

QED do ene zanke in renormalizacija

Peslin: 6.2, 6.3, 7.1, 7.5, 10.2, 10.3

QED - one-loop

Uvod in načrt dela

1.) $\mathcal{L}(e, m)$ s parametrioma gleja naboja in gole mase (Peslin: e_0, m_0)

2.) Najbolj tipično bomo dobili $\mathcal{M} = \infty$

3.) Regularizacija: redefinicija teorije, da izluščimo neskončnost na točno določen način.

• Dimenzacijska regularizacija: Rekli bomo da živimo v vesolju kjer namesto v 4D prostoru živimo v $d = 4 - \epsilon$, $\epsilon \rightarrow 0$ dim. prostoru.

↑
To bomo mi
uporabljali

$$\mathcal{M} = A \frac{1}{\epsilon} + \text{končni \u010drt}$$

• Cutoff regularizacija

• Pauli - Villars regularizacija

4.) Renormalizacija

↳ odpravimo neskončnosti ~~teorije~~ z izbiro golih parametrov e, m

Merjeni parametri:

$$e_R \left(\frac{e_R^2}{4\pi} \sim \frac{1}{137} \right), m_R (0.5 \text{ MeV}/c^2)$$

Ho\u010demo izbrati e, m tako, da bodo opazljivke kon\u010dne. Izlaze se, da morajo biti te goli parametri v ta namen neskon\u010dni.

Dobili bomo zvezo med parametri: $e, m \leftrightarrow e_R, m_R$

Opazljivke izrazimo z merjenimi parametri in dobimo končne opazljivke.

Renormalizacijske sheme:

- on-shell (na masni lupini) $\rightarrow$ pomeni, da je $p^2 = m_R^2$

- minimalna subtrakcija (Najbolj priročna)

5.) Ujemanje opazljivke z eksperimentom

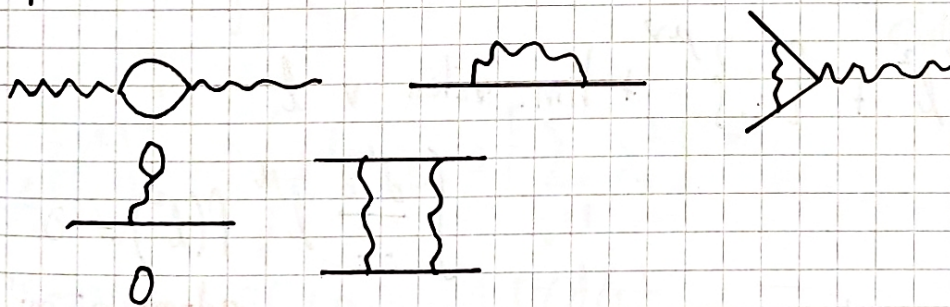
• Popravlch k magnetnemu momentu fermiona:

$$\mu = 2 + \frac{\alpha}{\pi} + \dots \quad \left. \vphantom{\mu = 2 + \frac{\alpha}{\pi} + \dots} \right\} \text{giromagnetno izmerje}$$

• EM potencial

$$U_c = \frac{e^2}{4\pi r} + \# \delta(r) e^4$$

6.) Popravki:



Popravlch k fotonshem propagatorju

$$i\Pi^{\mu\nu}(q^2) = (-1) \int \frac{d^4 k}{(2\pi)^4} \text{Tr} \left[(-ie\gamma^\mu) \frac{i(k+m)}{k^2 - m^2 + i\epsilon} \cdot (-ie\gamma^\nu) \frac{i(k+q+m)}{(k+q)^2 - m^2 + i\epsilon} \right] = N^{\mu\nu}$$

$$= (-1)(-1)(-ie)^2 \int \frac{d^4 k}{(2\pi)^4} \underbrace{m^2 g^{\mu\nu}}_B + \underbrace{k^\mu (k+q)^\nu + k^\nu (k+q)^\mu - g^{\mu\nu} k(k+q)}_A$$

zura di obravnave gibalne količine

$$\text{Tr} [1] = 4$$

$$m^2 \text{Tr} [\gamma^\mu \gamma^\nu] = m^2 4 g^{\mu\nu}$$

$$\text{Tr} [\gamma^\mu k_\alpha \gamma^\nu (k+q)_\beta \gamma^\mu] =$$

$$= k_\alpha (k+q)_\beta 4 (g^{\mu\alpha} g^{\nu\beta} + g^{\mu\beta} g^{\alpha\nu} - g^{\mu\nu} g^{\alpha\beta})$$

Prvi Intermezzo:

$$\frac{1}{AB} = \int_0^1 dx \frac{1}{[xA + (1-x)B]^2} = \int_0^1 dx \frac{1}{D^2}$$

x : Feynmannov parameter

$$D = x(A-B) + B = x(\cancel{k^2} + 2\cancel{k}q + q^2 - m^2 + i\epsilon - (\cancel{k^2} - m^2 + i\epsilon)) + \cancel{k^2} - m^2 + i\epsilon$$

$$= \cancel{k^2} + 2\cancel{k}q x + q^2 - m^2 + i\epsilon =$$

$$= \underbrace{(\cancel{k} + qx)^2}_{l^2} - \underbrace{q^2 x^2}_{-\Delta(q,x)} + q^2 - m^2 + i\epsilon$$

$$\Rightarrow i\bar{\Pi}^{\mu\nu} = -e^2 4 \int \frac{d^4 l}{(2\pi)^4} \int_0^1 dx \frac{N^{\mu\nu}}{[l^2 - \Delta(q,x) + i\epsilon]^2}$$

$$N^{\mu\nu} = 2l^\mu l^\nu - g^{\mu\nu} l^2 + \{...\}^{\mu\nu} + \text{lim. vilen v } l$$

$$\int \frac{d^4 l}{l^4} l^\mu f(l^2) = 0$$

Ker integriramo blizu polov rabimo:

Drugi Intermezzo: Wickova rotacija

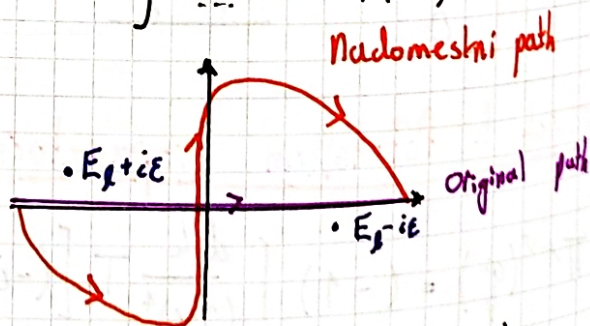
Pol: v integralu po l^0 :

$$l^2 - \Delta + i\epsilon = l_0^2 - \vec{l}^2 - \Delta + i\epsilon = (l_0 - (E_{\vec{l}} - i\epsilon))(l_0 + (E_{\vec{l}} + i\epsilon))$$

$$E_{\vec{l}} = \sqrt{\vec{l}^2 + \Delta}$$

$$\int_{-\infty}^{\infty} dl^0 f(l^0) + \int_{-\infty}^{\infty} dl^0 f(l^0) = \sum \text{Res} = 0$$

$$l^2 = l_0^2 - \vec{l}^2 = (il_E^0)^2 - \vec{l}^2 = -l_E^2 \leftarrow \int_{-\infty}^{\infty} dl^0 f(l^0) = \int_{-\infty}^{\infty} dl^0 f(l^0) = i \int_{-\infty}^{\infty} dl_E^0 f(l_E^0)$$



Tako dobimo:

$$i\Pi^{\mu\nu} = -4l^2 \int_0^1 dx i \int_{-\infty}^{\infty} dl_E \frac{\int d\vec{l}}{(2\pi)^4} \frac{N^{\mu\nu}}{[l_E^2 + \Delta]^2}$$

$$\int d\vec{l} \rightarrow \int d\Omega_3 \int dl l^2$$

$$\int \frac{d^4 l_E}{(2\pi)^4} \frac{1}{(l_E^2 + \Delta)^m} = \int_{-\infty}^{\infty} \int_{\text{kot}} \frac{d\Omega_4 l_E^3 dl_E}{(2\pi)^4 (l_E^2 + \Delta)^m} =$$

$$m=2 = \dots \int \frac{dl_E}{l_E} = \dots \ln l_E \Big|_{-\infty}^{\infty} = \infty$$

Tretji Intermezzo: Regularizacija

Najdes pa na proslojnicah.

$$\left(\int dx e^{-x^2} \right)^2 = \int dx_1 e^{-x_1^2} \int dx_2 e^{-x_2^2}$$

$$= \iint dx_1 dx_2 e^{-(x_1^2 + x_2^2)} = (\sqrt{\pi})^2, \text{ ker vemo koliko je vsebina obkrožen v na začeti.}$$

$$\left(\int dx e^{-x^2} \right)^d = \int d^d x \exp \left[\sum_{i=1}^d x_i^2 \right] =$$

$$= (\sqrt{\pi})^d = \int d\Omega_d \cdot \frac{1}{2} \Gamma(d/2)$$

Torej ponovimo integracijo po $d = 4 - \epsilon$ dimenzijah:

$$i\Pi^{\mu\nu} = -4e^2 \int_0^1 dx \int \frac{d^d l N^{\mu\nu}}{(2\pi)^d} \frac{1}{(l^2 - \Delta(q, x) + i\epsilon)^2}$$

$$N^{\mu\nu} = 2l^\mu l^\nu - g^{\mu\nu} l^2 + \{ \dots \}$$

$$\int \frac{d^d l}{(2\pi)^d} l^\mu l^\nu f(l^2) =$$

$$= \int \frac{d^d l}{(2\pi)^d} \frac{1}{d} g^{\mu\nu} l^2 f(l^2) /:$$

$$= \int \frac{d^d l}{(2\pi)^d} l^2 f(l^2) = \int \frac{d^d l}{(2\pi)^d} \underbrace{g^{\mu\nu} g_{\mu\nu}}_{=d} l^2$$

$$\int d\vec{l} = i \int d\vec{l}_E$$

$$l^2 = l^0^2 - \vec{l}^2 = -l_E^0^2 - \vec{l}^2 = -l_E^2$$

$$i\Pi^{\mu\nu} = -4e^2 \int_0^1 dx \ i \int \frac{d^d l_E}{(2\pi)^d} M^{4-d} \frac{N^{\mu\nu}}{(l_E^2 + \Delta + i\epsilon)^2}$$

