{-() \delta(\vec{p} - \vec{p}')} \right) = 0 = (*)$$

$$\phi(\vec{x}, 0) = \int \dots (a_{\vec{p}} + b_{-\vec{p}}^\dagger) e^{i \vec{p} \cdot \vec{x}}$$

$$\phi^\dagger(\vec{y}, 0) = \int \dots (a_{\vec{p}'}^\dagger + b_{-\vec{p}'}) e^{-i \vec{p}' \cdot \vec{y}}$$

$$(*) = \langle 0 | [\quad , \quad] | 0 \rangle = \langle 0 | \hat{\phi}(\vec{x}, 0) \hat{\phi}^\dagger(\vec{y}, 0) | 0 \rangle - \langle 0 | \hat{\phi}^\dagger(\vec{y}, 0) \hat{\phi}(\vec{x}, 0) | 0 \rangle$$

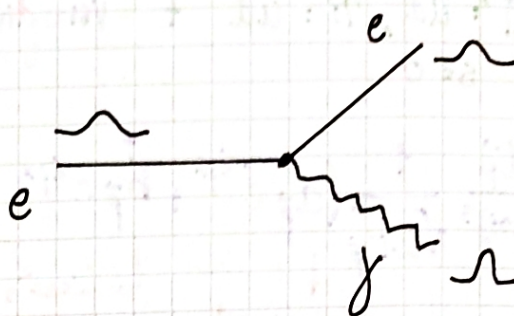


Karžalnost zahteva res prisotnost antidelecev.

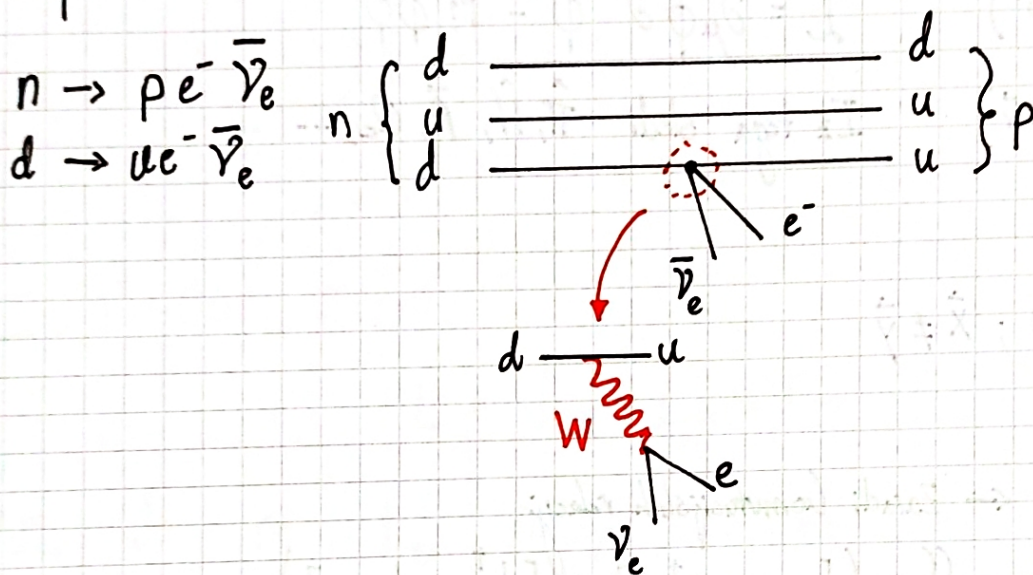
Interakcije med polji (ϕ, Ψ, V^μ) in delci

Uvedli bomo, da obe polji Interagirata na isti točki $(\vec{x}, t)$. Prej intro primer pred vizualizacijo.

$$\mathcal{H}_i(x) = e \bar{\Psi}(x) \gamma^\mu \Psi(x) A_\mu(x)$$



ali pa:



Načela

1.) Lokalnost, kausalnost

$$\mathcal{L}_i(x) \propto \phi^\dagger(x) \times x \text{ je okay.}$$

$$\phi^2(x) \phi^2(y) \rangle^* \rangle^* \text{ ni okay.}$$

2.) Komutacijske relacije

Če $\mathcal{L}_i$ ne vsebuje $\partial_0 \phi$: $\Pi = \frac{\partial \mathcal{L}}{\partial \partial_0 \phi} \Rightarrow \Pi$: Ostane nespremenjen

$$\hat{\Phi}(\vec{x}, t) = e^{iH_0 t} \hat{\Phi}(\vec{x}, 0) e^{-iH_0 t} \leftarrow \text{Zadošča KR}$$

↑
polje v interakcijski sliki

$$\hat{\Phi}(\vec{x}, t) = e^{iH t} \phi(\vec{x}, 0) e^{-iH t} \leftarrow$$

a zadošča tudi to?

$$\tilde{\phi} = \underbrace{e^{iH_0 t} e^{-iH_0 t}}_{U^{-1}} \phi_1(\vec{x}, t) \underbrace{e^{iH_0 t} e^{-iH_0 t}}_{U(t, 0)}$$

operator časovne evolucionje od 0 do t.

$$[\hat{\phi}(\vec{x}, t), \hat{\pi}(\vec{y}, t)] = [\hat{U}\hat{\phi}(\vec{x}, t)\hat{U}, \hat{U}^{-1}\hat{\pi}(\vec{y}, t)\hat{U}] = \hat{U}^{-1}[\hat{\phi}(\vec{x}, t), \hat{\pi}(\vec{y}, t)]\hat{U} = 0$$

3.) Lorentzova invarianca:

$$\mathcal{L}(x) \xrightarrow{\text{L.T.}} \mathcal{L}(\Lambda^{-1}x)$$

4.) Renormalizabilnost

→ QED: Samo dva parametra m_0, e_0

Peslin: 10.1

Teorija NI renormalizabilna če je ~~sklopitvena~~ enota sklopitvene konstante $[g] = eV^N$ in je $N < 0$,

Če $[g] = eV^N$ $N \geq 0$: lahko je ali ni renormalizabilna

5.) Simetrije

So lahko globalne ali pa lokalne/umeritvene.

Cel standardni model ima dodane samo 3 simetrije:

$$SU(3) \times SU(2) \times U(1)$$

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_i, \quad \mathcal{H} = \pi\dot{\phi} - \mathcal{L} = \underbrace{\pi\dot{\phi} - \mathcal{L}_0}_{H_0} - \underbrace{\mathcal{L}_i}_{H_i}; \quad \mathcal{H}_i = -\mathcal{L}_i$$

Masne dimenzije polj

$$\hbar = c = 1$$

$$[p] = [E] = \frac{1}{[x]} = eV$$

Kaj pa polja? $[\phi] = ?$
 $[\psi] = ?$
 $[A] = ?$

$$[\mathcal{L}] = ? \quad [H] = eV, \quad H = \int d^3x \mathcal{H} \quad [\mathcal{H}] = \frac{eV}{eV^{-3}} = eV^4 = [\mathcal{L}]$$

$$[\phi] = ? \quad \mathcal{L} = \partial_{x^\mu} \phi \partial_{x^\mu} \phi + \dots$$

$$eV^4 = eV[\phi]eV[\phi] \rightarrow [\phi] = eV$$

$$[\psi] = ? \quad \mathcal{L} = \bar{\psi} i \gamma^\mu \partial_{x^\mu} \psi + \dots$$

$$eV^4 = [\bar{\psi}] i \gamma^\mu [eV][\psi] \Rightarrow [\psi]^2 = eV^3$$

$$\downarrow$$

$$[\psi] = eV^{3/2}$$

$$[A^\mu]: \quad \mathcal{L} = -\frac{1}{4} \left(\partial_{x^\mu} A_\nu - \partial_{x^\nu} A_\mu \right)^2 =$$

$$= ([eV][A_\nu] - [eV][A_\mu])^2 = [eV^4]$$

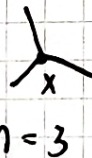
$$[A_\nu] = eV$$

Interakcije med skalarji

$$\mathcal{L}_i(x) = :A_n: \phi^n(x)$$

Sklopitvena konstanta

Da je teorija renormalizabilna: $[A_n] = eV^{4-n} \Rightarrow n=3, 4$



Interakcije med fermioni in skalarji

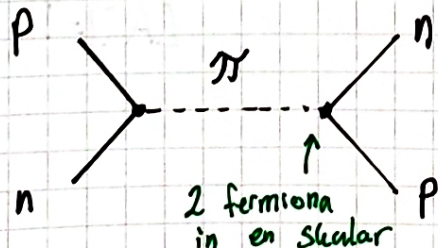
$$\mathcal{L}_i(x) = g \bar{\psi} \psi \phi$$

dva fermiona in en skalar

- 1.) ✓
- 2.) ✓
- 3.) ✓
- 4.)

$$eV^4 = [g] eV^3 eV \Rightarrow [g] = eV^0$$

ni drugih možnosti



Interakcija z Higgsovim poljem:

$$\mathcal{L}_i = g_f \bar{\Psi}(x) \Psi_f(x) \phi(x)$$

$$\phi(x) = v + \underbrace{h(x)}_{\text{Higgsov bozon}}$$

↑
Vakumska
pričakovana
vrednost

$$\Rightarrow \mathcal{L}_i = -g_f \bar{\Psi}_f(x) \Psi_f(x) v - g_f \bar{\Psi}_f(x) \Psi_f(x) h(x)$$

Interakcija $\hat{=}$ poljem
da ~~ga~~ masni člen m_f ,
ki da ~~maso~~ maso fermionu

→ pred tem so delci zato da so
določene simetrije, uvedeni brezmasni

Sklopitev je sorazmerna z maso fermiona $g_f \propto m_f$

Interakcije med fermioni in vektorji:

$$\Psi = e \quad V^\mu = A^\mu$$

$$\mathcal{H}_i = -\mathcal{L}_i = e \underbrace{\bar{\Psi} \gamma^\mu(x)}_{j_{EM}^\mu} A_\mu(x)$$

Načelo
(4) $j_{EM}^\mu \xrightarrow{LT} \Lambda^\mu_\nu j_{EM}^\nu \quad (3) \checkmark$

↓
 $eV^4 = [e] eV^3 \cdot eV$

$\Rightarrow [e] = eV^0$ kar je res. Spomni se $\frac{e^2}{4\pi} = \frac{1}{137}$

Kaj pa še kakšen drug člen, ki bi zadovoljeval temu?

$$\mathcal{L} \propto A_\mu(x) A^\mu(x), (A_\mu(x) A^\mu(x))^2 \dots$$

QED: Quantum electrodynamics

Postulira uimeritvene ~~gauge~~ (gauge) invariance:

bačdarna
gauge v
hrvačini

Transf.: $\Psi(x) \rightarrow e^{i\alpha(x)} \Psi(x)$, $A_\mu(x) \rightarrow A_\mu(x) - \frac{1}{e} \partial_\mu \alpha(x)$
 α je odvisen od x (lokalna transf.) $\vec{E} \rightarrow \vec{E}$, $\vec{B} \rightarrow \vec{B}$

$$\mathcal{L} = \bar{\Psi}(i\gamma^\mu D_\mu - m)\Psi; D_\mu = \partial_\mu + ieA_\mu$$

[Vaja: [Proof: $D_\mu \Psi \rightarrow e^{i\alpha(x)} D_\mu \Psi$]]

$$\begin{aligned} D_\mu \Psi &= (\partial_\mu + ieA_\mu)\Psi \rightarrow (\partial_\mu + ieA_\mu - i(\partial_\mu \alpha(x))) (e^{i\alpha(x)} \Psi(x)) = \\ &= \Psi(x) e^{i\alpha(x)} i \partial_\mu \alpha(x) + e^{i\alpha(x)} \partial_\mu \Psi(x) - ieA_\mu e^{i\alpha(x)} \Psi(x) = \\ &= e^{i\alpha(x)} (\partial_\mu + ieA_\mu) \Psi(x) = e^{i\alpha(x)} D_\mu \Psi \end{aligned}$$

Preverimo še, da velja: $\mathcal{L} \rightarrow \Psi^\dagger e^{i\alpha(x)} \gamma^0 (i\gamma^\mu e^{i\alpha} D_\mu \Psi - m e^{i\alpha} \Psi) =$
 $= \bar{\Psi} (iD_\mu \Psi - m\Psi) = \bar{\Psi} (iD_\mu - m)\Psi$

$$\mathcal{L}_{\text{QED}} = \bar{\Psi}(i\gamma^\mu D_\mu - m)\Psi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$

QED Lagrangian

V QCD je razlika ta, da bi pisali:

$$g_s \bar{q}_i(x) \gamma^\mu q_j(x) G_\mu^a(x)$$

$i=1,2,3$

$\downarrow$
 T_{ij}^a

in imamo dodatne simetrije
zaradi barve.

