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$$\boxed{dW_e = EVdP}$$

ker je V konst.

$\rho = 800\,kg/m^3$ pri $T_1 = 27°C$

$E = 10^7\,V/m$

$\chi_T = 2$ ... pri pogojih ↑↑

$$\frac{\chi}{\chi+3} \propto 1 + \frac{C}{T} \quad ; \quad C = 30K$$

P = polarizacija

$$= \frac{\sum \vec{P}_e}{V}$$

$$\vec{P} = \chi \varepsilon_0 \vec{E}$$

$V = konst.$

$c_p = 1700\,J/kgK$

Enačba stanja:

$c_E - c_p = ?$

$\chi_T - \chi_S = ?$

Termodinamske spremenljivke:

T, E, P

Adiabatno? $dS = 0$?

$$S = S(T,E)$$

$$dS = \left(\frac{\partial S}{\partial T}\right)_E dT + \left(\frac{\partial S}{\partial E}\right)_T dE = 0$$

$$= \frac{mc_E}{T} dT + \underbrace{\left(\frac{\partial S}{\partial E}\right)_T}_{\text{Išči transformacijo}} dE = 0$$

$$S = S(T,P)$$

$$dS = \left(\frac{\partial S}{\partial T}\right)_P dT + \left(\frac{\partial S}{\partial P}\right)_T dP$$

$$= \frac{mc_p}{T} dT + \underbrace{\left(\frac{\partial S}{\partial P}\right)_T}_{\text{Išči transformacijo}} dP$$

Maxwellove relacije:

$$dU = dQ + dW$$
$$= TdS + EVdP$$

Enačba stanja?? :

$$\frac{dE}{E} = \alpha dT + \chi dP$$

Tako zadevo bi rabil kmalu...

Jo imamo!

Torej potenciali:

$$dU = TdS + EVdP \Rightarrow \underline{\underline{U(S,P)}}$$

$$H = U - EVp$$
$$= TdS + \cancel{EVdp} - \cancel{EVdp} - PVdE$$
$$dH = TdS - PVdE \Rightarrow \underline{\underline{H(S,E)}}$$

$$F = U - TS$$
$$\cancel{= TdS - PVdE - TdS - SdT}$$
$$\cancel{dF = -SdT - PVdE \Rightarrow}$$
$$= TdS + EVdP - TdS - SdT$$
$$dF = -SdT + EVdP \Rightarrow \underline{\underline{F(T,P)}}$$

$$G = F - EVP$$
$$= -SdT + EVdP - EVdP - PVdE$$
$$= -SdT - PVdE \Rightarrow \underline{\underline{G(T,E)}}$$

1. $dF = -SdT + EVdP$

$$\frac{\partial^2 F}{\partial T \partial P} = \frac{\partial^2 F}{\partial P \partial T}$$

$$\frac{\partial}{\partial T}\left(\frac{\partial F}{\partial P}\right)_T = \frac{\partial}{\partial T}(EV) = \frac{\partial}{\partial P}\left(\frac{\partial F}{\partial T}\right)_P = \frac{\partial}{\partial P}(-S)$$

$$\Rightarrow \left(\frac{V \partial E}{\partial T}\right)_P = \left(\frac{-\partial S}{\partial P}\right)_T$$ Maxwellova relacija 1.

2. $dG = -SdT - PVdE$

$$\frac{\partial^2 G}{\partial T \partial E} = \frac{\partial^2 G}{\partial E \partial T}$$

$$\frac{\partial}{\partial T}\left(\frac{\partial G}{\partial E}\right)_T = \frac{\partial}{\partial T}(-PV) = \frac{\partial}{\partial E}\left(\frac{\partial G}{\partial T}\right)_E = \frac{\partial}{\partial E}(-S)$$

$$\Rightarrow \left(\frac{V \partial P}{\partial T}\right)_E = \left(\frac{\partial S}{\partial E}\right)_T$$

$$= \frac{mc_E}{T}\,dT + \left(\frac{\partial S}{\partial E}\right)_T dE \qquad\qquad = \frac{mc_p}{T}\,dT + \left(\frac{\partial S}{\partial p}\right)_T dP$$

$$= \frac{mc_E}{T}\,dT + V\underbrace{\left(\frac{\partial P}{\partial T}\right)_E}_{\text{Iz enačbe stanja}} dE \qquad\qquad = \frac{mc_p}{T}\,dT - V\underbrace{\left(\frac{\partial E}{\partial T}\right)_P}_{\text{Iz enačbe stanja}} dP$$