↑ Ta epsilon ni isti kot
tisti v $d=4-\epsilon$

$$N^{\mu\nu} = 2 \cdot \frac{1}{d} g^{\mu\nu} (-l_E^2) + g^{\mu\nu} l_E^2 + \{ \dots \}$$

Rezultat integrala smo že dobili (tisto z Gamma funkcijami):

$$i\Pi_2^{\mu\nu} = -4ie^2 \int dx M^{4-d} \left(\left[\left(1 - \frac{2}{d}\right) g^{\mu\nu} \frac{1}{(4\pi)^{d/2}} \frac{d}{2} \frac{\Gamma(2 - \frac{d}{2} - 1)}{1} \left(\frac{1}{\Delta}\right)^{2 - \frac{d}{2} - 1} \right. \right. \\ \left. \left. + \left\{ \right\}^{\mu\nu} \frac{1}{(4\pi)^{d/2}} \frac{\Gamma(2 - d/2)}{1} \left(\frac{1}{\Delta}\right)^{2 - \frac{d}{2}} \right] \right) =$$

↑
2nd order popravak
I think

$$\left(\frac{d}{2} - 1\right) \Gamma\left(1 - \frac{d}{2}\right) = -\Gamma\left(2 - \frac{d}{2}\right)$$

$$n \Gamma(n) = \Gamma(n+1)$$

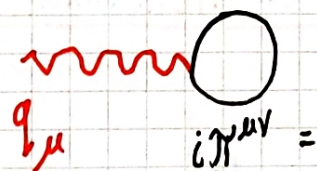
$$= -4ie^2 \int dx \left[M^{4-d} \frac{1}{(4\pi)^{d/2}} \frac{\Gamma(2 - \frac{d}{2})}{\Delta^{2-d/2}} \right] \underbrace{(-g^{\mu\nu} \Delta + \{ \}^{\mu\nu})}_{\text{...}}$$

Ko vstavimo, dobimo (ni trivalno dobiti)

$$2x(1-x) \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right)$$

Mora veljati: (Posplošena) Wordova Identiteta:

$$q_\mu \Pi^{\mu\nu} = 0$$



$$q_\mu i\Pi^{\mu\nu} =$$

$$= i(q^2 g^{\mu\nu} - q^\mu q^\nu) \Pi_2(q^2)$$

~~$$q_\mu \dots (q^2 g^{\mu\nu} - q^\mu q^\nu)$$~~

$$q_\mu \dots (q^2 g^{\mu\nu} - q^\mu q^\nu) = (q^2 q^\nu - q^2 q^\nu) = 0$$

$$\Pi_2(q^2) = -4ie^2 \int dx [\dots] 2x(1-x) =$$

$$\mu^{4-d} \frac{1}{(4\pi)^{d/2}} \frac{\Gamma(2-\frac{d}{2})}{\Lambda^{2-d/2}} = \frac{1}{(4\pi)^2} \frac{\Gamma(\frac{\epsilon}{2})}{\left(\frac{\Lambda}{4\pi\mu^2}\right)^{\epsilon/2}}$$

$$d=4-\epsilon$$

$$\hookrightarrow \lim_{\epsilon \rightarrow 0} a^{-\epsilon/2} = 1 - \frac{\epsilon}{2} \ln a ; \quad \Gamma\left(\frac{\epsilon}{2}\right) = \frac{2}{\epsilon} - \gamma + \mathcal{O}(\epsilon)$$

Euler-Mascheroni constant
 $\gamma \simeq 0.577$

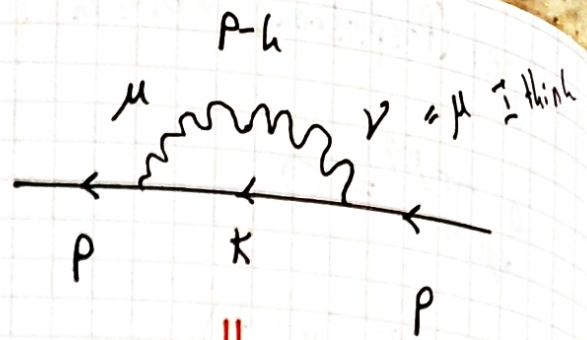
$$[\dots] = \frac{\lim_{\epsilon \rightarrow 0}}{(4\pi)^2} \left(1 - \frac{\epsilon}{2} \ln\left(\frac{\Lambda}{4\pi\mu^2}\right) + \dots \right) \left(\frac{2}{\epsilon} - \gamma + \dots \right) =$$

$$= \frac{1}{(4\pi)^2} \left(\frac{2}{\epsilon} - \ln \frac{\Lambda}{\mu^2} - \gamma + \ln(4\pi) \right)$$

$$\Delta(x, q^2)$$

Popravek k elektronskemu propagatorju

$$i\Sigma_2(p) = \int \frac{d^d k}{(2\pi)^d} M^{4-d} \underbrace{(-ie\gamma^\mu)}_B \frac{i\cancel{k} + m}{k^2 - m^2 + i\epsilon} \cdot \underbrace{(ie\gamma_\mu)}_A \frac{-i(\cancel{p-k})}{(p-k)^2 + i\epsilon}$$



$$\parallel$$

$$i\Sigma_2(p)$$

$$\gamma^\mu \gamma_\alpha \gamma^\alpha \gamma_\mu =$$

$$= -(d-2)\gamma^\mu \gamma_\mu; \gamma_\mu \gamma^\mu = d$$

$$D = xA + (1-x)B = x(p^2 - 2pk + k^2 + i\epsilon) +$$

$$+ (1-x)(k^2 - m^2 + i\epsilon) =$$

$$= k^2 - 2xpk + xp^2 + xm^2 - m^2 + i\epsilon =$$

$$= \underbrace{(k - xp)^2}_{l} - \underbrace{x^2 p^2 + xp^2 + xm^2 - m^2 + i\epsilon}_{-\Delta(x, m, p)}$$

Torej:

$$i\Sigma_2 = \int \frac{d^d l}{(2\pi)^d} M^{4-d} \int dx \frac{-(d-2)(\cancel{l} + x\cancel{p}) + dm}{[l^2 - \Delta + i\epsilon]^2} =$$

$$= \underbrace{i}_{\text{Wickova rotacija}} \int \frac{d^d l_E}{(2\pi)^d} \int dx M^{4-d} \frac{(-e^2) + ((d-2)x\cancel{p} + md)}{(-l_E^2 - \Delta)^2} =$$

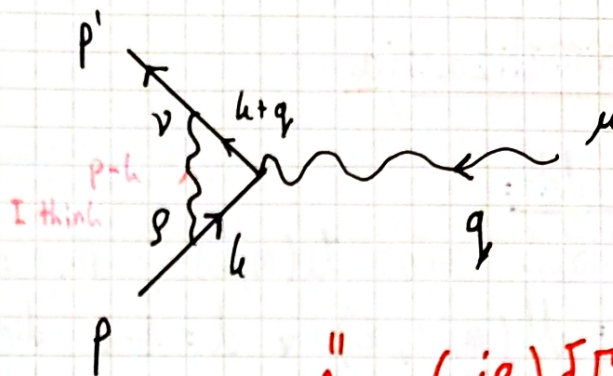
$$\underbrace{l_E^0 \text{ rotira za } 90^\circ}_{\text{Wickova rotacija}} = -ie^2 \int dx \left[M^{4-d} \frac{1}{(4\pi)^{d/2}} \Gamma(2-d/2) \left(\frac{1}{\Delta}\right)^{2-d/2} \right] (-(d-2)x\cancel{p} + md)$$

To smo poravnali že prej:

$\frac{2}{\epsilon} + \text{končni členi}$

Popravljeni h vozlišča

$$\star = \int \frac{d^4 l}{(2\pi)^4} M^{4-d} \bar{u}(p') (-ie \gamma^\nu)$$



$$\star = (-ie) \delta \Gamma^\mu$$

$$\cdot \left(\frac{i((k+q) + m)}{(k+q)^2 - m^2 + i\epsilon} \right) (-ie \gamma^\mu)$$

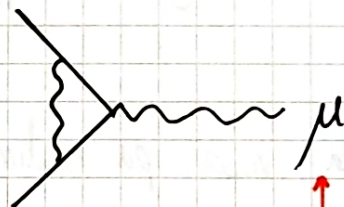
$$\cdot \left(\frac{i(k+m)}{k^2 - m^2 + i\epsilon} \right) (-ie \gamma^\rho) u(p) \frac{-ig_{\nu\rho}}{(p-k)^2 - m^2 + i\epsilon}$$

Nadaljuje se na prisojnicah zaradi kompleksnosti.

$$\bar{u}(p') \dots u(p)$$

$$\downarrow$$

$$\gamma^\mu, p^\mu, p'^\mu$$



↑ zmanjši indeks

↳ Te tri lahko izrazimo z dvema neodvisnima kombinacijama

$$\bar{u}(p') \left[\gamma^\mu F_1(q^2) + i \frac{\delta^{\mu\nu} q_\nu}{2m} F_2(q^2) \right] u(p)$$

↑ Dve funkciji, ki ju moramo ven dobiti

Izkaže se:

$$F_2: \delta^{\mu\nu} q_\nu : \int d^4 l \frac{1}{l^6 + \dots} \sim \int dl \cdot l^3 \frac{1}{l^6} \dots \Leftarrow \text{Konvergirata}$$

Schwinger dobil Nobela (mag. mom. f. oz. giramag. fer.) (Regularizacija ni potrebna)

$$\delta F_1: \gamma^\mu : \int d^4 l \frac{(l^2 + \dots)}{l^6 + \dots} = \infty \Leftarrow \text{Divergirata}$$