Interakcija med velostji in skalarji

Invariantne simetrije:

$$\phi \rightarrow e^{i\alpha(x)} \phi, V^\mu \rightarrow V^\mu - \frac{1}{g} \partial^\mu \alpha$$

$$\mathcal{L} = \partial_\mu \phi \partial^\mu \phi^\dagger - m^2 \phi \phi^\dagger \text{ ni inv.}$$

$$\hookrightarrow \mathcal{L} = D_\mu \phi D^\mu \phi^\dagger - m^2 \phi^\dagger \phi = !$$

$$= (\partial_\mu \phi + ig V_\mu \phi) (\partial^\mu \phi^\dagger - ig V^{\mu\dagger} \phi^\dagger) - m^2 \phi^\dagger \phi$$

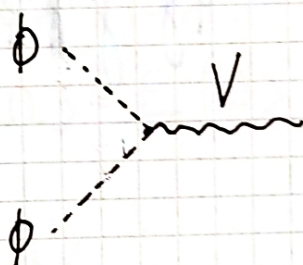
$$D^\mu \phi \rightarrow e^{i\alpha} D^\mu \phi$$

$$\partial^\mu \rightarrow D^\mu = \partial^\mu + ig V^\mu$$

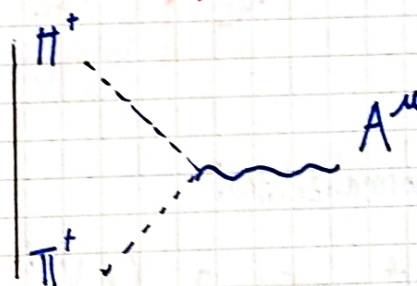
The authenticity of this book is guaranteed by the author.

To pa je.

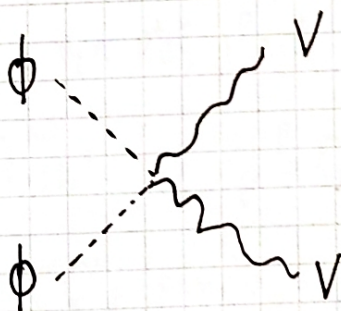
$$\mathcal{L}_i: g V_\mu \phi \partial^\mu \phi^\dagger$$



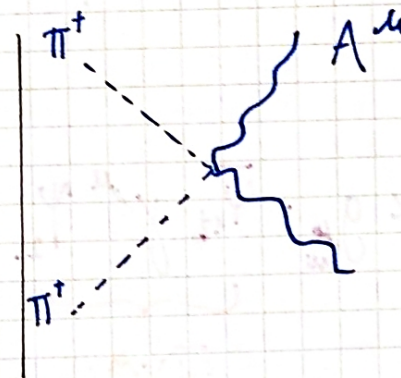
(npr. interakcija piona z em poljem)



$$: g^2 V_\mu V^{\mu\dagger} \phi \phi^\dagger$$



(npr. kot prej ali pa interakcija z Higgsovim poljem)



Povzetek:

QM: $V(\vec{r})$ poljuben

QFT: Načela in obdelali smo vse pare interakcij, ki so realizirane v naravi.

$$\mathcal{L}_i \propto \phi(x)^4$$

$$\mathcal{L}_i \propto A_\mu A^\mu x, W_\mu W^\mu x$$

$$V^\mu \rightarrow V^\mu - \frac{1}{g} \partial^\mu \alpha$$

T Vaja: [4 fermionska interakcija]

$$\mathcal{L}_i = G_F \bar{\Psi}_1 \gamma^\mu (1 - \gamma_5) \Psi_2 \bar{\Psi}_3 \gamma_\mu (1 - \gamma_5) \Psi_4$$

$$\propto \bar{\Psi}_{1L} \gamma^\mu \Psi_{2L}$$

$$\bar{\Psi} \gamma^\mu \Psi : 1 = \gamma^0$$

$$\bar{\Psi} \gamma^\mu \gamma^5 \Psi : 1 = \gamma^0$$

Izkaže se, da je to efektivna nizko-energijska teorija.

Ali je renormalizabilno?

$$eV^4 = [G_F] \cdot (eV^{3/2})^4$$

$$\text{ampak } [G_F] = eV^{4-6} = eV^{-2} \Rightarrow \text{ni normalizabilna}$$

Pravilna je:

$$\mathcal{L}_i \propto g_w \cdot \bar{\Psi}_{1L} \gamma^\mu \Psi_{2L} W_\mu$$

$$\bar{\Psi}_1 \gamma^\mu (1 - \gamma_5) \Psi_2$$

$$eV^4 = [g_w] eV^3 \cdot eV \Rightarrow [g_w] = eV^0 \text{ to je ok!}$$

Intermezzo: Masni člen fermionov:

$$\mathcal{L} : m \bar{\Psi} \Psi = m (\bar{\Psi}_L \Psi_R + \bar{\Psi}_R \Psi_L)$$

Levorotacijske lahko ~~ne~~ vrtimo, desnorotacijske pa ne!

$$\checkmark \text{ Šibke simetrije: } \Psi_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \rightarrow e^{i\vec{\alpha}(x) \cdot \vec{\tau}} \Psi_L$$

$$g_w (\bar{\nu}_e e^\dagger)_L \gamma^\mu \tau^+ W_\mu \begin{pmatrix} \nu \\ e \end{pmatrix}_L$$

Zato napišemo par, ki se vrti preko Higgsovega mehanizma:

$$\mathcal{L} : \begin{matrix} \phi^\dagger & \bar{\Psi}_L & \Psi_R \\ & \bar{\Psi}_L & \Psi_R \end{matrix}$$

Motivacija za izračun korelacijskih funkcij C :

1.) Izluščimo lahko lastne energije E_n

$$\hat{H} = \hat{H}_0 + \hat{H}_1; \quad \hat{H}|n\rangle = E_n|n\rangle$$

$$H_0|0\rangle \propto |0\rangle \rightarrow \text{neintegrirajoči ne deli}$$

$$\hat{H}|\Omega\rangle = E_0|\Omega\rangle$$

$$C(t) = \langle \Omega | \hat{\mathcal{O}}(t) \hat{\mathcal{O}}^\dagger(0) | \Omega \rangle$$

Primeri $\mathcal{O}$:

- $\phi(x)$
- $\bar{\Psi} \gamma^\mu \Psi$: npr. e^+e^- , $j^P = 1^-$
- $\bar{\Psi}_d \gamma_5 \Psi_u$: $j^P = 0^-$

- kombinacija $\Psi_u \Psi_u \Psi_d$ (proton): $j^P = \frac{1}{2}^+$

Torej:

$$\rightarrow e^{iHt} \hat{\mathcal{O}}(0) e^{-iHt}$$

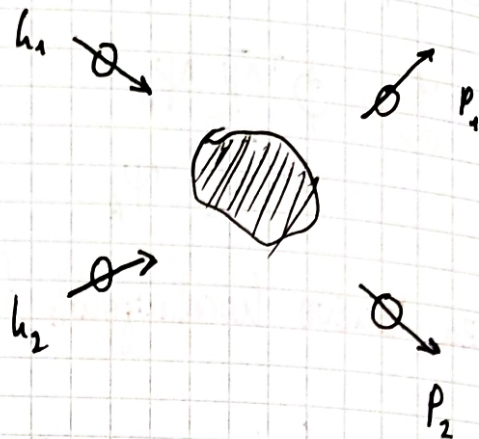
$$C(t) = \sum_n \langle \Omega | \hat{\mathcal{O}}(t) | n \rangle \langle n | \hat{\mathcal{O}}(0) | \Omega \rangle =$$

$$= \sum_n \langle \Omega | e^{i\mathcal{O}t} \hat{\mathcal{O}}(0) e^{-iE_n t} | n \rangle \langle n | \hat{\mathcal{O}}(0) | \Omega \rangle =$$

$$= \sum_n |\langle \Omega | \hat{\mathcal{O}}(0) | n \rangle|^2 e^{-iE_n t}$$

$$C(t) \rightarrow E_n$$

2.) Siplni preseki σ in Siplne matrike S, U



Interakcijska slika (Diracova slika)

$$\phi_i(t) = e^{iH_0(t-t_0)} \phi_i(t_0) e^{-iH_0(t-t_0)}$$

$$|\Psi_i(t)\rangle = e^{iH_0(t-t_0)} |\Psi_s(t)\rangle$$

$$= e^{\frac{iH_0(t-t_0) - iH(t-t_0)}{e}} |\Psi_i(t_0)\rangle$$

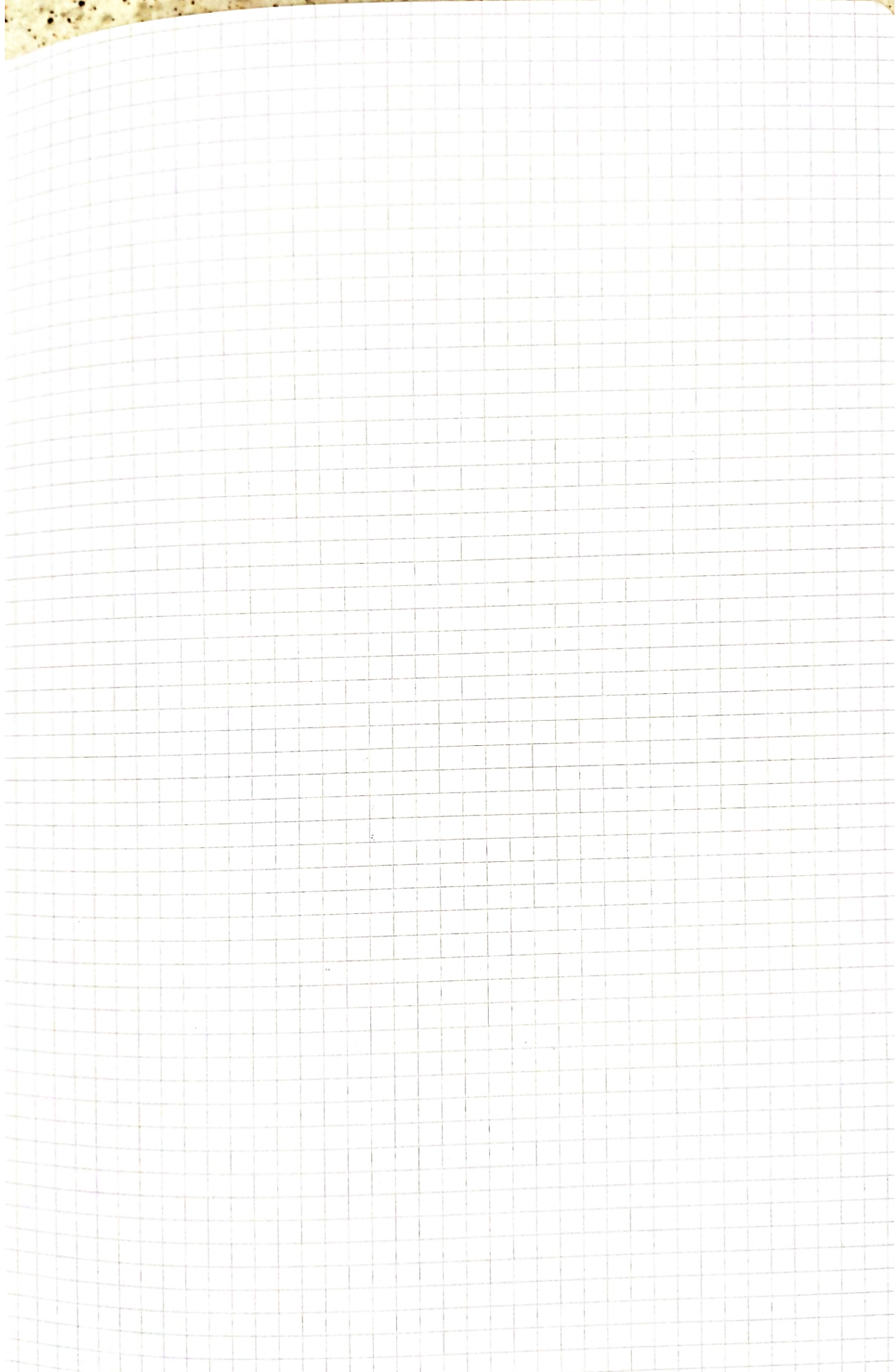
npr.