Enačba stanja:

$$P(T,E) = \chi(T)\,\varepsilon_0 E$$

$$\frac{\chi}{\chi+3} \propto 1 + \frac{C}{T} \quad \Rightarrow \chi = \frac{3\left(k + \frac{kC}{T}\right)}{\left(1 - k - \frac{kC}{T}\right)} \; ; \; k = \frac{4}{11}$$

Enačba stanja je torej:

$$P(T,E) = \frac{3\left(k + \frac{kC}{T}\right)}{\left(1 - k - \frac{kC}{T}\right)}\,\varepsilon_0 E$$

$$\left(\frac{\partial P}{\partial T}\right)_E = -\frac{3\varepsilon_0 E kC}{(T - kT - kC)^2} \qquad\qquad \left(\frac{\partial E}{\partial T}\right)_P = \frac{PkC}{3\varepsilon_0 (kT + kC)^2}$$

$$dT\,\frac{mC_E}{T} = V\frac{3\varepsilon_0 E kc}{(T-kT-kc)^2}\,dE \qquad dT\,\frac{mC_p}{T} = \frac{VPKC}{3\varepsilon_0(kT+kc)^2}\,dP$$

$$\frac{C_E}{C_p} = \frac{\cancel{V\frac{T}{m}}\,\frac{3\varepsilon_0 E kc}{m(T-kT-kc)^2}\,dE}{\cancel{V\frac{T}{m}}\,\frac{PKC}{3\varepsilon_0(kT+kc)^2}\,dp} = \frac{(3\varepsilon_0 E kc)(3\varepsilon_0(kT+kc)^2)\,dE}{PKC\,(T-kT-kc)^2\,dp} =$$

$$= \frac{3^2\varepsilon_0^2 E\,\cancel{kc}\,(kT+kc)^2}{P\,\cancel{kc}\,(T-kT-kc)^2}\left(\frac{\partial E}{\partial P}\right)_S \qquad \ldots??$$

$$C_E = \frac{T}{m}\left(\frac{\partial S}{\partial T}\right)_E \qquad S = (T, P(T,E))$$

$$= \underbrace{\frac{T}{m}\Big[\left(\frac{\partial S}{\partial T}\right)_p}_{C_p} + \left(\frac{\partial S}{\partial p}\right)_T \cdot \left(\frac{\partial p}{\partial T}\right)_E \Big]$$

$$C_E - C_p = \frac{T}{m}\left(\frac{\partial S}{\partial p}\right)_T \left(\frac{\partial p}{\partial T}\right)_E = \frac{T}{m}\left(\frac{-V\partial E}{\partial T}\right)_p \left(\frac{\partial p}{\partial T}\right)_E =$$

$$= \frac{T}{m}(\cancel{-}V)\frac{Pkc}{\cancel{3\varepsilon_0}(kT+kc)^2}\cdot\frac{\cancel{-3\varepsilon_0}Ekc}{(T-kT-kc)^2} =$$

$$= \frac{TV}{m}k^2c^2PE\frac{1}{(kT+kc)^2(T-kT-kc)^2} =$$

$$= \frac{T\cancel{V}}{\cancel{V}\rho}k^2c^2\frac{PE}{(kT+kc)^2(T-kT-kc)^2} =$$

$$C_E - C_p = \frac{TPEk^2c^2}{\rho(kT+kc)^2(T-kT-kc)^2} = 0{,}000169\ \frac{J}{kg\,K}$$

$$\frac{mC_E}{T}\,dT = V\cdot\frac{3\varepsilon_0 E kc}{(T-kT-kc)^2}\,dE \qquad \left(\frac{\partial P}{\partial T}\right)_E$$