(Potrebna je regularizacija)

$$\Rightarrow \delta F_1 = \frac{2}{\epsilon} + \text{končni členi}$$

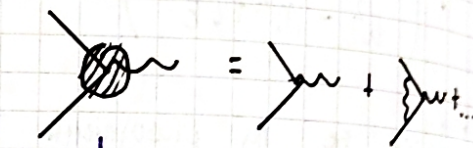
Renormalizacija

Načrt dela:

- Določili smo $\mathcal{L}(m, e)$, m, e : goli parametri
- $\mathcal{M} = \mathcal{M}(m, e, \epsilon)$ v $d = 4 - \epsilon$ dimenzij
= $\# \frac{1}{\epsilon} +$ končni členi

- Uvedemo renormalizacijske pogoje

Ekspiriment meri popolnoma popravljeno vozlišče,
kjer je: $-ie_R \gamma^+ \Rightarrow e_R^2 / 4\pi = \frac{1}{137}$



(popolnoma popravljeno vozlišče)

Za pogoj za maso pa sledimo iz propagatorja:

$$\frac{i(\not{p} + m_R)}{p^2 - m_R^2} = \frac{i}{\not{p} - m_R} \quad \text{meaning} \quad \frac{i}{(\not{p} - m_R)(\not{p} + m_R)} = \frac{i(\not{p} + m_R)}{p^2 - m_R^2}$$

$\propto$ 

$\underbrace{p^2 - m_R^2}_{p^2 I}$

Pol popavljenega elektronskega propagatorja pri $p^2 = m_R^2$. Hočemo, da je propagator Greenova funkcija naše gibalne enačbe (Diracova enačba):

$$(i \gamma^\mu \frac{\partial}{\partial x^\mu} - m_R) P_{\text{prop}}(x-y) \delta(x-y): R.P : m \leftrightarrow m_R$$

Potem bomo zapisali gibalne amplitude in upamo, da ne divergirajo:

$$-\mathcal{M} = \mathcal{M}(e_R, m_R)$$

popravljen propagator elektrona

$$A_{\text{free}}(p) = \text{---} + \text{---} \circlearrowleft \text{---} + \text{---} \circlearrowleft \circlearrowleft \text{---} + \dots$$

↑
one particle irreducible diagram

$$\text{---} \circlearrowleft \text{---} = \text{---} \text{---} + \text{---} \text{---} + \text{---} \text{---} + \dots$$

$$= \frac{i}{\not{p} - m} + \frac{i}{\not{p} - m} \left[(-i \Sigma) \frac{i}{\not{p} - m} \right] + \dots$$

$$= \frac{i}{\not{p} - m} \sum_{n=0}^{\infty} \left(\frac{\Sigma}{\not{p} - m} \right)^n = \frac{i}{\not{p} - m} \frac{1}{1 - \frac{\Sigma}{\not{p} - m}} =$$

$$= \frac{i}{\not{p} - m - \Sigma(\not{p}, m)}$$

$$\text{---} \circlearrowleft \text{---}^{-1} = \text{---}^{-1} - \text{---} \circlearrowleft \text{---}$$

Dyson-Schwinger-Lippman

Renorm. pogoj. (R.P.): Hočemo pol pri $p^2 = m_R^2 \rightarrow \not{p} = m_R$:

$$\not{p} - m - \Sigma(\not{p}, m) \Big|_{\not{p} = m_R} = 0$$

$$\Rightarrow \boxed{m_R - m - \Sigma(\not{p}_R, m) = 0}$$

Zveza med m_R in m

$$e^2 \rightarrow e_R^2$$

↓

$$e = e_R [1 + \mathcal{O}(e_R^2)]$$

$$m = m_R + \mathcal{O}(e_R^2)$$

Pr.

$$e^2 + e_R^2 \dots (m_R \neq m_R e_R^2)$$

↓
neskončen

↓

tudi gola masa je neskončna,

ampak s temi neskončnostmi delujemo

prejšnje.

↳ že reda e_R^2

Razvoj $\text{Prop}_e(p)$ za $p \simeq m_R$:

$$\text{Prop}_e(p) = \frac{i}{\underbrace{p - m - \Sigma(m_R, m)}_{-m_R} - \frac{\partial \Sigma}{\partial p} \Big|_{p=m_R} (p - m_R)} =$$

$$= \frac{i}{(p - m_R)} \underbrace{\frac{1}{\left(1 - \frac{\partial \Sigma}{\partial p} \Big|_{p=m_R}\right)}}_{Z_2} =$$

Z_2 renormalizacijska el. polja.
field renorm. strength

$$= i \frac{Z_2}{\underbrace{p - m_R}_{\Gamma_e^{(2)}(p)}} + \dots \quad \psi_R \equiv Z_2^{-\frac{1}{2}} \psi$$

Naprej na preosojnicah (Renormalization of elektron field, slide 31)

Popravljen fotonski propagator

Uvod na preosojnicah slide 33.

$$\begin{aligned} \text{Prop}_\gamma(q) &= \text{diagram with wavy line and shaded circle} = \text{diagram with wavy line} + \text{diagram with wavy line and 1PI loop} + \text{diagram with wavy line and 2PI loops} + \dots \\ &= \frac{-ig_{\mu\nu}}{q^2} + \frac{-ig_{\mu\sigma}}{q^2} \left[i(q^2 g^{\sigma\beta} - q^\sigma q^\beta) \Pi(q^2) \right] \frac{-ig_{\beta\nu}}{q^2} + \dots \\ &= \left(-\frac{ig_{\mu\nu}}{q^2} \right) \left(1 + \Pi(q^2) + \Pi^2(q^2) + \dots \right) = \left(-\frac{ig_{\mu\nu}}{q^2} \right) \frac{1}{1 - \Pi(q^2)} \end{aligned}$$

Posledica Wardovih identitet

Renorm. pogoj: $m_\gamma = 0$, $q^2 = 0$ za realni foton

Torej hočemo pol $\text{Prop}_\gamma(q)$ pri $q^2 = 0$.

In other words je elektron Greenova funkcija Maxwellovih enačb:

$$\square \text{Prop}_\gamma(x-y) = \delta(x-y)$$

Ta pogoj je avtomatično zadoščen:

$$q^2 (1 - \Pi(q^2)) \big|_{q^2=0} = 0 \quad \checkmark$$

Razvoj $\text{Prop}_\gamma(q)$ za $q \approx 0$:

$$\text{Prop}_\gamma(q) = \left(-\frac{i g_{\mu\nu}}{q^2} \right) \frac{1}{1 - \Pi(q^2)} = \frac{-i g_{\mu\nu}}{q^2 (1 - (\underbrace{\Pi(q^2) - \Pi(0)}_{\hat{\Pi}(q^2) : \text{konino}}))} \underbrace{\frac{1}{1 - \Pi(0)}}_{Z_3}$$

$$\text{Prop}_\gamma(q^2) = \frac{-i g_{\mu\nu}}{q^2 (1 - \hat{\Pi}(q^2))} Z_3 \quad A_R^\mu = Z_3^{-1/2} A^\mu$$

Naprej na prosojnici 34.

RP za $\Gamma_{\gamma, R}^{(2)}$: i) $\Gamma_{\gamma, R}^{(2)}(q^2 = 0) = 0$ avtomatsko po Ward Identity

$$\text{ii) } \frac{\partial \Gamma_{\gamma, R}^{(2)}}{\partial q^2}(q^2 = 0) = 1$$

Popravljeno vozlišče

Uvod na slide 35.

$$\text{Diagram: } \Gamma_{R,\mu}^{(3)} = \sqrt{Z_2} \sqrt{Z_2} \sqrt{Z_3} \Gamma_{\mu}^{(3)} = Z_2 \sqrt{Z_3} (-ie) \left[\gamma^\mu (1 + \delta F_1(q^2)) + i \frac{\partial^{\mu\nu} q_\nu}{2m} F_2(q^2) \right]$$

Re. Po.: $\Gamma_{R,\mu}^{(3)}(q=0) = -i \gamma^\mu e_R$

$$\Gamma_{R,\mu}^{(3)}(q=0) = Z_2 \sqrt{Z_3} (-ie) \gamma^\mu (1 + \delta F_1(0))$$

$$\left(Z_2 (1 + \delta F_1(0)) \right) \cdot \frac{1}{Z_1} = (1 + \delta Z_2) (1 + \delta F_1(0)) = 1$$

$$\delta Z_2 = -\delta F_1(0)$$

$$e_R = e \sqrt{Z_3} ; e = e_R \sqrt{1 - \Pi_2(0)} = \infty$$

Imamo:

$$\begin{aligned} \Gamma_{R,\mu}^{(3)}(q) &= Z_2 (-ie_R) \left[\gamma^\mu (1 + \delta F_1(q^2) - \delta F_1(0) + \delta F_1(0)) + \frac{i \partial^{\mu\nu} q_\nu}{2m} F_2(q^2) \right] \\ &= (-ie_R) \left[\gamma^\mu (1 + \delta \hat{F}_1(q^2)) + \frac{i \partial^{\mu\nu} q_\nu}{2m_R} F_2(q^2) \right] \end{aligned}$$

Vse e_R^2 zanemajemo

$$\frac{e_R^2}{m_R (1 + \delta(e_R^2))} = \frac{e_R^2}{m_R} (1 - \delta e_R^2)$$

Opazljive posledice

Na prosojnici 39.