$$\phi_i = \int \frac{d\vec{p}}{c} \frac{1}{\sqrt{c}} \left(a_{\vec{p}} e^{-ipx} + b_{\vec{p}}^\dagger e^{ipx} \right)$$

Se žc transformira/razvija talno.

$U(t, t_0)$: Operator časovne evolucije

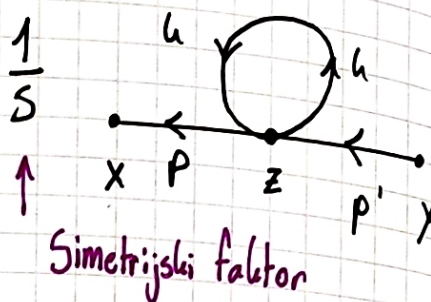
{ Želimo izraziti korelacijske funkcije z polji v ~~korelacijski~~ interakcijski sliki.



Feynmanova pravila v momentnem prostoru

$$C_2^*(x-y) = -i\lambda \int d^4z \cdot D(x-z) D(z-z) D(z-y) \frac{1}{S}$$

↑
Propagator iz z v x



$$D(x-z) = \langle 0 | T \{ \phi(x) \phi^\dagger(z) \} | 0 \rangle = \int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2 - m^2 + i\epsilon} e^{-ip(x-z)}$$

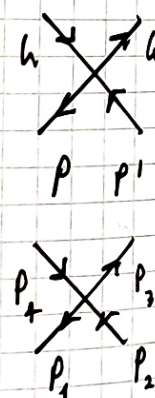
Torej:

$$C_2(x-y) = -\frac{i\lambda}{S} \int d^4z \int \frac{d^4p}{(2\pi)^4} \frac{i e^{ip(x-z)}}{p^2 - m^2 + i\epsilon} \int \frac{d^4k}{(2\pi)^4} \frac{i e^{-ik(z-z)}}{k^2 - m^2 + i\epsilon} \int \frac{d^4p'}{(2\pi)^4} \frac{i e^{-ip'(z-y)}}{p'^2 - m^2 + i\epsilon}$$

$$\lim_{T \rightarrow \infty} \int_{-T}^T \int_{-\infty}^{\infty} d^4z e^{iz(p-p'+k-k)} = (2\pi)^4 \delta(p-p'+k-k)$$

↳ To ignoriramo
for now (*)

$$(2\pi)^4 \delta(p_1 + p_3 - p_2 - p_4) \\ (2\pi)^4 \delta(\sum p_i) \rightarrow$$



$\int d^4z \rightarrow$ Ohranitev četverca gibalne
količine v vsakem vozlišču

$$C_2(x, y) = \int \frac{d^4p}{(2\pi)^4} C_2^{\text{mom.}}(p, x, y)$$

$$C_2^{\text{mom.}}(p, x, y) = (-i\lambda) \left(\frac{1}{S} \right) \int \frac{d^4k}{(2\pi)^4} \left(\frac{i}{p^2 - m^2 + i\epsilon} \right) \frac{i}{k^2 - m^2 + i\epsilon} e^{-ipx} e^{ipy}$$

Prepiši pravila:

↓

(*) Povezava med $i\epsilon$ in časovno urejenostjo

Slides 02. peshin p. 95

$$Z_0' = (1+i\epsilon) Z_0$$

$$Z_0' : -\infty (1-i\epsilon)(1+i\epsilon) = -\infty (1+\mathcal{O}(\epsilon^2)) = -\infty \Rightarrow \text{Delta funkcija}$$

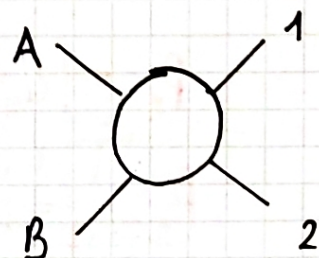
A je $p^0 = \text{Re}(p)(1+i\epsilon)$ upravičen?

Verjetnosti za procese S-matrike, 2

Radi bi korelacijske funkcije povezali z S-matrikami in preseki.

i) Verjetnost za proces

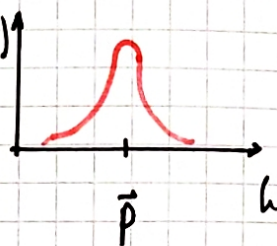
Če hočemo govoriti o verjetnostih je pomembno, da so začetna stanja normirana.



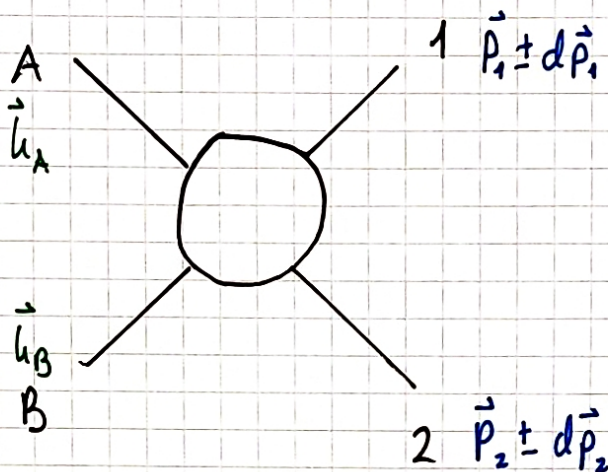
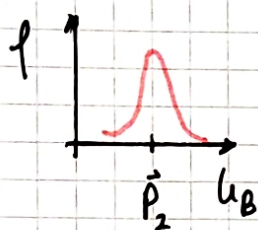
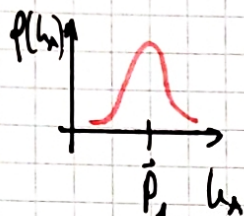
Recimo:

Ravni valovi: $\langle \vec{u} | \vec{u} \rangle \propto \delta(0)$ // Ni normirano

Valovni paketi: $|\phi\rangle = \int \frac{d\vec{k}}{(2\pi)^3} \frac{1}{\sqrt{2E_k}} \phi(\vec{k}) |\vec{k}\rangle$



$$\langle \phi | \phi \rangle = 1$$



$$|\phi_A \phi_B\rangle_{in}$$

$$|\vec{p}_1 \vec{p}_2\rangle_{out}$$

$|\phi_A \phi_B\rangle_{in}$: lastno stanje $\hat{H}$, ki

ob $t = -\infty$, Ustrezno ~~neinteragirajo~~ neinteragirajočima A in B

$|12\rangle_{out}$: lastno stanje $\hat{H}$, ki ob

$t = +\infty$ Ustrezno neinteragirajočima A in B

kjer:

$$\langle \phi_A \phi_B | \phi_A \phi_B \rangle_{in} = 1$$

•) Če bi bile $|n\rangle_{out}$ norm. $|\phi_A \phi_B\rangle_{in} = \sum_{n'} |n'\rangle_{out} \cdot c_n' / \cdot \langle n |_{out}$

Dobimo:

$$\langle n | \phi_A \phi_B \rangle_{in} = \sum_{n'} c_n' \underbrace{\langle n | n' \rangle_{out}}_{\delta_{nn'}} = c_n$$

$$P_n = |c_n|^2 = |\langle n | \phi_A \phi_B \rangle|^2$$

$$P_{n \rightarrow 12} = |\langle 12 | \phi_A \phi_B \rangle|^2$$

ii) Ampak ker niso lastna stanja niso normalizirana

$$dP_{AB \rightarrow 12} = |\langle 12 | \phi_A \phi_B \rangle|^2 \cdot d\vec{p}_1 d\vec{p}_2 \cdot C ; C = ?$$

$$\langle \phi_A \phi_B | \phi_A \phi_B \rangle = 1 = P(AB \rightarrow \text{kar koli}) = (*)$$

$$1 = \int \frac{d\vec{p}}{(2\pi)^3} \frac{1}{2E_p} |\vec{p} \times \vec{p}| + \int \frac{d\vec{p}_1}{(2\pi)^3} \frac{1}{2E_1} \int \frac{d\vec{p}_2}{(2\pi)^3} \frac{1}{2E_2} |\vec{p}_1 \vec{p}_2 \rangle_{\text{out}} \langle \vec{p}_1 \vec{p}_2 | + \dots$$

3 delena
+ delene
etc.