$$\frac{mC_p}{T}\,dT = V\cdot\frac{Pkc}{3\varepsilon_0(kT+kc)^2}\,dP$$

$$\cancel{\frac{C_p}{C_E} = \frac{\frac{mC_E}{T}dT}{\frac{mC_p}{T}dT}}$$

$$\frac{C_p}{C_E} = \frac{\cancel{V}\cancel{\frac{1}{m}}\,\frac{PKC}{3\varepsilon_0(kT+kc)^2}\,dP}{\cancel{V}\cancel{\frac{1}{m}}\,\frac{3\varepsilon_0 EKC}{(T-kT-kc)^2}\,dE}$$

$$\frac{C_p}{C_E} = \frac{P\cancel{kc}(T-kT-kc)^2}{9\varepsilon_0^2 E\cancel{kc}(kT+kc)^2}\left(\frac{\partial P}{\partial E}\right)_S$$

$$\Rightarrow \left(\frac{\partial P}{\partial E}\right)_S = \frac{C_p}{C_E}\,\frac{9\varepsilon_0^2 E(kT+kc)^2}{P(T-kT-kc)^2}$$

χ

$$P(T,E) = \chi(T)\,\varepsilon_0 E$$

$$dP = \chi(T)\,dT\,\varepsilon_0 E + \chi(T)\,\varepsilon_0\,dE$$

$$-\chi_S + \chi_T = ?$$

$$\chi_T = \frac{1}{\varepsilon_0}\left(\frac{\partial P}{\partial E}\right)_T \qquad \chi_S = \frac{1}{\varepsilon_0}\left(\frac{\partial P}{\partial E}\right)_S$$

$$dP = \underbrace{\left(\frac{\partial P}{\partial T}\right)_E}_{???} dT + \underbrace{\left(\frac{\partial P}{\partial E}\right)_T}_{\chi(T)\,\varepsilon_0} dE \rightarrow \chi_T = \frac{1}{\varepsilon_0}\left(\frac{\partial P}{\partial E}\right)_T$$

Po isti fori =>

$$\chi_S = \frac{1}{\varepsilon_0}\left(\frac{\partial P}{\partial E}\right)_S$$

$$\chi_T - \chi_S =$$

$$= \frac{1}{\varepsilon_0}\underbrace{\left(\frac{\partial P}{\partial E}\right)_T}_{\text{Zdi se mi, da } \chi(T)\varepsilon_0} - \frac{1}{\varepsilon_0}\,\frac{C_p}{C_E}\,\frac{9\varepsilon_0^2 E(\mu T + \mu C)^2}{P(T - \mu T - \mu C)^2}$$

$$\chi_T - \chi_S =$$

$$= \frac{1}{\cancel{\varepsilon_0}} \frac{3(h+\frac{hc}{T})}{(1-h-\frac{hc}{T})} \cancel{\varepsilon_0} - \frac{1}{\cancel{\varepsilon_0}} \frac{C_p}{C_E} \frac{9\varepsilon_0^{\cancel{2}} E (hT+hc)^2}{P(T-hT-hc)^2} =$$

$$PT^2\left(1-h-\frac{hc}{T}\right)^2$$

$$= \frac{3(h+\frac{hc}{T})}{(1-h-\frac{hc}{T})} - \frac{9\varepsilon_0 E C_p (hT+hc)^2}{PT^2 C_E (1-h-\frac{hc}{T})^2} =$$

$$= \frac{3(h+\frac{hc}{T})(1-h-\frac{hc}{T}) PT^2 C_E - 9\varepsilon_0 E C_p (hT+hc)^2}{PT^2 C_E (1-h-\frac{hc}{T})^2} =$$

$$= \frac{(\cancel{3h} - 3h^2 - 3\frac{h^2 c}{T} + \cancel{3\frac{hc}{T}} - \cancel{3h} - \cancel{3\frac{hc}{T}}) PT^2 C_E - 9\varepsilon_0 E C_p (hT+hc)^2}{-\|-}$$

$$= \frac{-3h^2(1-\frac{c}{T}) PT^2 C_E - 9\varepsilon_0 E C_p (hT+hc)^2}{PT^2 C_E (1-h-\frac{hc}{T})^2}$$