Vaja: $[e^-(p) \mu^-(k) \rightarrow e^-(p') \mu^-(k')] \text{ to } \mathcal{O}(e^4)]$

$$i\mathcal{M} = (\sqrt{Z_{2e}})^2 (\sqrt{Z_{2\mu}})^2 \cdot \left[\text{Diagram 1} + \text{Diagram 2} \right] =$$

$$\star = \frac{-ig_{\mu\nu} \hat{Z}_3}{q^2(1-\hat{\Pi}_2(q^2))} \bar{u}(p')$$

Trotočlovna Γ

↑ Tega smo zanemarili: za zdaj

$$= (\sqrt{Z_{2e}})^2 (\sqrt{Z_{2\mu}})^2 \cdot (-ie) \left[\gamma^\mu (1 + \delta F_1(q^2) - \delta F_1(0) + \delta F_1(0)) + \frac{i\delta g_{\mu\alpha}^{\mu\alpha}}{2m_R^e} F_2^e(q^2) \right] u(p)$$

$$\bar{u}(k') (-ie) \left[\gamma^\nu (1 + \delta F_1(q^2) - \delta F_1(0) + \delta F_1(0)) + \frac{i\delta g_{\nu\beta}^{\nu\beta}}{2m_R^\mu} F_2^\mu(q^2) \right] u(k) \star =$$

$$= \bar{u}(p') (-ie_R) \left\{ \gamma^\mu + (1 + \delta F_1(q^2)) + \dots \right\} u \cdot$$

$$(-ie)(-ie) \dots (1 + \delta Z_2)(1 + \delta F_1)$$

$\propto e^4$
Zanemarimo ker reda 6.

$$\bar{u}(k') (-ie_R) \left\{ \gamma_\mu + (1 + \delta F_1^\mu(q^2)) + \dots \right\} u.$$

$$e^2 Z_3 = e_R^2$$

$$Z_2(1 + \delta F_1(0)) = 1 \quad \uparrow$$

$$\frac{-i}{q^2(1-\hat{\Pi}_2(q^2))}$$

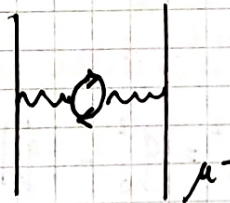
Opazljive posledice popravka $\sim Q^2$

Posledica $\hat{\Pi}_2(0) = \hat{\Pi}_2(q^2) - \Pi_2(0) = + \frac{2\alpha_R}{\pi} \int_0^1 dx (1-x) \ln \frac{m_R^2 - x(1-x)q^2}{m_R^2}$

• Lambov premik (pri nizkih energijah)

$$|q|^2 \ll m_R^2$$

$$q^2 = q_0^2 - |\vec{q}|^2 = -|\vec{q}|^2$$



$$\Rightarrow \hat{\Pi}_2 = \frac{2\alpha_R}{\pi} \int dx (1-x) x(1-x) \frac{|q|^2}{m_R^2} = \neq \propto_R \frac{|q|^2}{m_R^2}$$

$$V(\vec{r}) = \underbrace{\int \frac{d\vec{q}}{(2\pi)^3} e^{i\vec{q}\vec{r}}}_{\text{F.T.}} \tilde{V}(\vec{q})$$

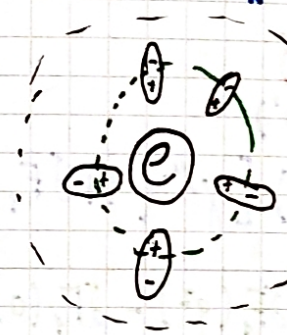
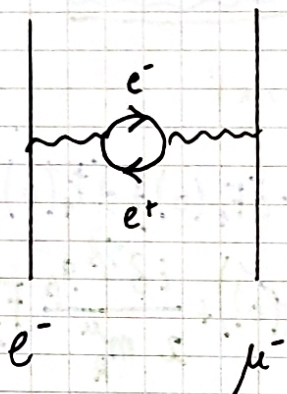
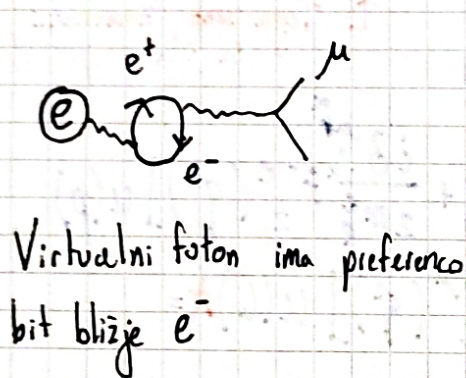
drevesni led
= najnižji red brez
zanka

dr. red.

$$V(\vec{r}) = \text{F.T.} \frac{+|e_R|^2}{|\vec{q}|^2} = + \frac{|e_R|^2}{4\pi r}$$

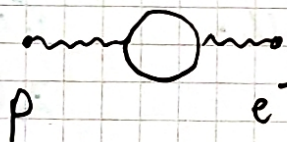
$$V(\vec{r}) = \int \frac{d\vec{q}}{(2\pi)^3} e^{i\vec{q}\vec{r}} \cdot \frac{|e_R|^2}{|\vec{q}|^2} (1 + \# |\vec{q}|^2 \alpha_R \frac{1}{m_R^2})$$

$$= \frac{\alpha_R}{r} + \frac{4}{15} \frac{\alpha_R^2}{m_R^2} \delta(\vec{r})$$



Merjan naboj
izmerimo
Izmerimo goli
naboj, ki je
neskončen, zato $\delta(\vec{r})$

Dokaz za ta efekt dobimo iz atomskih spektrov (Slide 53)



Razlika $\approx 10^{-7} \text{ eV}$

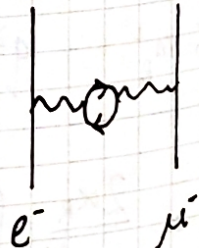
Merimo razliko med 2s in 2p orbital. Če tega efekta ne bi bilo
 $\uparrow$ čuti δ $\uparrow$ nič v izhodišču
in ne čuti δ
bi ta dva stanja imela enako energijo.

• • Tehnica (running) / shlopitvena konstanta $e_R(q^2)$ (visoke energije)

$-q^2 \gg m_R^2$
 $\uparrow$
Velika
Virtualnost

$$q^2 = q_0^2 - |\vec{q}|^2$$

$$\frac{+|e_R|^2}{q^2} \rightarrow \frac{|e_R|^2}{q^2 (1 - \hat{\Pi}_2(q^2))} = \overset{\#M}{\tilde{V}(q^2)}$$



$$\ln \frac{m_R - x(1-x)q^2}{m_R} \rightarrow \ln[x(1-x)] + \ln \frac{-q^2}{m_R^2}$$

$$\hat{\Pi}_2(q^2) = \frac{2\alpha_R}{\pi} \int dx x(1-x) \left(\ln[x(1-x)] + \ln\left[-\frac{q^2}{m_R^2}\right] \right) =$$

$$= \frac{2\alpha_R}{\pi} \left[-\frac{5}{18} + \frac{1}{6} \ln \frac{-q^2}{m_R^2} \right] :$$

$$\tilde{V}(q^2) = \frac{|e_R|^2}{q^2 \left(1 - \frac{2\alpha_R}{6\pi} \ln \frac{-q^2}{m_R^2 A} \right)}$$

$$e_R^{\text{eff}}(q^2)$$

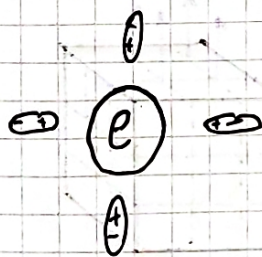
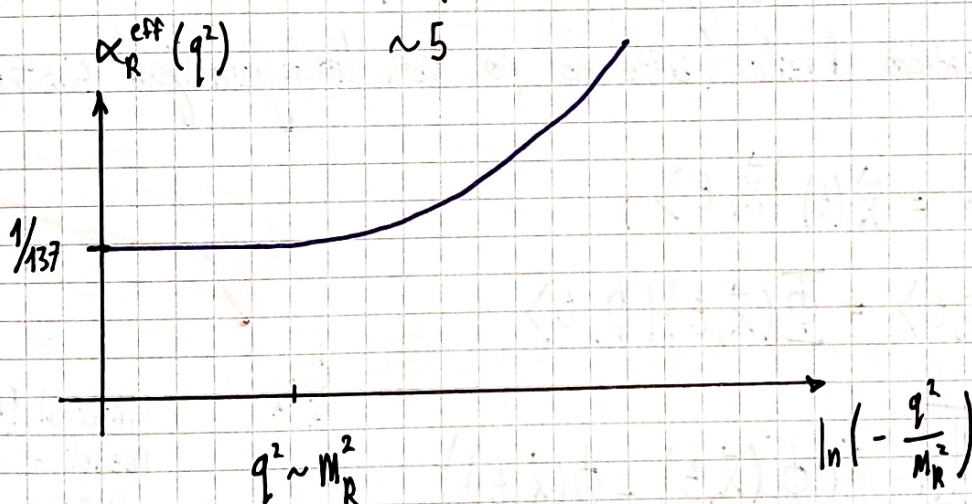
$$e_R^{\text{eff}}(0) = e_R$$

Pr:

$$-q^2 \simeq (30 \text{ GeV})^2 :$$

$$\hat{\Pi}_2(q^2=0) = 0$$

$$\frac{2 \cdot \frac{1}{137} \cdot 2}{6\pi} \ln \frac{30 \cdot 1000 \text{ MeV}}{0.5 \text{ MeV}} \approx 5\% \text{ popravch}$$



Ko imamo majhne valovne dolžine tipljemo merjeni /sončni naboj. Ko gremo k bolj dolgim sežemo globlje in bližje gledamo naboju in α_R^{eff} raste.

Kvantizacija z Feynmanovim Popotnim integralom

Source:
Schwartz
Hartman

Motivacija:

- .) Kvantizacije: kanonične ali pa s popotnim integralom
- .) Je brez operatorjev (ampak so funkcije polja)
- .) Edini pristop, ki omogoča neperturbativni izračun do poljubnega reda (računalništvo)
- .) V kanonični kvantizaciji je Hamiltonian glavni objekt:

$\exp(-i \int \mathcal{H} dt)$: pomembne simetrije $\mathcal{H}$.