Identiteta
lastnih
stanj

Vstavimo to identiteto: (ker nas zanima 2 delena smo dali 2 delena del)

$$(*) = \int \frac{d\vec{p}_1}{(2\pi)^3 2E_1} \int \frac{d\vec{p}_2}{(2\pi)^3 2E_2} \langle \phi_A \phi_B | \vec{p}_1 \vec{p}_2 \rangle \langle \vec{p}_1 \vec{p}_2 | \phi_A \phi_B \rangle + \dots$$

$$dP_{AB \rightarrow 12} = \left\{ \prod_{f=1,2} \frac{d\vec{p}_f}{(2\pi)^3 2E_f} \right\} |\text{out} \langle \vec{p}_1 \vec{p}_2 | \phi_A \phi_B \rangle|^2$$

Velikost faz. pr. Matricni element

ii) S-matrika, T-matrika

$$\text{out} \langle \vec{p}_1 \vec{p}_2 | \vec{u}_A \vec{u}_B \rangle_{\text{in}} = S_{fi} = \langle \vec{p}_1 \vec{p}_2 | e^{-i2TH} | \vec{u}_A \vec{u}_B \rangle = \hat{S}$$

Matricni element
S-matrike $T = +\infty$ $T \rightarrow -\infty$

$$= \langle \vec{p}_1 \vec{p}_2 | U(T, -T) | \vec{u}_A \vec{u}_B \rangle = \hat{S}$$

$\uparrow$ V interakcijski sliki

$$\hat{S} = e^{-i2TH}, \quad \hat{S} \hat{S}^\dagger = e^{-i2TH} e^{i2TH} = I ; \text{ unitaren operator}$$

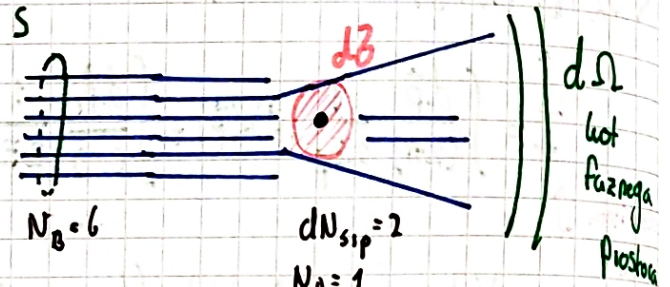
$$\hat{S} = I + i\hat{T} ; \hat{T} ; \text{ Operator, } T_{fi}$$

$$\hat{S} \hat{S}^\dagger = I = (I + i\hat{T})(I - i\hat{T}^\dagger) = I + i(\hat{T} - \hat{T}^\dagger) + \hat{T} \hat{T}^\dagger ;$$

$$\Rightarrow i(\hat{T} - \hat{T}^\dagger) = -\hat{T} \hat{T}^\dagger \quad \text{Optični teorem}$$

$$\langle \vec{p}_1 \vec{p}_2 | i\hat{T} | \vec{h}_A \vec{h}_B \rangle = (2\pi)^4 \delta^4(p_1 + p_2 - h_A - h_B) i\mathcal{M}(h_A h_B \rightarrow p_1 p_2)$$

iii) Sipalni preseki σ
(Strnad 4):

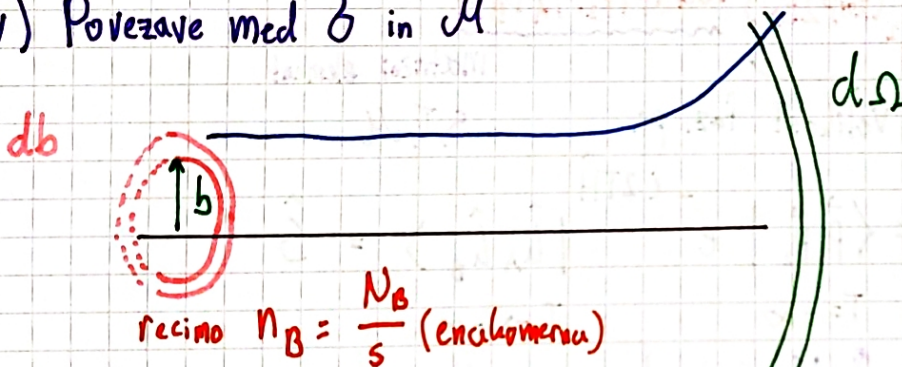


$$N_A = 1 \quad \frac{d\sigma}{S} = \frac{dN_{sip}}{N_B} \quad dN_{sip} = \frac{d\sigma}{S} N_B$$

OZ:

$$dN_{sip} = \frac{d\sigma}{S} N_B N_A \quad \frac{dN_{sip}}{t} = d\sigma \cdot \frac{N_A N_B}{S \cdot t}$$

iv) Povezave med σ in $\mathcal{L}$



Luminosity =
luminoznost ($\mathcal{L}$)

$$\mathcal{L} = 10^{34} \text{ 1/cm}^2 \cdot \text{s}$$

$$\int \mathcal{L}(t) dt \approx 150 \frac{1}{fb}$$

$$b = 10^{-28} \text{ m}^2$$

$d^2\sigma(b)$: preseki, ki ustrezajo kolobarju $2\pi b db$ v $d\Omega$

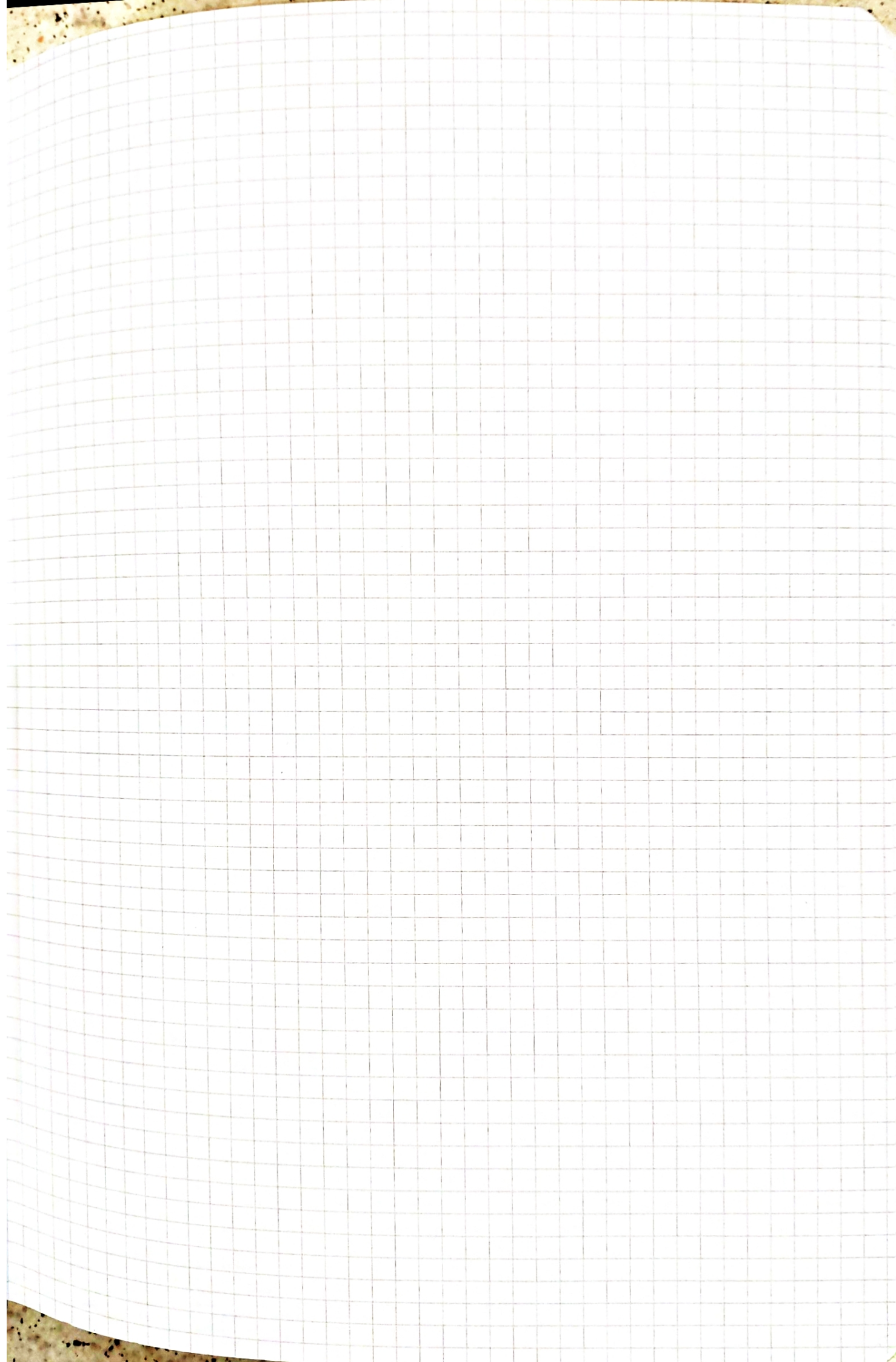
$$d^2\sigma(b) = \frac{d^2 N_{sip} S}{N_B N_A} = \frac{d^2 N_{sip}}{n_B}$$

Za $N_A = 1, N_B = 1 \rightarrow dN_{sip} = dP_{AB \rightarrow 12}$ ravno verjetnost

Za $N_A = 1, N_B = 2\pi b db n_B \rightarrow dN_{sip} = dP_{AB \rightarrow 12} dN_B$

$$d^2\sigma = \frac{dP_{AB \rightarrow 12}(b) 2\pi b db n_B}{n_B}$$

$$\Rightarrow d^2\sigma = dP_{AB \rightarrow 12} 2\pi b db$$



Popravek:

$$\overline{\phi_I(z)|p\rangle} \equiv \phi_I^-|p\rangle$$

$$\langle p|\phi_I(z) \equiv \langle p|\phi_I^+$$

Ponovitev:

$\times \uparrow^f; iM = -i\lambda$

Feyn. pr. za iM

$\times -i\lambda$

$\frac{1}{5}$

$\frac{i}{p^2 - m^2 + i\epsilon}$

Simetrijski faktor 1

Rešitev DN:

$$\frac{d\sigma}{d\Omega} = \frac{|\hat{P}_1|}{16\pi^2 E_{cms}} |M|^2 \frac{1}{2E_A E_B |v_A - v_B|} = \frac{P_1}{16\pi^2 2E_1} \lambda^2 \frac{1}{4E_1^2 \frac{2P_1}{E_1}} =$$

$E_A = E_B = E_1$

$$= \frac{\lambda^2}{64\pi^2 (2E_1)^2}$$

$N_A \rightarrow v_B$

$|v_A - v_B| = 2v_A = \frac{2P_1}{E_1}$

$E_{cms} = 2E_1$

$$\sigma = \int \frac{d\sigma}{d\Omega} d\Omega = \int_0^{\pi/2} \sin\theta d\theta \int_0^{2\pi} d\phi \frac{\lambda^2}{64\pi^2 (2E_1)^2}$$

$$= \frac{4\pi}{2} \frac{\lambda^2}{64\pi^2 (2E_1)^2}$$

Ker 2 identična delca

Wickov teorem za fermione

$$N(a_p a_q^\dagger a_l) = -a_q^\dagger a_p a_l = +a_q^\dagger a_l a_p$$

$$T(\hat{\psi}(x_1) \hat{\psi}(x_2) \hat{\bar{\psi}}(x_3) \psi(x_4)) = X_2^0 > X_1^0 > X_4^0 > X_3^0$$

$$= (-1)^2 \psi(x_2) \psi(x_1) \psi(x_4) \bar{\psi}(x_3)$$

Torej za polja v interakcijski sili velja:

$$T(\hat{\psi}_{i_1}(x_1) \dots \hat{\psi}_{i_n}(x_n)) = N(\hat{\psi}_{i_1}(x_1) \dots \hat{\psi}_{i_n}(x_n) + \text{vse kontrakcije})$$

$$T(\psi_I^\alpha(x) \bar{\psi}_I^\beta(y)) = N(\psi_I^\alpha(x) \bar{\psi}_I^\beta(y)) + \overbrace{\psi_I^\alpha(x) \bar{\psi}_I^\beta(y)}$$

hjer je:

$$\overbrace{\Psi_I^\alpha(x) \bar{\Psi}_I^\beta(y)} = S^{\alpha\beta}(x-y) = \begin{cases} \langle 0 | \Psi_I^\alpha(x) \bar{\Psi}_I^\beta(y) | 0 \rangle; & x^0 > y^0 \\ -\langle 0 | \bar{\Psi}_I^\beta(y) \Psi_I^\alpha(x) | 0 \rangle; & y^0 > x^0 \end{cases}$$

$$= \int \frac{d^4 p}{(2\pi)^4} \frac{i(p+m)^{\alpha\beta}}{p^2 - m^2 + i\epsilon} e^{-ip(x-y)}$$

Vaja:

Vežemo lahko tako par ψ in $\bar{\psi}$

$$T(\Psi_I(x_1) \Psi_I(x_2) \bar{\Psi}_I(x_3) \bar{\Psi}_I(x_4)) =$$

$$= T(1 \ 2 \ \bar{3} \ \bar{4}) =$$

$$= N(1 \ 2 \ \bar{3} \ \bar{4} + 1 \ 2 \ \bar{3} \ \bar{4} + 1 \ 2 \ \bar{3} \ \bar{4} + 1 \ 2 \ \bar{3} \ \bar{4} + 1 \ 2 \ \bar{3} \ \bar{4} + 1 \ 2 \ \bar{3} \ \bar{4})$$

$$N(\Psi_I^\alpha(x_1) \Psi_I^\beta(x_2) \bar{\Psi}_I^\gamma(x_3) \bar{\Psi}_I^\delta(x_4)) =$$

$$= -S_F(x_1 - x_3)^{\alpha\gamma} N(\hat{\Psi}_I^\beta(x_2) \hat{\Psi}_I^\delta(x_4))$$

Razpisano ena tu kontrola

Yukavina teorija za interakcijo med fermioni in skalarki (ϕ)

$$\mathcal{H}(x) = \mathcal{H}_0^\psi(x) + \mathcal{H}^\phi(x) + g \bar{\Psi}(x) \Psi(x) \phi(x)$$

Vaje: $\mathcal{H}$ je $f(p, s) + f(l, r) \rightarrow f(p', s') + f(l', r')$

(merimo, da nista enaki)

Ampak prej če sta enaka:

$$\langle f | i T | i \rangle = \langle f | T \exp(-ig \int d^4 z \bar{\Psi}_2 \Psi_2 \phi_2) | i \rangle =$$

časovna ureduje

$$= (-ig)^2 \frac{1}{2!} \int d^4x \int d^4y \underbrace{\langle f | T(\bar{\psi}_x \psi_x \phi_x \bar{\psi}_y \psi_y \phi_y) | i \rangle}_{(*)}$$

$\psi: a, b^\dagger$

$\bar{\psi}: a^\dagger, b$

$$(*) = \underbrace{\langle f(u, r') | \langle f(p', s') | N(\bar{\psi}_x^\alpha \psi_x^\alpha \phi_x \bar{\psi}_y^\beta \psi_y^\beta \phi_y) | f(p, s) \rangle}_0 \underbrace{| f(u, r) \rangle}_0 \cdot \underbrace{f^{(1)}}_2$$

Zamenjamo $x \leftrightarrow y$

V Zamenjavi ni slučajev, kar pomeni, da je edina možnost, da jih skontahiramo. Zdaj moramo še narediti to kontrakcijo.

remind: $\bar{\psi} = \psi \gamma_0$

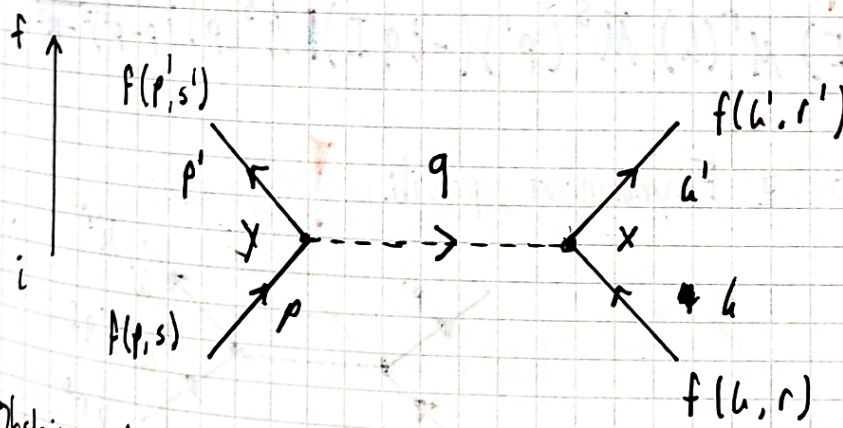
$$\overline{\psi^\alpha} |p, s\rangle \equiv \psi^{\alpha-}(x) |p, s\rangle = \int \frac{d^3q}{(2\pi)^3 2E_q} \sum_r u_r^\alpha(q) a_q^\dagger e^{-iqx} \sqrt{2E_p} a_{\vec{p}}^{s\dagger} |0\rangle =$$

$$= \int \frac{d^3q}{(2\pi)^3 2E_q} \sum_r u_r^\alpha(q) (2\pi)^3 \delta(\vec{p}-\vec{q}) \delta_{rs} e^{-iqx} \sqrt{2E_p} |0\rangle$$

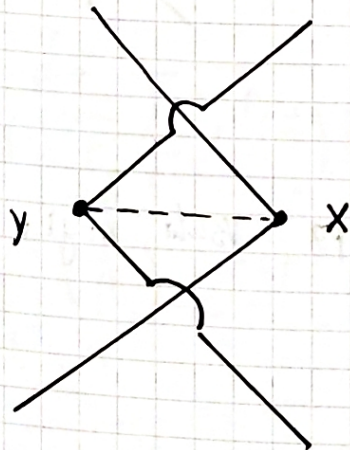
$$= e^{-ipx} u_s^\alpha(\vec{p}) |0\rangle$$

$e^{-ipx} u_s(\vec{p}) = \overline{\psi^\alpha}(x) |p, s\rangle$

$e^{ipx} \bar{u}_s^\alpha(\vec{p}) \langle p, s | \overline{\psi^\alpha}(x) = \langle p, s | \bar{\psi}^{\alpha\dagger}(x)$



Obstaja tudi možnost če zamenjamo $x \leftrightarrow y$ od tod $f^{(1)}$



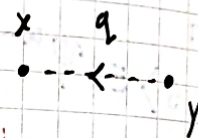
$$\langle f | i T | i \rangle^{(1)} = (-iq)^2 \frac{1}{2!} \int d^4x \int d^4y \underbrace{\bar{M}_\alpha^{r'}(h')}_{e^{i h' x}} \underbrace{M_\alpha^r(h)}_{e^{-i h x}} \underbrace{\bar{M}_\beta^{s'}(p')}_{e^{i p' y}} \underbrace{M_\beta^s(p)}_{e^{-i p y}}$$

Predznak porzno
2 antikomutacije
(Wickovik kontrakcija)

$$= \int \frac{d^4q}{(2\pi)^4} \frac{c e^{-iq(x-y)}}{q^2 - m^2 + i\epsilon}$$

$$= \int \frac{d^4q}{(2\pi)^4} \underbrace{\int d^4x e^{-ix(h-h'+q)}}_{(2\pi)^4 \delta^4(h-h'+q)} \underbrace{\int d^4y e^{-iy(p-p'-q)}}_{(2\pi)^4 \delta^4(p-p'-q)} = (2\pi)^4 \delta^4(h-h'+p-p')$$

Ohranitev momenta



Določimo še predznaki:

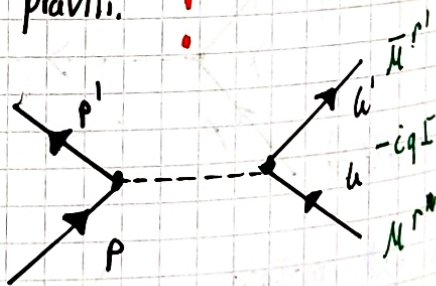
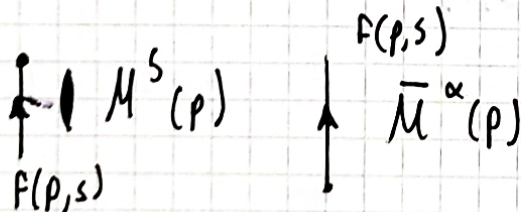
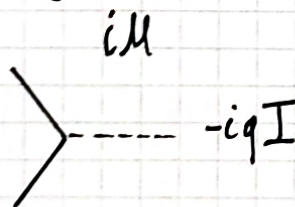
$$P_w^{(1)} : - \langle f(h') | \bar{\Psi}_\alpha \langle f(p) | \Psi_\alpha \bar{\Psi}_\beta \Psi_\beta | f(p) \rangle | f(h) \rangle$$

je podobna tabeli

Zapišimo zdaj bipolarno amplitudo:

$$iM^{(1)} = P_w^{(1)} \frac{1}{s} \bar{M}^{r'}(h) \underbrace{(-iqI)}_{\text{Identity}} M^r(h) \bar{M}^{s'}(p') \underbrace{(-iqI)}_{\text{Identity}} M^s(p) \frac{i}{(p-p')^2 - m^2}$$

Do tega lahko tudi pridemo z Feynmanovi pravili. !



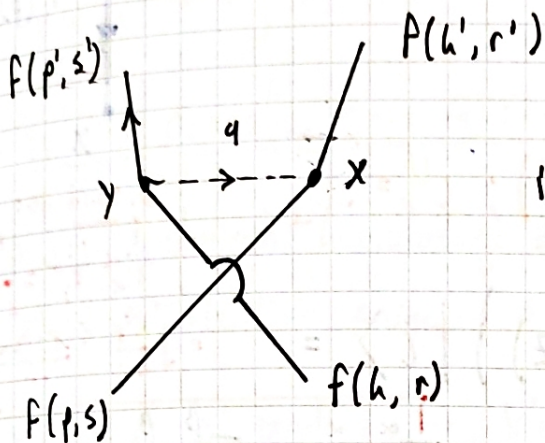
polke puščice
označijo fermione
tola

Gremo v obratni smeri fermionskega tola po vsaki nogi:

$$\bar{M}^{r'}(h) (-iqI) M^r(h)$$

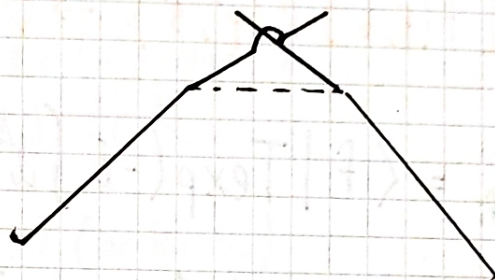
Še ena možnost za (*):

$$(*) = \langle f(k') | \langle f(p') | N(\bar{\psi}_x^\alpha \psi_x^\alpha \phi_x \bar{\psi}_y^\beta \psi_y^\beta \phi_y) | f(p) \rangle | f(k) \rangle \cdot f^{(2)}$$



$$k = q + p'$$

$$q = k - p'$$



$$\Rightarrow iM^{(2)} = p_w^0 \bar{M}_\alpha^{r'}(k') (-iqI) M_\alpha^s(p) \bar{M}_\beta^{s'}(p') (-iqI) M_\beta^r(k) \frac{i \cdot 1/5^{(2)}}{(k-p')^2 - m_s^2 + i\epsilon}$$

-1

(pogledaj sem, Amal
ker f₂ obravnava)

$$\frac{1}{5} = \frac{1}{N!} \text{ (f)}$$

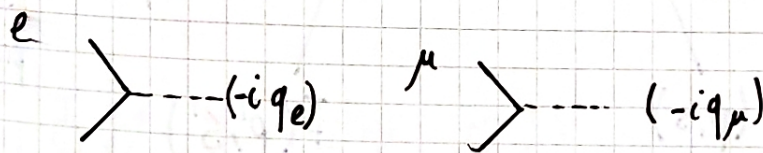
→ pride N! če so f enaki medu

če je red interakcije

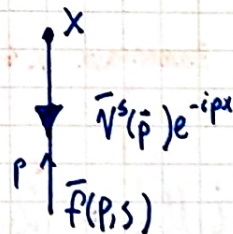
$$\mathcal{O}(g^N)$$

DU:

$$\mathcal{H}_i = g_e \bar{\psi}_e \psi_e \phi + g_\mu \bar{\psi}_\mu \psi_\mu \phi \quad e^- \mu^- \rightarrow e^- \mu^-$$



$$\langle \bar{\psi}(x) | \bar{f}(p, s) \rangle = \int \frac{d\vec{q}}{(2\pi)^3 E_q} \sum_r \bar{v}^r(\vec{q}) b_{\vec{q}}^r e^{-iqx} \sqrt{2E_p} b_{\vec{p}}^{s\dagger} |0\rangle =$$