Popotni integral pa: $\exp(i S/\hbar)$: pomembne simetrije $\mathcal{L}$

- .) Analogija s Statistično fiziko (seštevamo po vseh konfiguracijah sistema)

Izpeljava:

QM: $\hat{X}(t) |\vec{X}, t\rangle = \vec{X}(t) |\vec{X}, t\rangle$;

QFT: $\hat{\Phi}(\vec{x}, t) |\phi, t\rangle = \Phi(\vec{x}, t) |\phi, t\rangle$

$$\langle \phi', t | \phi, t \rangle = \prod_{\vec{x}} \delta(\phi'(\vec{x}, t) - \phi(\vec{x}, t))$$

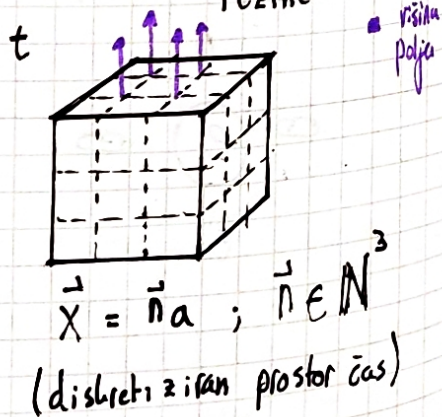
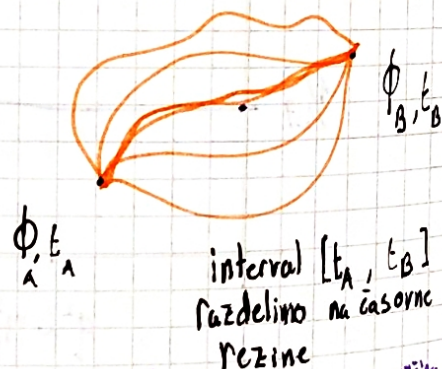
↳ Ortogonalnost

V vsaki časovni slice vstavimo identiteto:

$$\mathbb{1}(t) = \prod_{\vec{x}} \int d\phi(\vec{x}, t) |\phi, t\rangle \langle \phi, t| = (*)$$

Funkcionalni integral $\int \mathcal{D}^x \phi(\vec{x}, t)$

$$(*) = \int \mathcal{D}^x \phi(\vec{x}, t) |\phi, t\rangle \langle \phi, t|$$



→ Tvoji popolno bazo polja (Vsebuje vse možne oblike)

$$U \propto \int D^* \phi_{N+1}(t_N) \dots \int D^* \phi_1(t_1) \langle \phi_B, t_B | e^{-i\hat{H}\Delta t} | \phi_N, t_N \rangle \langle \phi_N, t_N | \dots | \phi_{N-1}, t_{N-1} \rangle \dots$$

$$\dots \langle \phi_1, t_1 | \langle \phi_1, t_1 | e^{-i\hat{H}\Delta t} | \phi_A, t_A \rangle$$

Spinnen &:

$$\hat{H} = \sum_{\vec{x}} \left(\frac{1}{2} m^2 \hat{\Phi}^2(\vec{x}) + \frac{1}{2} (\nabla \hat{\Phi})^2 + \frac{\lambda}{4} \hat{\Phi}^4 + \frac{1}{2} \hat{\pi}^2(\vec{x}) \right) \Delta \vec{x}$$

$$\Rightarrow \langle \phi', t+\Delta t | e^{-i\hat{H}\Delta t} | \phi, t \rangle =$$

$$\hat{H}_\phi: \hat{H}_\phi | \phi, t \rangle = \sum_{\vec{x}} \left(\frac{1}{2} m^2 \Phi(\vec{x}, t) + \frac{1}{2} (\nabla \Phi)^2 + \frac{\lambda}{4!} \Phi^4(\vec{x}, t) \right) \vec{x}$$

$$= H_\phi(\Phi) | \phi, t \rangle$$

$$\Rightarrow \langle \phi', t+\Delta t | e^{-i\hat{H}_\phi \Delta t} | \phi, t \rangle = e^{-iH_\phi(\phi) \Delta t} \mathcal{J}(\phi' - \phi)$$

$$\hat{H}_\pi: \langle \phi', t+\Delta t | e^{-i\hat{H}_\pi \Delta t} | \phi, t \rangle = \langle \phi', t+\Delta t | e^{-i\hat{H}_\pi \Delta t} \int D^* \pi(\vec{x}, t) / \pi, t \rangle \langle \pi, t | \phi, t \rangle =$$

$$\stackrel{\uparrow}{1} \quad \quad \quad \underbrace{\int D^* \pi(\vec{x}, t) / \pi, t \rangle \langle \pi, t | \phi, t \rangle}_{e^{-iH_\pi(\pi) \Delta t}} = (**)$$

$$\langle \vec{x}, \vec{p} \rangle = e^{i\vec{p}\vec{x}} \quad \langle \phi, t | \pi, t \rangle \propto e^{\sum_{\vec{x}} i \pi(\vec{x}) \phi(\vec{x}) \Delta \vec{x}}$$

$$\sum_{\vec{x}} \frac{1}{2} \pi^2(\vec{x}) \Delta \vec{x}$$

$$[\vec{x}, \vec{p}] = i\hbar \quad [\hat{\Phi}(\vec{x}, t), \hat{\pi}(\vec{x}', t)] = \delta(\vec{x} - \vec{x}') i\hbar$$

$$(**) = \int D^* \pi(\vec{x}, t) \langle \phi' | \pi \rangle \langle \pi | \phi \rangle e^{-iH_\pi(\pi) \Delta t}$$

$$e^{i \sum_{\vec{x}} \Delta \vec{x} \pi(\vec{x}, t) \cdot \underbrace{(\phi'(\vec{x}, t+\Delta t) - \phi(\vec{x}, t))}_{\hat{\Phi}(\vec{x}) \Delta t}} =$$

$$= \int D^x \Pi(\vec{x}) e^{-i H_\pi(\Pi) \Delta t} e^{i \underbrace{\sum_{\vec{x}} \Delta \vec{x} \Delta t \Pi(\vec{x}) \dot{\phi}(\vec{x})}_{\text{"smrdi" po Lagrangianu}}}$$

Od prej (en izraz):

$$\delta(\phi' - \phi) = \prod_{\vec{x}} \delta(\phi'(\vec{x}) - \phi(\vec{x})) \propto \underbrace{\prod_{\vec{x}} \int_{-\infty}^{\infty} d\Pi(\vec{x}) e^{i(\phi'(\vec{x}) - \phi(\vec{x})) \Pi(\vec{x}) \Delta \vec{x}}}_{\int D^x \Pi(\vec{x})}$$

Zdaj lahko obe združimo:

$$\hat{H}_\pi + \hat{H}_\phi: \quad \langle \phi, t + \Delta t | e^{-i H \Delta t} | \phi, t \rangle \propto \int D^x \Pi(\vec{x}) e^{-i \sum_{\vec{x}} H_{\phi, \pi}(\phi, \Pi) \Delta \vec{x} \Delta t} =$$

expon. $\rightarrow \sum_{\vec{x}} i \Delta \vec{x} \Delta t (\dot{\phi}(\vec{x}) \Pi(\vec{x}))$

$$= \int D^x \Pi e^{i \sum_{\vec{x}} \Delta \vec{x} \Delta t (\dot{\phi} \Pi - \mathcal{H})} = \left(\begin{smallmatrix} * \\ * \\ * \end{smallmatrix} \right)$$

Integral po $D^x \Pi(\vec{x})$

Pri danem $\vec{x}, t$ dobimo:

$$\int_{-\infty}^{\infty} d\Pi(\vec{x}) e^{i \Delta \vec{x} \Delta t \left(\dot{\phi} \Pi - \frac{1}{2} \Pi^2 \right)} =$$

(drugi neodrisani od Π in
nas tu ne zanimajo)

na pulce kvadrata

$$= \int_{-\infty}^{\infty} d\Pi(\vec{x}) e^{i \Delta \vec{x} \Delta t \left(-\frac{1}{2} (\Pi - \dot{\phi})^2 + \frac{1}{2} \dot{\phi}^2 \right)}$$

$\underbrace{\Pi - \dot{\phi}}_{\pi'}$

neha številk

(prop. faktor nas tu ne zanima, bolj zoprno)

$$= \text{oz.} \propto e^{+ \Delta \vec{x} \Delta t \frac{1}{2} \dot{\phi}(\vec{x})};$$

$$\Rightarrow \left(\begin{smallmatrix} * \\ * \\ * \end{smallmatrix} \right) = \text{oz.} \propto e^{i \sum_{\vec{x}} \Delta \vec{x} \Delta t \left(-\mathcal{H}(\phi + \frac{1}{2} \dot{\phi}^2) \right)}$$

$\mathcal{L}(\phi, \partial_\mu \phi)$

$$\left[\frac{1}{2} \dot{\phi}^2 - \frac{1}{2} (\nabla \phi)^2 - \frac{m^2}{2} \phi^2 - \frac{\lambda}{4!} \phi^4 \right]$$

$$= e^{i \sum_{\vec{x}} \Delta \vec{x} \Delta t \mathcal{L}(\vec{x})} !$$

Torej: $\int \mathcal{D}\phi(t) \dots$ Vsota po vseh konf. v prostoru in času

$$U_{\alpha} \int \mathcal{D}^* \phi_0 \dots \int \mathcal{D}^* \phi_1(t_1) \langle \phi_0, t_0 | e^{-i \hat{H} \Delta t} | \phi_1, t_1 \rangle \langle \phi_1, t_1 | \dots | \phi_{N-1}, t_{N-1} \rangle \dots \langle \phi_N, t_N | e^{-i \hat{H} \Delta t} | \phi_N, t_N \rangle$$

$$\cdot e^{i \sum_{\vec{x}} \Delta \vec{x} \sum_{t_0}^{t_N} \Delta t \mathcal{L}(\phi(\vec{x}, t), \partial_{\mu} \phi(\vec{x}, t))}$$

$$U_{\alpha} \int \mathcal{D}\phi(\vec{x}, t) e^{i S[\phi(\vec{x}, t)] / \hbar}$$

$$\phi(\vec{x}, t_A) = \phi_A(\vec{x})$$

$$\phi(\vec{x}, t_B) = \phi_B(\vec{x})$$

Schwartz 14.2.2

Remember: Za zvebit operatorjev

$$\hat{\phi}(\vec{x}, t) |\phi, t\rangle = \phi(\vec{x}, t) |\phi, t\rangle$$

↓
lastno stanje

Klasična limita

$$\hbar \rightarrow 0, [\hat{x}, \hat{p}] \Rightarrow 0$$

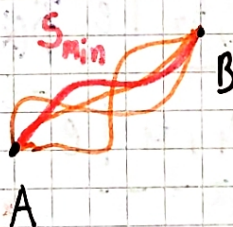
QM:

bliznja pot minimalni:

$$\vec{x}(t) \approx \vec{x}_d(t) + \delta \vec{x}(t); \quad \frac{\delta S}{\delta x} = 0, e^{i S / \hbar} \text{ prispevki se koherentno seštevajo}$$

Za oddaljene poti:

$$\vec{x}(t) \neq \vec{x}_d(x) + \delta \vec{x}(t); \quad \frac{\delta S}{\delta x} \neq 0, \delta \vec{x} \text{ prispevki se pohajšajo in odštejejo}$$

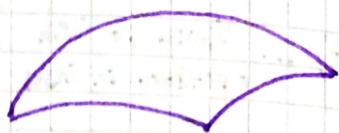


QFT:

$$S(\phi)$$

$$S_{\min}[\phi]$$

za: $\phi = \phi_d + \delta\phi, \frac{\delta S}{\delta\phi} = 0, e^{iS[\phi]/\hbar}$



min konfiguracija

↳ prispevki se seštevajo

$\phi \neq \phi_d, \frac{\delta S}{\delta\phi} \neq 0 \rightarrow$ prispevki se polnirajajo

Naprej proučujmo 7:

$$\phi(\vec{x}_1, t_1) |\phi_1, t_1\rangle = \hat{\phi}(\vec{x}_1, t_1) |\phi_1, t_1\rangle \quad \left. \vphantom{\phi(\vec{x}_1, t_1)} \right\} \text{plan kot prej}$$

Identiteta

Torej:

$$C_{DS} \propto \int D\phi_1 \int D\phi_2 \langle \phi_B, t_B | e^{-i\hat{H}(t_B - t_2)} \hat{\phi}(\vec{x}) | \phi_2, t_2 \rangle \cdot$$

right side

$$\cdot \langle \phi_2, t_2 | e^{-i\hat{H}(t_2 - t_1)} \hat{\phi}(\vec{x}) | \phi_1, t_1 \rangle \cdot$$

Identiteta

$$\cdot \langle \phi_1, t_1 | e^{-i\hat{H}(t_1 - t_A)} | \phi_A, t_A \rangle$$

*Velja:

$$e^{-i\hat{H}t} \hat{\phi}(\vec{x}) e^{i\hat{H}t} = \hat{\phi}(\vec{x}, t)$$

$$\Rightarrow C_{DS} \propto \langle \phi_B, t_B | e^{-i\hat{H}t_B} T(\hat{\phi}(\vec{x}, t_2) \hat{\phi}(\vec{x}, t_1)) e^{i\hat{H}t_A} | \phi_A, t_A \rangle$$

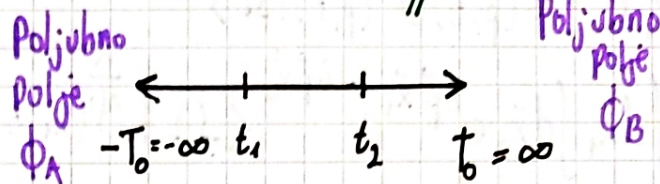
ker mora biti
visti red časov

Limita ko $t_A \rightarrow -\infty(1-i\epsilon)$, to bo dalo korelacijsko f.:

$$\lim_{t_A \rightarrow -\infty(1-i\epsilon)} e^{i\hat{H}t_A} | \phi_A, t_A \rangle = \sum_n e^{-iE_n \infty(1-i\epsilon)} | n \rangle \langle n | \phi_A, -\infty \rangle \propto | \Omega \rangle$$

$$\Rightarrow \lim_{\substack{t_A \rightarrow -\infty(1-i\epsilon) \\ t_B \rightarrow +\infty(1-i\epsilon)}} C_{05} \propto \langle \Omega | T(\hat{\phi}(x_1) \hat{\phi}(x_2)) | \Omega \rangle$$

Veljati mora
 $\langle \Omega | \phi_A \rangle \neq 0$
 $\langle \phi_B | \Omega \rangle \neq 0$



$$\langle \Omega | T(\hat{\phi}(x_1) \hat{\phi}(x_2)) | \Omega \rangle \propto \int \mathcal{D}\phi \phi(x_1) \phi(x_2) e^{iS/\hbar} ; S = \int_{-T_0}^{T_0} dt \int d\vec{x} \mathcal{L}[\phi(x)]$$

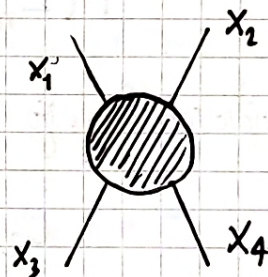
$\uparrow$
 že normalizacija $\frac{1}{\int \mathcal{D}\phi e^{iS/\hbar}}$

Za 0-točkovno funkcijo:

$$\langle \Omega | 1 | \Omega \rangle = \frac{\int \mathcal{D}\phi 1 e^{iS/\hbar}}{\int \mathcal{D}\phi e^{iS/\hbar}} = 1$$

$$Z_{T_0}[\phi] = \int \mathcal{D}\phi e^{iS[\phi]/\hbar} \quad \left. \vphantom{\int \mathcal{D}\phi e^{iS[\phi]/\hbar}} \right\} \text{Particijska funkcija / fazna vsota}$$

Vaja: $\langle \Omega | T(\hat{\phi}(x_1) \hat{\phi}(x_2) \hat{\phi}(x_3) \hat{\phi}(x_4)) | \Omega \rangle =$
 $= \frac{1}{Z_0} \int \mathcal{D}\phi \phi(x_1) \phi(x_2) \phi(x_3) \phi(x_4) e^{iS[\phi]/\hbar}$



V QED: določi maso pozitronija $\bar{e}e^+ m(j^P = 1^-) = ?$

Imamo: $\mathcal{L}_{QED}, \psi, A^\mu$

$$\langle \Omega | \sigma(t) \sigma^\dagger(t) | \Omega \rangle = \sum_n A_n e^{-iE_n t}$$

$\uparrow$
 $\sum_n |n\rangle \langle n|$

Pa recimo $p=0$ ker
 potem $E_n = mc^2$

Recimo primerni operator (primerna kvantna števila):

$$\hat{\sigma}(\vec{x}, t) = \hat{\bar{\psi}}_{i\alpha} \gamma_3 \hat{\psi}(\vec{x}, t)$$

$$\hat{\sigma}(\vec{p}, t) = \int d\vec{x} \hat{\sigma}(\vec{x}, t) e^{i\vec{x} \cdot \vec{p}}$$

$\uparrow$
 operator z dobro lega

FT
 $\rightarrow$

$\uparrow$
 operator z dobro globalno uveljavljeno

Torej:

$$\gamma_3 = \gamma_2$$

$$\begin{aligned} & \langle \Omega | \hat{\sigma}(\vec{p}, t) \hat{\sigma}^\dagger(\vec{p}, t) | \Omega \rangle = \\ & = \int d\vec{x}_1 \int d\vec{x}_2 e^{i\vec{p}\vec{x}_1} e^{-i\vec{p}\vec{x}_2} \langle \Omega | \hat{\Psi}(x_1, t) \gamma_3 \hat{\Psi}^\dagger(x_1, t) [\hat{\Psi}^\dagger(x_2, 0) \gamma_3 \hat{\Psi}(x_2, 0)]^\dagger | \Omega \rangle. \\ & = \int d\vec{x}_1 \int d\vec{x}_2 e^{i(\vec{x}_1 - \vec{x}_2) \cdot \vec{p}} \frac{1}{Z_0} \int \mathcal{D}\Psi \int \mathcal{D}\bar{\Psi} \int \mathcal{D}A^\mu \bar{\Psi}(x_1, t) \gamma_3 \Psi(x_1, t) (\bar{\Psi}(x_2, 0) \gamma_3 \Psi(x_2, 0))^\dagger \\ & = \sum_{\vec{p}=0} A_n e^{-im_n t / \hbar} \end{aligned}$$

↑ različna masna lastna stanja pozitronija

QFT pri končni temperaturi T

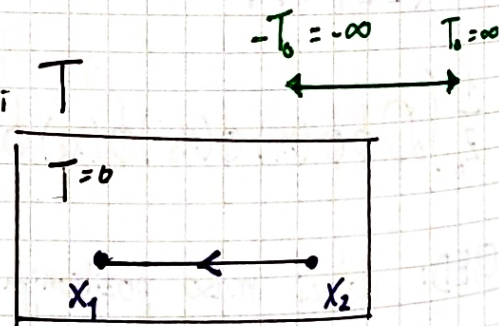
Termično ravnovesje. Imamo ensambel stanj,

$$P(E_n) \propto e^{-E_n / kT} \rightarrow \begin{array}{l} \text{energija stanja} \\ \text{(oz. frekvenca zanj)} \\ \propto \text{z Boltzmann porazdelitvijo} \end{array}$$

Imejmo dvotočkovno funkcijo v skalarni teoriji:

$$C_2(x_1, x_2) \dots \text{ pri končni } T$$

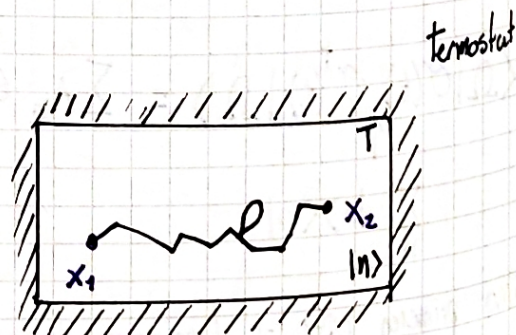
$$C_2^{T=0}(x_1, x_2) = \langle \Omega | T(\hat{\phi}(x_1) \hat{\phi}(x_2)) | \Omega \rangle$$