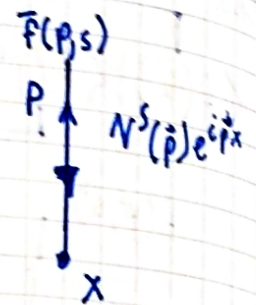


$$= \bar{v}^s(\vec{p}) e^{-ipx} |0\rangle = \bar{\psi}(x) | \bar{f}(p, s) \rangle$$

$$\psi = u a e^{-ipx}, \quad \bar{\psi} = \bar{u} a^\dagger e^{ipx}$$

$$\psi = v b e^{-ipx}, \quad \bar{\psi} = \bar{v} b^\dagger e^{ipx}$$

$$\langle \bar{\Psi}(p, s) | \Psi(x) \rangle \equiv \langle \bar{\Psi}(p, s) | \Psi^\dagger(x) = v^s(\vec{p}) e^{ipx} \langle 0 |$$



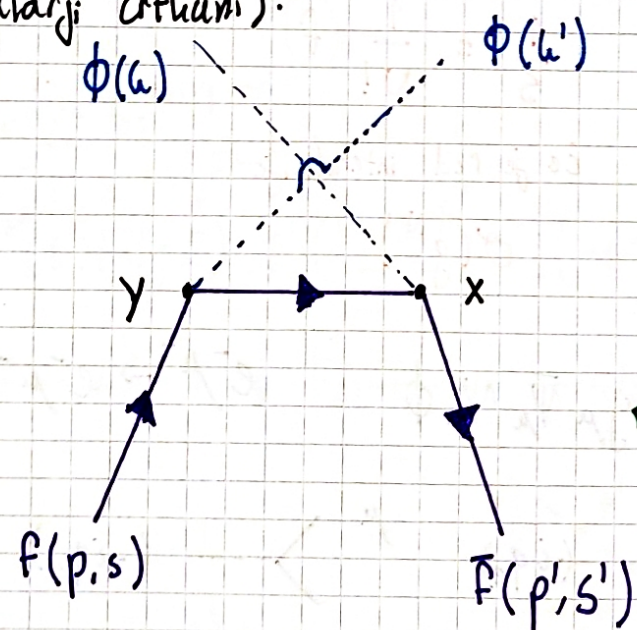
Vaja [$f(p, s) \bar{f}(p', s') \rightarrow \phi(k) \phi(k')$; $\mathcal{M} = ?$]

Spomni se:
 $\Psi_x^\alpha \bar{\Psi}_y^\beta = \delta_{xy} \delta_{\alpha\beta}$
 $= \int \frac{d^4 p}{(2\pi)^4} e^{ip(x-y)} e^{-ip(x-y)}$

$$i T_{fi} = \langle f | T \exp(-i \int d^4 z \mathcal{H}_I(z)) | i \rangle =$$

$$\stackrel{x \leftrightarrow y}{=} 2 \cdot \langle \phi(k) \phi(k') | \frac{1}{2!} (-ig)^2 \int d^4 x d^4 y \underbrace{\bar{\Psi}_x^\alpha \Psi_x^\alpha \phi_x}_{T(\dots)} \underbrace{\bar{\Psi}_y^\beta \Psi_y^\beta \phi_y}_{T(\dots)} | f(p, s) \bar{f}(p', s') \rangle$$

Narišimo diagram (skalarni črtni):



Spomni se:
 $\frac{i(\not{q} + m)}{q^2 - m^2 + i\epsilon}$

proti fermionskemu
 toku za Feynmanova
 pravila

Zamenjaj $x \leftrightarrow y$

$$i \mathcal{M}_{fi} = 2 \cdot \frac{1}{2!} (-ig)^2 \underbrace{\bar{v}_\alpha^{s'}(\vec{p}')}_{\text{Vozlišča}} \frac{i(\not{q} + m)_{\alpha\beta}}{q^2 - m^2 + i\epsilon} u_\beta^s(\vec{p}) (+1)$$

$$\stackrel{s=1}{=} \frac{1}{5} \underbrace{\bar{v}^{s'}(\vec{p}')}_{\bar{f}(p', s')} \underbrace{(-igI)}_{\text{Vozlišča}} \underbrace{\frac{i(\not{q} + m)}{q^2 - m^2 + i\epsilon}}_{\text{Propagator}} \underbrace{(-igI)}_{\text{Vozlišča}} \underbrace{u^s(\vec{p})}_{f(p, s)} P_\omega$$

$$\frac{d}{dx} [f\bar{f} \rightarrow f\bar{f}]$$

$\phi\phi \rightarrow \phi\phi$ $\mathcal{O}(g^4)$ v tej teoriji ne ϕ^4

Teorem, ki splošno povezuje spin in statistiko (Spin statistics theorem)

Pravi, da moramo postulirati komutatorje za delce z celim spinom in antikomutatorje z polštevilčnim spinom $s = (2n + 1/2) \quad n \in \mathbb{N}$.

Za poljuben spin je to izpeljano v Weinberg, 5.7 (ne-trivialno)

Srednichi, 4.4 (za spin 0) $\rightarrow$ ma tudi obratno metriko

Argument: $[H(x), H(y)] = 0$ za $(x-y)^2 < 0 \leftarrow$ kausalnost

$[H_i(x), H_i(y)] = 0$ za $(x-y)^2 < 0 \leftarrow T_{fi}$ da je to Lorentzovo invariantno.

$$\langle f | T(\exp(-i \int \mathcal{H}_i(z))) | i \rangle$$

časovno urejanje ni Lorentzovo invariantno
Za dogodek krajnjega tipa

$\rightarrow$ da imamo invarianco zahtevamo isto prejš.

torij:

$$\mathcal{H}_i = f(\psi, \psi^\dagger; \phi, \phi^\dagger, \dots) = f(\psi_{ab}, \psi_{ab}^\dagger)$$

$$[\psi_{ab}(x), \psi_{ab}(y)]_{\mp} = 0,$$

$$[\psi_{ab}(x), \psi_{ab}^\dagger(y)]_{\mp} = 0$$

Zgornji predznaki: komutator

Spodnji predznaki: anti-komutator

To pa dosežemo tako, da imamo:

$$\Psi_{ab} = \int \frac{d\vec{p}}{(2\pi)^3} [\mathcal{X} a^\# + \lambda b^\#]$$

$$|\mathcal{X}|^2 = |\lambda|^2 = 1$$

$$- \frac{1}{4} (-1)^{2j} \leftarrow \text{Spin pri Weinbergu}$$

$$j = 0 \rightarrow \oplus$$

$$j = \frac{1}{2} \rightarrow \oplus, \ominus$$

Kvantna elektrodinamika (Q.E.D.)

Zapišimo Hamiltonian:

$$e = -e_0$$

$$\mathcal{L}_{\text{QED}} = \bar{\Psi} (i \gamma^\mu (\partial_\mu + i e A_\mu) - m) \Psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$-\mathcal{L}_i = \mathcal{H}_i = e \bar{\Psi} \gamma^\mu \Psi A_\mu$$

Tu se veliko govori o umeritveni teoriji:

$$\Psi(x) \rightarrow e^{i\alpha(x)} \Psi(x)$$

$$A_\mu(x) \rightarrow A_\mu(x) - \frac{1}{e} \partial_\mu \alpha(x)$$

: Oboje da isti $\vec{E}, \vec{B}$

Elektromagnetno polje: A_μ

$$\text{E-L: } \partial_\mu F^{\mu\nu} = 0 \quad \text{dobimo M. eq.}$$

6. Ker imamo preveč prostostnih stopenj ureamo umeritev ("Gauge-Fixing"). Zelo popularna je Lorentz Gauge:

$$\partial_\mu A^\mu = 0$$

Tudi:

$$\partial_\mu (\partial^\mu A^\nu - \partial^\nu A^\mu) = 0 \rightarrow \partial_\mu \partial^\mu A^\nu = 0$$

$$A^\nu(x) = \epsilon_r^\nu e^{-ikx}, \quad (-ik)^\mu (-ik)_\mu \epsilon_r^\nu e^{-ikx} = 0$$

$k = (k, 0, 0, k) \rightarrow k^2 = 0$ za realne fotone
 $\hookrightarrow$ za virtualne to ne velja

Polarizacije imamo:

$$\epsilon_{r=0}^\nu = (1, 0, 0, 0), \quad \epsilon_{r=1}^\nu = (0, 1, 0, 0), \quad \epsilon_{r=2}^\nu = (0, 0, 1, 0), \quad \epsilon_{r=3}^\nu = (0, 0, 0, 1)$$

Skalarna polarizacija Transverzalna Transverzalna Longitudinalna

Kakšne polarizacije bo okay?

$$\partial_\mu A^\mu = 0 \rightarrow \epsilon_r^\mu (-i) k_\mu e^{-ikx} = 0 \rightarrow \epsilon_{r=1,2} \quad \text{in}$$

$$\epsilon_{r=S+L} = (1, 0, 0, 1)$$

Imamo 3 možne polarizacije, foton ima pa samo dve. Uporabimo umeritveno transformacijo:

$$A_\mu(x) \rightarrow A'_\mu(x) = A_\mu(x) - \frac{1}{e} \partial_\mu \alpha(x)$$

$$\epsilon_r^\mu e^{-ikx} \rightarrow \epsilon_r'^\mu e^{-ikx} = \epsilon_r^\mu e^{-ikx} - \frac{1}{e} C \cdot (-i) k_\mu e^{-ikx}$$

Problem je da sama umeritev ni Lorentz. inv.

Umeritvena transformacija: $\epsilon_\mu \xrightarrow{\text{u.t.}} \epsilon'_\mu = \epsilon_\mu + D k_\mu$

$$\partial_\mu A^\mu = 0 \rightarrow \epsilon_r^\mu (-i) k_\mu e^{-ikx} = 0 \rightarrow \epsilon_{r=1,2}$$

$$\nabla \cdot \vec{A} = 0$$

Coulombova umeritev

$$\epsilon_{r=S+L}^\mu \xrightarrow{\text{u.t.}} \epsilon_{r=S+L}^\mu + D k^\mu = 0$$

$\hookrightarrow D$ izberemo da bo to 0

Kvantizacija EM polja

$$\hat{A}_\mu(x), \hat{\Pi}_\nu(y); \quad \Pi_\nu \equiv \frac{\partial \mathcal{L}}{\partial \dot{A}_\nu}$$

↓ konjugiran moment

$$\Rightarrow \Pi_i = \frac{\partial \mathcal{L}}{\partial \dot{A}_i} \neq 0$$

$$\boxed{\Pi_0 = \frac{\partial \mathcal{L}}{\partial \dot{A}_0} = 0}$$

! To pa zelo
zakomplicirata stvari

Rigorozne kvantifikacije EM polja:

• Peskin 9.4

• Tong 6

Feynmanova umeritev (Lorentzova + ...)

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \rightarrow \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2\xi} (\partial_\mu A^\mu)^2; \quad \xi=1$$

Feynmanova umeritev.

Tu dobimo:

$$\Pi_0 = \frac{\partial \mathcal{L}}{\partial \dot{A}_0} \neq 0 \quad E-L: \partial_\mu \partial^\mu A_\nu = 0; \quad A_\nu = \epsilon_\nu^r e^{-ikx}$$

↑ To smo pa že rešili.

Zapišimo operator EM polja:

$$\partial_\mu \partial^\mu \hat{A}_\nu(x) = 0$$

$$\hat{A}_\nu(x) = \int \frac{d\vec{k}}{(2\pi)^3 \sqrt{2E_k}} \sum_{r=0, \dots, 3} \left[\epsilon_\nu^r(\vec{k}) e^{-ikx} a_{\vec{k}}^r + \epsilon_\nu^{r\dagger} e^{ikx} a_{\vec{k}}^{r\dagger} \right] = \hat{A}_\nu^\dagger(x)$$

!