Kaj pa pri $T > 0$?

$$C_2^T(x_1, x_2) = \sum_n \langle n | \hat{\phi}(x_1) \hat{\phi}(x_2) | n \rangle e^{-E_n / kT}$$

$\frac{1}{\sum_n \langle n | n \rangle e^{-E_n / kT}}$



Evklidski čas

$$t_E = 0$$

$$t_E = 1/kT$$

$$t = -it_E$$

Kakšna je particijska funkcija?

$$Z_T \equiv \sum_n \langle n | e^{-\hat{H}/\hbar T} | n \rangle$$

↳ Lastna stanja
Hamiltoniana

↳ To zgleda kot
sled operatorja

$$\text{Tr}[\hat{O}] = \sum_m \langle e_m | \hat{O} | e_m \rangle$$

neodvisno od baze, ki jo
vzamemo

Spomni se:

$$\text{Unitary: } U^\dagger = U^{-1}$$

$$Z_T = \text{Tr}[e^{-\hat{H}/\hbar T}] \quad \hat{H}|n\rangle = E_n|n\rangle, \quad |n\rangle \rightarrow |\phi, t\rangle$$

Pri fiksnem času t :

$$Z_T = \int \mathcal{D}^x \phi_0 \langle \phi_0, t | e^{-\hat{H}/\hbar T} | \phi_0, t \rangle$$

lastna
stanja
polja

$$\rightarrow \text{analogno: } \langle \phi_B, t_B | e^{-i\hat{H}(t_B-t_A)} | \phi_A, t_A \rangle$$

ko:

$$|\phi_A\rangle \rightarrow |\phi_0\rangle$$

$$|\phi_0\rangle \rightarrow |\phi_0\rangle$$

$$\frac{1}{\hbar T} = i(t_B - t_A)$$

$$\begin{array}{cc} \phi_0 & \phi_0 \\ \hline t_E^A = 0 & t_E^B = 1/\hbar T \end{array}$$

↳ ravnovesna stanja
lastne vrednosti so časovno neodvisne

Matematični trik:

→ podobno kot evolucija v
imaginarnem evklidskem času

izberemo si pa:

$$t_E^A = 0 \quad t_E^B = 1/\hbar T$$

$$t_E = -it$$

$$\Rightarrow Z_T = \int \mathcal{D}^x \phi_0 \int \mathcal{D} \phi e^{iS}$$

basically
periodični
robni pogoji

$$\begin{cases} \phi(0) = \phi_0 \\ \phi(t_E = 1/\hbar T) = \phi_0 \end{cases}$$

evklidska akcija

$$= \int \mathcal{D} \phi e^{-S_E/\hbar}$$

$$\phi(0) = \phi_0 = \phi(1/\hbar T)$$

$$S = i \int_0^{1/\hbar T} dt \int d\vec{x} \mathcal{L}$$

Več o evklidski aluciji na prosopnicah slide 10.

$$S = -i \int dt^E \int d\vec{x} (-\mathcal{L}_E) = i S_E$$

Ta evklidski čas etc. je zgolj matematični trik, lahko lahko izrazimo / izračunamo popolni integral.

$$C_T(x_1, x_2) = \frac{1}{Z_T} \int \mathcal{D}\phi \phi(x_1) \phi(x_2) e^{-S_E[\phi]}$$

Spomni se:

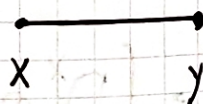
$$\int \mathcal{D}\phi = \prod_{x,t} d\phi(x,t) \quad \rightarrow \text{oz. } t \in \mathbb{E}$$

LSZ teorem

Povezava med $C_2, Z, E_n, |n\rangle$
↑
dvotična

Hočemo pokazati: $\sqrt{Z} = \langle \Omega | \hat{\phi}(0) | S(\vec{p}=0) \rangle$

brez interakcij: $Z=1$



$$C_2(x, y) = \langle \Omega | \hat{\phi}(x) \hat{\phi}(y) | \Omega \rangle = (*)$$

$$1 = \sum_{\vec{q}} \frac{d\vec{q}}{(2\pi)^3} \frac{1}{2E_{\vec{q}}} |\lambda_{\vec{q}}\rangle \langle \lambda_{\vec{q}}|$$

$\vec{q}$ ↙ lastna stanja ↘ Osvetl
 $|\lambda_{\vec{q}}\rangle: |S(\vec{q})\rangle, |B\rangle, |S(\vec{q}-\vec{q}')q(\vec{q}')\rangle, \dots$: Fockov prostor stanj
(SS) (tudi več delna stanja)

$\vec{q} \neq 0$ dobimo z potiskom iz stanj z $\vec{q} = 0$.

$$|\lambda_{\vec{q}}\rangle$$

$$|\lambda_0\rangle$$

$$m_H: m_R, m_B, \dots \quad E_{\vec{q}} = \sqrt{m_R^2 + \vec{q}^2}$$

$$(*) = \sum_{\lambda} \int \frac{d\vec{q}}{(2\pi)^3} \frac{1}{2E_{\vec{q}}} \langle \Omega | \hat{\phi}(x) | \lambda_{\vec{q}} \rangle \langle \lambda_{\vec{q}} | \phi(y) | \Omega \rangle = (**)$$

$$\begin{aligned} \langle \Omega | \hat{\phi}(x) | \lambda_{\vec{q}} \rangle &= \langle \Omega | \underbrace{e^{+i\hat{p}_x}}_{\langle \Omega |} \hat{\phi}(0) \underbrace{e^{-i\hat{p}_x}}_{e^{-iqx} | \lambda_{\vec{q}} \rangle} | \lambda_{\vec{q}} \rangle = \\ &= e^{-iqx} \langle \Omega | \hat{\phi}(0) | \lambda_{\vec{q}} \rangle = \end{aligned} \quad q^0 = E_{\vec{q}}$$

Naredimo Lorentzov premik nazaj v mirujoči sistem:

$$= e^{-iqx} \langle \Omega | U^{-1} U \hat{\phi}(0) U^{-1} U | \lambda_{\vec{q}} \rangle = \langle \Omega | \hat{\phi}(0) | \lambda_0 \rangle e^{-iqx}$$

$$(**) = \sum_{\lambda} \int \frac{d\vec{q}}{(2\pi)^3} \frac{1}{2E_{\vec{q}}} \underbrace{|\langle \Omega | \hat{\phi}(0) | \lambda_0 \rangle|^2}_{\int \frac{d\vec{q}}{(2\pi)^3} \frac{1}{q^2 - m_\lambda^2 + i\epsilon}} e^{(-iq(x-y))}$$

Kakšna je korelacijska funkcija v momentnem prostoru?

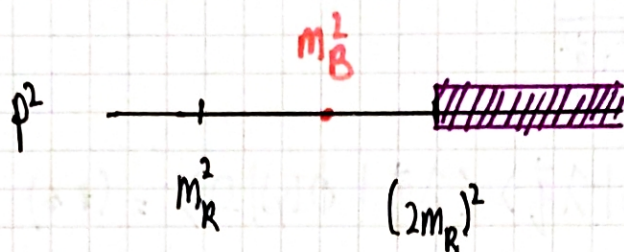
$$C_2(p) = \int d^4x e^{ipx} C_2(x, 0)$$

$$= \int d^4x e^{ipx} \sum_{\lambda} \frac{d^4\vec{q}}{(2\pi)^4} \frac{i}{q^2 - m_\lambda^2 + i\epsilon} e^{-iqx} |\langle \Omega | \hat{\phi}(0) | \lambda_0 \rangle|^2 =$$

$$= \sum_{\lambda} \frac{i}{p^2 - m_\lambda^2 + i\epsilon} |\langle \Omega | \hat{\phi}(0) | \lambda_0 \rangle|^2 =$$

Za en sam delce od tistih, ki so eno ven:

$$\begin{aligned} &= \frac{i |\langle \Omega | \hat{\phi}(0) | s(\vec{p}=0) \rangle|^2}{p^2 - m_\lambda^2 + i\epsilon} + \sum_{\lambda \neq s} \frac{i}{p^2 - m_\lambda^2} |\langle \Omega | \hat{\phi}(0) | \lambda_0 \rangle|^2 \end{aligned}$$



$$C_2(p) \approx \frac{i\sqrt{Z}}{p^2 - m_R^2 + i\epsilon} \quad \longleftrightarrow \quad C_2(p) = \frac{i\sqrt{Z}}{p^2 - m_R^2 + i\epsilon}$$

analogno

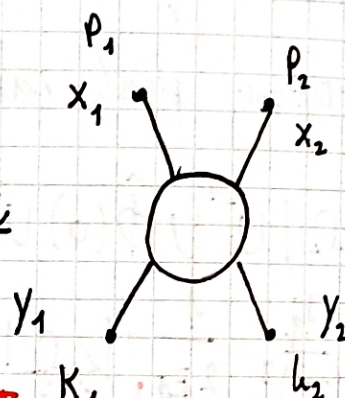
dobili smo pa

$$\hookrightarrow \frac{i |\langle \Omega | \hat{\phi}(0) | S(\vec{p}=0) \rangle|^2}{p^2 - m_R^2 - i\epsilon} + \dots \Rightarrow \sqrt{Z} = K |\Omega | \hat{\phi}(0) | S(\vec{p}=0) |$$

LSZ theorem

$$\prod_{i=1,2} \int d^4x_i e^{ip_i x_i} \prod_{j=1,2} \int d^4y_j e^{-ik_j y_j} \langle \Omega | T(\hat{\phi}(x_1) \hat{\phi}(x_2) \hat{\phi}(y_1) \hat{\phi}(y_2)) | \Omega \rangle \approx$$

↑
izhodni vhodni zato - v exp!



$$\approx \prod_{i=1,2} \frac{i\sqrt{Z}}{p_i^2 - m_R^2 + i\epsilon} \prod_{j=1,2} \frac{i\sqrt{Z}}{k_j^2 - m_R^2 + i\epsilon} \langle \vec{p}_1, \vec{p}_2 | \hat{S} | \vec{k}_1, \vec{k}_2 \rangle$$

Sipalna matrika

$$\hat{S} = e^{-2i\Gamma H}$$

Prvo pokazimo:

$$FT_1 C_4 = \int d^4x_1 e^{ip_1 x_1} \langle \Omega | \hat{\phi}(x_1) \phi\phi\phi | \Omega \rangle =$$

↑
ideja je da dobimo en delec

$$= \int d^4x_1 e^{ip_1 x_1} \sum_{\vec{q}} \int \frac{d\vec{q}}{(2\pi)^4} \frac{1}{2E_{\vec{q}}} \langle \Omega | \hat{\phi}(x_1) | \lambda_{\vec{q}} \rangle \langle \lambda_{\vec{q}} | \phi\phi\phi \rangle =$$

$\underbrace{\int \frac{d^4q}{(2\pi)^4} \frac{i}{q^2 - m_\lambda^2 + i\epsilon}}_{\text{red}} \underbrace{e^{-iqx_1} \langle \Omega | \hat{\phi}(0) | \lambda_0 \rangle}_{\text{purple}}$

$$= \sum_{\lambda} \frac{i \langle \Omega | \hat{\phi}(0) | \lambda_0 \rangle}{p_1^2 - m_\lambda^2 + i\epsilon} \langle \lambda_{\vec{p}_1} | \phi(x_2) \phi \phi | \Omega \rangle \approx$$

Zanima nas izraz, ko gre $p_1^2 \rightarrow m_R^2$ oz misljeno $E_{p_1} = p_1^0$

edini, ki bo
zdaj prispeval

renormalizirana masa

$$|\lambda\rangle = |s(\vec{p}_1)\rangle$$

Skalarni delci z gibalno
količino $\vec{p}_1$

$$\approx \frac{i \sqrt{Z} \langle \Omega | \hat{\phi}(0) | s(\vec{0}) \rangle}{p_1^2 - m_R^2 + i\epsilon} \cdot \underbrace{\langle s(\vec{p}_1) | \hat{\phi}(x_2) \hat{\phi}(y_1) \hat{\phi}(y_2) | \Omega \rangle}_{\vec{p}_1} + \dots$$

Iz polja smo naredili delec.

Zdaj bi radi iz dvodelčnega stanja prideli dva delca.

$$FT_{x_1, x_2} C_4 = \int d^4x_1 e^{ip_1 x_1} \int d^4x_2 e^{ip_2 x_2} \int \frac{d^4q_1}{(2\pi)^4} \frac{i}{q_1^2 - m_R^2} \int \frac{d^4q_2}{(2\pi)^4} \frac{i}{q_2^2 - m_R^2} \cdot$$

4. točkorna
korelacijska
funkcija

$$I = \int \frac{d\vec{q}_1}{(2\pi)^3} \frac{1}{2E_{\vec{q}_1}} \int \frac{d\vec{q}_2}{(2\pi)^3} \frac{1}{2E_{\vec{q}_2}} |s(\vec{q}_1) s(\vec{q}_2)\rangle \langle s(\vec{q}_1) s(\vec{q}_2)|$$

$$\cdot \underbrace{\langle \Omega | \hat{\phi}(x_1) \hat{\phi}(x_2) | s(q_1) s(q_2) \rangle \langle s(q_1) s(q_2) | \phi \phi | \Omega \rangle}_{e^{-iq_1 x} \sqrt{Z} e^{-iq_2 x_2} \sqrt{Z}} =$$

$$= \frac{i\sqrt{Z}}{p_1^2 - m_R^2} \frac{i\sqrt{Z}}{p_2^2 - m_R^2} \langle \vec{p}_1 \vec{p}_2 | \phi \phi \rangle$$

$$\text{če bi vstavili } \langle p_1 p_2 | I | \phi \phi \rangle \rightarrow \underbrace{\text{out} \langle \vec{p}_1 \vec{p}_2 | \vec{u}_1 \vec{u}_2 \rangle}_{\text{to je v Heisenbergovi sliki}}$$

to je v Heisenbergovi sliki

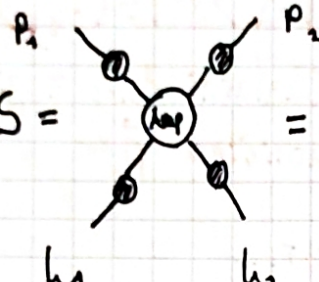
$$\rightarrow \langle \vec{p}_1 \vec{p}_2 | \hat{S} | \vec{u}_1 \vec{u}_2 \rangle \} \text{ ista reč v Schrödingerjevi sliki}$$

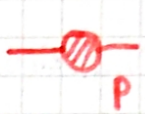
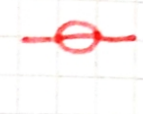
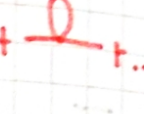
Zveza med golo $\Gamma^{(n)}$ in renormalizirano $\Gamma_R^{(n)}$

$$\Gamma_R^{(4)} = (\sqrt{Z})^4 \Gamma^{(4)}$$

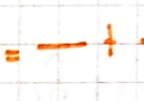
oz. $\Gamma_R^{(n)} = (\sqrt{Z})^n \Gamma^{(n)}$

Vzemimo levo stran LSZ teorema:

LS =  = ~~XXXX~~ (X)

 =  +  +  + ... = $\frac{iZ}{p^2 - m_R^2}$

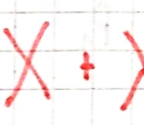
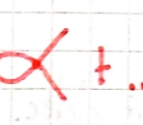
DN.

=  + ...

$Z = ?$
 $m_R = ?$

One particle irreducible

Vsi popravki

 =  +  + ... = $\Gamma^{(4)}$

$\overset{p_i^2 \approx m_R^2}{(X)} \overset{\uparrow}{\approx} \Gamma^{(4)} \prod_{i=1,2} \frac{iZ}{p_i^2 - m_R^2 + i\epsilon} \prod_{j=1,2} \frac{iZ}{h_j^2 - m_R^2 + i\epsilon} (2\pi)^4 \delta(\sum p) =$

Primerjamo to zdaj z desno stranjo

= $\prod \frac{i\sqrt{Z}}{\dots} \prod \frac{i\sqrt{Z}}{\dots} \underbrace{\langle \vec{p}_1 \vec{p}_2 | i\hat{T} | \vec{h}_1 \vec{h}_2 \rangle}_{i\mathcal{M} (2\pi)^4 \delta(\sum p)}$

$i\mathcal{M} (2\pi)^4 \delta(\sum p)$

$\Gamma_R^{(4)}$

LS

LSZ

DS

=>

$\Gamma^{(4)} / Z^4$

= $Z^2 / \Gamma_R^{(4)}$

$\hookrightarrow \Gamma_R^{(4)} = Z^2 \Gamma^{(4)}$

Renormalizacija z metodo protičkenov

$$\mathcal{L}_{QED} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\Psi}(i\gamma - m)\Psi - e \bar{\Psi} \gamma^\mu \Psi A_\mu; \quad \Psi = \sqrt{Z_2} \Psi_R$$

$$A_\mu = \sqrt{Z_3} A_{\mu R}$$

$$= -\frac{1}{4} Z_3 F_{\mu\nu}^R F_R^{\mu\nu} + Z_2 \bar{\Psi}_R (i\gamma - m) \Psi_R - e Z_2 \sqrt{Z_3} \bar{\Psi}_R \gamma^\mu \Psi_R A_{\mu R} = \text{Delamo se, da ne vemo:}$$

$$Z_2 = 1 + \delta_2 \quad \swarrow \text{protičken}$$

$\mathcal{L}_0$

$\mathcal{L}_{\text{Finite}}$

$$Z_3 = 1 + \delta_3$$

$$= \left(-\frac{1}{4} F_{\mu\nu}^R F_R^{\mu\nu} + \bar{\Psi}_R (i\gamma - m_R) \Psi_R - e_R \bar{\Psi}_R \gamma^\mu \Psi_R A_{\mu R} \right)$$

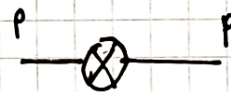
$$Z_1 = 1 + \delta_1$$

$$Z_2 m = m_R + \delta_m$$

$$+ \left(-\frac{1}{4} \delta_3 F_{\mu\nu}^R F_R^{\mu\nu} + \bar{\Psi}_R (i\delta_2 \gamma - \delta_m) \Psi_R - \delta_1 e_R \bar{\Psi}_R \gamma^\mu \Psi_R A_{\mu R} \right)$$

$\mathcal{L}_{\text{CounterTerms}}$

Vaja: $\langle e(p) | \bar{\Psi}_R (i\delta_2 \gamma - \delta_m) \Psi_R(x) | e(p) \rangle$



$$i \mathcal{L}_{\text{int}} = -i \mathcal{H}_{\text{int}}$$

$$\langle e(p) | i \bar{\Psi}_R (i\delta_2 \frac{\partial}{\partial x^\mu} \gamma_\mu - \delta_m) \Psi_R(x) | e(p) \rangle =$$

$$= \langle e(p) | i \bar{\Psi}_R(x) (i\delta_2 \int \frac{d\vec{q}}{(2\pi)^3} \frac{1}{\sqrt{2E_q}} e^{-iqx} \underbrace{a_{\vec{q}}(-iq)^\mu \gamma_\mu - \delta_m}_{\text{zaradi odzoda}} \hat{\Psi}_R(x) \sqrt{2E} a_p^\dagger | 0 \rangle =$$

$$= \underbrace{e^{+ipx}}_{\text{Prispevek ene noge}} \underbrace{\bar{u}(p) (i\delta_2 (-i\not{p}) - \delta_m)}_{\text{Prispevek vozlišča}} \underbrace{e^{-ipx} u(p)}_{\text{prispevek druge noge}}$$

$$\downarrow$$

$$i(\delta_2 \not{p} - \delta_m)$$

Zdaj je treba določiti še protičken...