Postuliramo še komutatorje:

$$[a_{\vec{k}}^r, a_{\vec{k}'}^{r'}] = (2\pi)^3 \delta(\vec{k} - \vec{k}') \delta^{rr'}$$

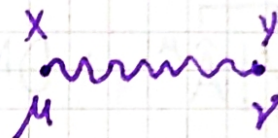
$$r = 1, 2, 3$$

$$r = 0 : \text{težave}$$

Zapišimo propagator:

$$D_{\mu\nu}(x-y) = \langle 0 | T(A_\mu(x) A_\nu(y)) | 0 \rangle = \overbrace{A_\mu(x) A_\nu(y)}$$

$$D_{\mu\nu}(x-y) = \int \frac{d^4 k}{(2\pi)^4} \frac{i \sum_{r=0}^3 \epsilon_\mu^r(k) \epsilon_\nu^{r*}(k)}{k^2 + i\epsilon} e^{-ik(x-y)} =$$



Spomni se, da je to lastni

propagatorju elektrona:

$$\sum_{s=1} \alpha^s(\vec{p}) \bar{\alpha}^s(\vec{p})$$

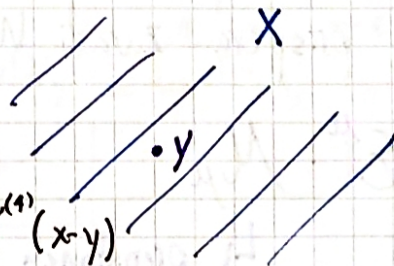
$$\sum_{r=0}^3 \epsilon_\mu^r \epsilon_\nu^{r*} = -g_{\mu\nu}$$

$$= \int \frac{d^4 k}{(2\pi)^4} \frac{-i g_{\mu\nu}}{k^2 + i\epsilon} e^{-ik(x-y)}$$

Ta propagator je tudi Greenova funkcija naših Maxwellovih enačb.

$$\partial_\alpha \partial^\alpha D_{\mu\nu}(x-y) \propto \delta^\alpha_\alpha(x-y)$$

$$\partial_\alpha \partial^\alpha D_{\mu\nu} \propto \int \frac{d^4 k}{(2\pi)^4} g_{\mu\nu} \frac{k^2}{k^2 + i\epsilon} e^{-ik(x-y)} \propto \delta^{(\mu\nu)}(x-y)$$



↳ Res je rešitev za točkast izvor.

Moramo še definirati metriko, kar običajno naredimo z zahtevo, da za lastna stanja velja:

$$\langle \Psi' | \partial_\mu A^\mu | \Psi \rangle = 0$$

$$|\Psi\rangle = |\Psi_T\rangle |\phi\rangle$$

$$(a_{\vec{R}}^{r=0} - a_{\vec{R}}^{r=3}) |\phi\rangle = 0$$

Kontrakcije z zunanjimi delci

$$\hat{A}^\mu(x) |\gamma(l, r)\rangle = A^\mu(x) |\gamma(l, r)\rangle =$$

$$= \int d^4k \sum_\mu \epsilon^\mu(k') a_{k'}^{r'} e^{-ik'x} \sqrt{2E} a_k^{r+} |0\rangle =$$

$$= \epsilon^\mu(k) e^{-ikx} |0\rangle$$

$$\langle \gamma | \hat{A}^\mu : DN$$

$$\begin{array}{c} \mu \\ \gamma(l, r) \end{array} \left\{ \begin{array}{c} \epsilon_r^{\mu*}(k) \\ \mu \end{array} \right\} \quad \begin{array}{c} \gamma(l, r) \\ \epsilon_r^\mu(k) \end{array} \quad \begin{array}{c} \epsilon_r^\mu(k) \\ \mu \end{array}$$

$$- \frac{i g_{\mu\nu}}{k^2 + i\epsilon}$$

Polarizacije zunanjih fotonov

V Lorentzovi umeritvi: $\partial_\mu A^\mu = 0$, $k_\mu \epsilon^\mu = 0$

Temu ustrezajo 3 različne:

$$\epsilon_{r=1,2}^T; \quad \epsilon^{SL} = (1, 0, 0, 1) \propto k^\mu$$

Trditve:

ϵ^{SL} ne prispeva zaradi Wordove identitete.

$$\mathcal{M} = \epsilon^\mu \cdot \mathcal{M}_\mu$$

Umer. inv: $\mathcal{M} \xrightarrow{u.t.} \mathcal{M}$

$\hookrightarrow$ preostane, ki je neodvisen od polarizacije ϵ^μ

$$\mathcal{M} = \epsilon^\mu \mathcal{M}_\mu \xrightarrow{u.t.} (\epsilon^\mu + D K^\mu) \mathcal{M}_\mu = \epsilon^\mu \mathcal{M}_\mu$$

Wordova identiteta: $k^\mu \mathcal{M}_\mu = 0$

$\hookleftarrow$ Wordova identiteta

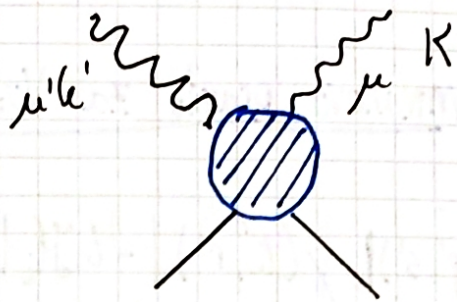
če so reči fotonov:

$$\mathcal{M} = \epsilon^\mu \epsilon'^{\mu'} \mathcal{M}_{\mu\mu'}$$

$$k^\mu \mathcal{M}_{\mu\mu'} = 0$$

$$k'^{\mu'} \mathcal{M}_{\mu\mu'} = 0$$

tudi Wardove identitete



Torej:

Dokaz:

$$\mathcal{M} = \epsilon_\mu^{\text{str}} \mathcal{M}^\mu \propto k_\mu \mathcal{M}^\mu = 0 \quad \text{Torej res nič ne prispeva.}$$

Vaja: [Doloci $\mathcal{M}$ za $e^-(p, s) \mu(k, r) \rightarrow e^-(p', s') \mu(k', r')$]

$$iT_{fi} = \langle f | T(\exp(-i \int \mathcal{H}_I(z))) | i \rangle = (*)$$

$$\exp(\quad) = \dots + \frac{1}{2!} (-ie)^2 \left(\int d^4x \bar{\Psi}_x^\mu \gamma^\mu \Psi_x^\mu A_x^\mu + \int d^4y \bar{\Psi}_y^e \gamma^\nu \Psi_y^e A_y^\nu \right)^2$$

$$\mathcal{H}_I = e \bar{\Psi}_\mu \gamma^\mu \Psi_\mu A_\mu + e \bar{\Psi}_e \gamma^\nu \Psi_e A_\nu$$

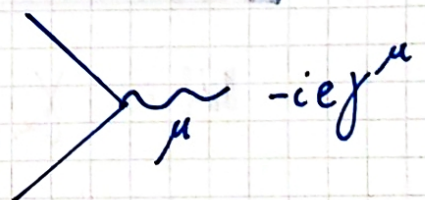
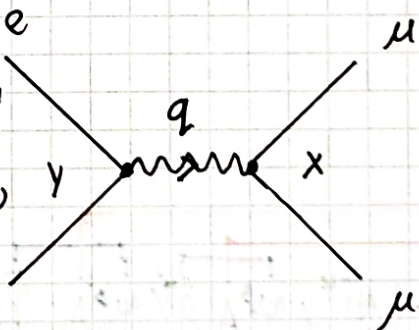
$$(*) = \langle \mu(k', r') | \langle e(p', s') | \frac{1}{2!} (-ie)^2 \cdot 2 \int d^4x \int d^4y \underbrace{\bar{\Psi}_x^\mu \gamma^\mu \Psi_x^\mu A_x^\mu}_{\text{mešani člen}} \underbrace{\bar{\Psi}_y^e \gamma^\nu \Psi_y^e A_y^\nu}_{\text{mešani člen}} | e(i) \rangle | \mu(i) \rangle$$

Menjava $x \leftrightarrow y$ NI možna (zaradi različnih delcev)

$$i\mathcal{M} = (-ie)^2 \frac{1}{s} \bar{u}_\alpha(k') \gamma_\mu u_\alpha(k) \bar{u}_\beta(p') \gamma_\nu u_\beta(p) \frac{-ig_{\mu\nu}}{q^2 + i\epsilon} \rho_\omega^{\alpha\beta}$$

$$= \frac{1}{s} \rho_\omega^{\alpha\beta} \bar{u}_\alpha(k') (-ie \gamma^\mu) u_\alpha(k) \bar{u}_\beta(p') (-ie \gamma^\nu) u_\beta(p) \left\{ e \frac{-ig_{\mu\nu}}{q^2 + i\epsilon} \right\}$$

Vozlišče



Feynmanovo pravilo za Vozišče

Gremo v prvi red in pogledamo primer interakcije, da dobimo vozišče.

$$\langle e(p', s') | \gamma(u', r') | -i \mathcal{H}_i | e(p, s) \rangle =$$

$i \mathcal{L}_i$ če nimamo Hamiltoniana

$$= \langle e(p', s') | \gamma(u', r') | -ie \bar{\Psi} \gamma^\mu \Psi A_\mu | e(p, s) \rangle =$$

$$= -ie \underbrace{\bar{u}^{s'}(p')} \gamma^\mu \underbrace{u^s(p)} \epsilon_\mu^{r'}(u')$$

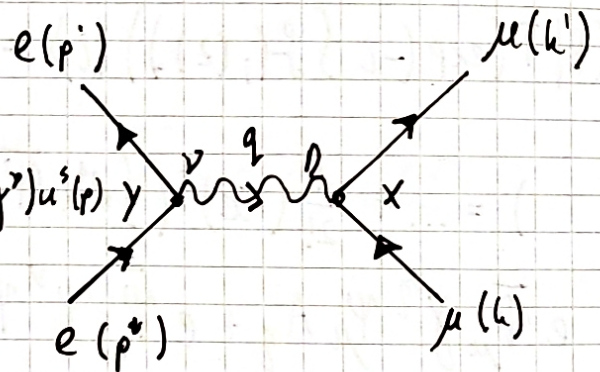
↓
Vozišče!

↓
Kontrakcija z zunanjiimi stanji

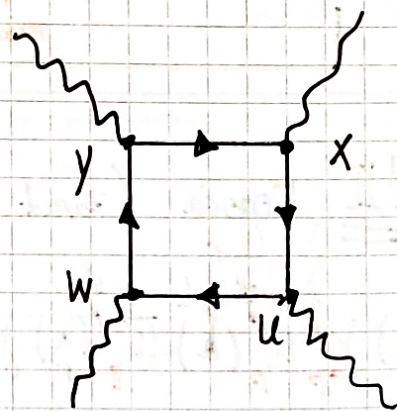
DN: $e^- e^+ \rightarrow e^- e^+$
 $e^- e^- \rightarrow e^- e^-$

Še z Feynmanovimi pravili:

$$i\mathcal{M} = \frac{1}{3} p_w \cdot \bar{u}^{r'}(u') (-ie \gamma^\mu) u^r(u) \bar{u}^{s'}(p') (-ie \gamma^\nu) u^s(p) \gamma_\mu \gamma_\nu$$



Vaja: [Doloci $\mathcal{M}$ za $\gamma\gamma \rightarrow \gamma\gamma$]



$$\langle \gamma\gamma | \bar{\Psi}_x \gamma_\mu \Psi_x A_\mu \bar{\Psi}_y \gamma_\nu \Psi_y A_\nu \bar{\Psi}_w \gamma_\rho \Psi_w A_\rho \bar{\Psi}_u \gamma_\sigma \Psi_u A_\sigma | \gamma\gamma \rangle$$

$$\text{Dolci } \alpha. \quad \gamma_{\alpha\beta} S^{\beta\alpha'} \gamma_{\alpha'\beta'} S^{\beta'\gamma'} \gamma_{\gamma'\delta'} S^{\delta'\gamma} \gamma_{\gamma\delta} S^{\delta\alpha}$$

Prosta indeksa le: $\text{Tr}[\gamma_5 \gamma_5 \gamma_5 \gamma_5]$
 $\propto \propto$

$P_w = -1$ Za zaključeno fermionsko zanko dobimo vedno $P_w = -1$

• Šestevch po polarizacijah fermionov

$$\mathcal{M} \mathcal{M}^* \rightarrow \sum_s \mathcal{M}^s(\vec{p}) \bar{u}^s(\vec{p}) = \not{p} + mI$$

$$\sum_s v^s(\vec{p}) \bar{v}^s(\vec{p}) = \not{p} - mI$$

$\mathcal{M}$ could be u?

• Šestevch po polarizacijah fotonov

$$\mathcal{M} \mathcal{M}^* : \sum_{r=1,2} \epsilon_\mu^{r*}(\vec{k}) \epsilon_\nu^r(\vec{k}) \rightarrow (-g_{\mu\nu})$$



Zakaj to lahko velja?

$$\mathcal{M} = \epsilon_\mu^{r*}(\vec{k}) \mathcal{M}^\mu$$

$$\Rightarrow \mathcal{M} \mathcal{M}^* = \left(\sum_{r=1,2} \epsilon_\mu^{r*} \mathcal{M}^\mu \right) \left(\sum_{r=1,2} \epsilon_\nu^r \mathcal{M}^{\nu*} \right) = (-g_{\mu\nu}) \mathcal{M}^\mu \mathcal{M}^{\nu*}$$

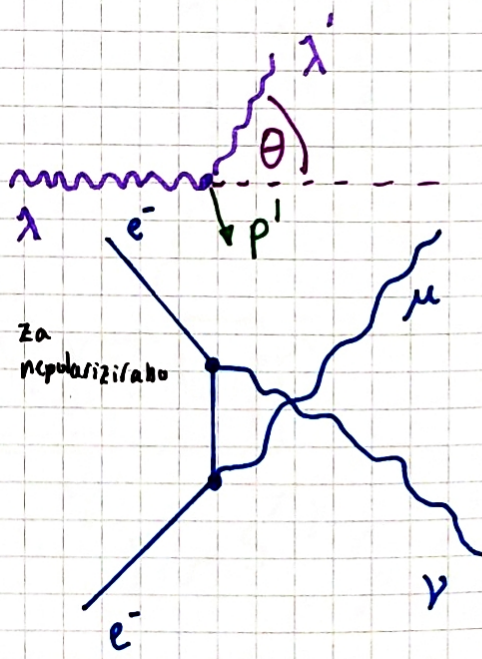
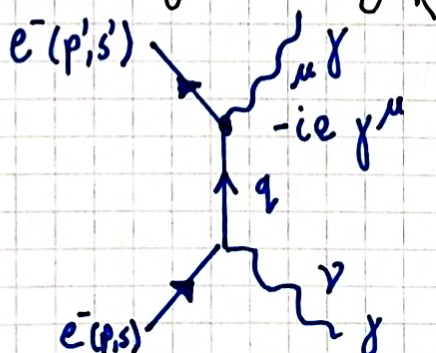
Wardova identiteta

$$k^\mu \mathcal{M}_\mu = 0$$

Vaja: [Comptonovo sipanje]

$$|\mathcal{M}|^2 = ? \quad \text{če} \quad \gamma e^- \rightarrow e^- \gamma$$

Diagrami:



za nepolarizirano

$$i\mathcal{M} = \bar{u}^s(\vec{p}')(-ie\gamma^\mu) \frac{i(\not{p} + mI)}{q^2 - m^2 + i\epsilon} (ie\gamma^\nu) u^s(\vec{p}) \epsilon_\nu^r(k) \epsilon_\mu^{r'*}(k') + \dots =$$

$$S=1, P_W=1$$

$$= -ie^2 \epsilon_\nu^r(k) \epsilon_\mu^{r'}(k') \bar{u}_\alpha^s(p') \Gamma_{\alpha\beta}^{\mu\nu} u_\beta^s(p)$$

$$\tilde{\Gamma} = \gamma^0 \Gamma^\dagger \gamma^0$$

$$-i\mathcal{M}^* = ie^2 \epsilon_\nu^{r'}(k) \epsilon_\mu^r(k') \bar{u}_{\alpha'}^{s'}(\vec{p}') \tilde{\Gamma}_{\alpha'\beta'}^{\mu'\nu'} u_{\beta'}^s(p')$$

To se naredi pri
FJOD in si tam pogledaj

Zlaj lahko izračunamo: povprečanje po polarizaciji

$$|\mathcal{M}|_{\text{npolarizirano}}^2 = e^4 \underbrace{\frac{1}{2} \sum_{r=1,2} \epsilon_\nu^r(k) \epsilon_\nu^{r'}(k)}_{(-g_{\nu\nu'})} \underbrace{\sum_{r'=1,2} \epsilon_\mu^{r'}(k') \epsilon_\mu^{r''}(k')}_{(-g_{\mu\mu'})} \underbrace{\frac{1}{2} \sum_{s=1,2} u_\beta^s(\vec{p}) \bar{u}_{\alpha'}^{s'}(\vec{p}')}_{(\not{p} + mI)_{\beta\alpha'}}$$

$$\cdot \tilde{\Gamma}_{\alpha'\beta'}^{\mu'\nu'} \sum_{s'} u_{\beta'}^s(p') \bar{u}_\alpha^s(p') \Gamma_{\alpha\beta}^{\mu\nu} =$$

$$(\not{p}' + mI)_{\beta'\alpha}$$

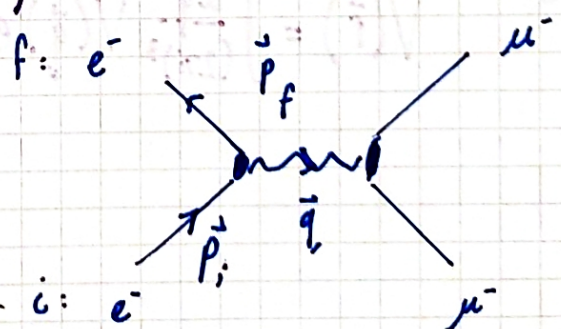
$$\text{Tr}(\gamma^\mu \gamma^\nu) = 4g^{\mu\nu}$$

$$= e^4 \frac{1}{4} \text{Tr}((\not{p} + mI) \tilde{\Gamma}^{\mu\nu} (\not{p}' + mI) \Gamma^{\mu\nu})$$

Povezave med $T_{fi}, \mathcal{M}$ in $V(r), \tilde{V}(\vec{q})$ (v neel. limiti)

a) $\mathcal{M}_{\text{QFT}}$ v NR limiti recimo $e^- \mu^- \rightarrow e^- \mu^-$

b) T_{fi} izrazimo z Fermijevim zlatim pravilom kjer



$$T_{fi}^{HM} = -i \int dt \int d\vec{x} \psi_f^*(\vec{x}, t) V(\vec{x}, t) \psi_i(\vec{x}, t)$$

↓
Hobson-Martin
definicija
= i T Peskin

$$= -i \int dt e^{iE_f t} e^{-iE_i t} \int d\vec{x} \frac{1}{\sqrt{V}} e^{-i\vec{p}_f \cdot \vec{x}} V(\vec{x}) e^{i\vec{p}_i \cdot \vec{x}} \frac{1}{\sqrt{V}} =$$

Elastično sipanje

$$= -i (2\pi) \delta(E_f - E_i) \frac{1}{V} \tilde{V}(\vec{q}) = i T_{fi}^{Ps}$$

$$\frac{1}{V} \int d\vec{x} e^{-i\vec{q} \cdot \vec{x}} V(\vec{x}) = \frac{1}{V} \tilde{V}(\vec{q})$$

↑
Volumen ↑
potencial ↑
FT potenciala

To smo že naredili:

$$\langle \mu | \langle e^- | | e^- | | \mu \rangle$$

$$a) i\mathcal{M}_{fi} = (-ie)^2 \bar{u}^s(\vec{0}) \gamma^\mu u^{s'}(\vec{0}) \frac{-ig_{\mu\nu}}{q^2} \bar{u}^r(\vec{0}) \gamma^\nu u^{r'}(\vec{0}) =$$

$$\delta_{\mu 0} \underline{2m_e} \delta^{ss'} \frac{-ig_{00}}{0 - |\vec{q}|^2} \delta_{\nu 0} \underline{2m_\mu} \delta^{rr'}$$

za relativistično normalizacijo

$$= -ie^2 (2m_e)(2m_\mu) \left(\frac{1}{|\vec{q}|^2} \right)$$

$$\tilde{V}(\vec{q}) = -\frac{e^2}{|\vec{q}|^2}$$

$$i\mathcal{M} \propto \langle \mu | \langle e^- | \bar{\psi}_x^\mu \psi_x^\mu \dots \bar{\psi}_y^e \psi_y^e | e^- \rangle | \mu \rangle \Rightarrow P_w = 1$$

Torej:

$$V(\vec{r}) = \int \frac{d\vec{q}}{(2\pi)^3} \tilde{V}(\vec{q}) e^{i\vec{q} \cdot \vec{r}} = \dots = \frac{e^2}{4\pi r}$$

Sipanje $e^- \mu^+ \rightarrow e^- \mu^+$

$$i\mathcal{M} = \dots \langle \mu^+ | \langle e^- | \bar{\psi}_x^\mu \psi_x^\mu \bar{\psi}_y^e \psi_y^e | e^- \rangle | \mu^+ \rangle, P_w = -1$$

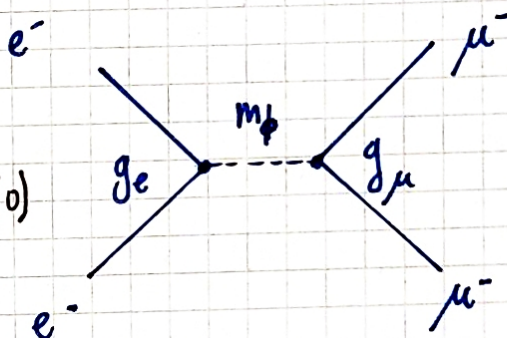
$$i\mathcal{M}_{fi} = -1(-ie^2)(2m_e)(2m_\mu) \frac{1}{|\vec{q}|^2}; \quad \tilde{V}(\vec{q}) = -\frac{e^2}{|\vec{q}|^2}$$

$$V(\vec{r}) = -\frac{e^2}{4\pi r}$$

Sipanje preko izmenjave skalarjev

$$\mathcal{M} \propto g_e g_\mu \bar{u}(\vec{0}) I u(\vec{0}) \frac{i}{q^2 - m_\phi^2} \bar{u}(0) I u(0)$$

$$\tilde{V}(\vec{q}) = -g_e g_\mu \frac{1}{|\vec{q}|^2 + m_\phi^2}$$



Yukavin
potencial

$$V(\vec{r}) = -g_e g_\mu \frac{1}{4\pi r} e^{-m_\phi r}$$

če so prenašalci
interakcije masivni,
težji kot so, krajši
ima doseg ta interakcija