$C_p = 1700\ J/kgK$

$C_E = 1700......$

$P = 0{,}000177$

$h = \frac{4}{11}$

$c = 30$

$T = 300$

$$\chi_T - \chi_S = -2{,}9917....$$

Pomožni list

$$\frac{x}{x+3} \propto 1+\frac{C}{T} \quad =?$$

$$\frac{x}{x+3} = k\left(1+\frac{C}{T}\right)$$

sorazmernostna konstanta!

$$x = \left(1+\frac{C}{T}\right)(x+3)$$

$$x = x + \frac{xC}{T} + 3 + \frac{3C}{T}$$

$$\cancel{x} = \cancel{x}\left(1+\frac{C}{T}+\frac{3}{x}+\frac{3C}{x\tau}\right)$$

$$-1-\frac{C}{T} = \frac{3}{x}+\frac{3C}{xT}$$

$$-x\left(1+\frac{C}{T}\right) = 3+3\frac{C}{T} = 3\left(1+\frac{C}{T}\right)$$

$$-x = \frac{3+3\frac{C}{T}}{1+\frac{C}{T}} \quad , 3 ???$$

???

$T = 27°C$ $C = 30K$ $\Rightarrow x = 2$
$= 300K$

$$\frac{2}{2+3} = k\left(1+\frac{30K}{300K}\right)$$

Testing

$$\frac{2}{5} = k \cdot \frac{11}{10}$$

$$\cancel{\frac{2}{5} \cdot \frac{10}{11} \cdot k = \frac{20}{77} = 0{,}2597}$$

$$\boxed{\frac{2}{5} \cdot \frac{10}{11} = k = \frac{4}{11} = \underline{\underline{0{,}36\overline{36}}}}$$

$$k = 0{,}36\overline{36} = \frac{4}{11}$$

$$x = k\left(1+\frac{C}{T}\right)(x+3)$$

$$x = \left(k+\frac{kC}{T}\right)(x+3)$$

$$= kx + \frac{kC}{T}x + 3k + 3\frac{kC}{T}$$

$$x - kx - \frac{kC}{T}x = 3k + 3\frac{kC}{T}$$

$$x\left(1-k-\frac{kC}{T}\right) = 3\left(k+\frac{kC}{T}\right)$$

$$x = \frac{3\left(k+\frac{kC}{T}\right)}{\left(1-k-\frac{kC}{T}\right)}$$

Pomožni list

$$\left(\frac{\partial p}{\partial T}\right)_E = \varepsilon_0 E \frac{\left(-3kc\frac{1}{T^2}\right)\left(1-k-\frac{kc}{T}\right) - 3\left(k+\frac{kc}{T}\right)\frac{kc}{T^2}}{\left(1-k-\frac{kc}{T}\right)^2} = \qquad \left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

$$f = 3\left(k+\frac{kc}{T}\right)$$

$$f' = -3kc\frac{1}{T^2}$$

$$g = \left(1-k-\frac{kc}{T}\right)$$

$$g' = \frac{kc}{T^2}$$

$$= 3\varepsilon_0 E \frac{-\frac{kc}{T^2}\left(1-k-\frac{kc}{T}\right) - \left(k+\frac{kc}{T}\right)\frac{kc}{T^2}}{\left(1-k-\frac{kc}{T}\right)^2} =$$

$$= 3\varepsilon_0 E \frac{-\frac{kc}{T} + \cancel{\frac{k^2c}{T^2}} + \cancel{\frac{k^2c^2}{T^3}} - \cancel{\frac{k^2c}{T^2}} - \cancel{\frac{k^2c^2}{T^3}}}{-//-} =$$

$$= \frac{3\varepsilon_0 E\left(-\frac{kc}{T}\right)}{\left(1-k-\frac{kc}{T}\right)^2} = -\frac{3\varepsilon_0 E kc}{(T-kT-kc)^2}$$

Pomožni list

$dF = -\cancel{SdT} + EVdp$ $dF = -SdT + \cancel{EVdp}$

$dF = EVdp$

$\frac{dF}{dp} = EV$

$dG = \cancel{-SdT} - PVdE$ $dG = -SdT - \cancel{PVdE}$

$$P = \underbrace{\frac{3\left(h + \frac{hc}{300k}\right)}{\left(1 - \cancel{3} \frac{hc}{300k}\right)}}_{x=2} \quad \varepsilon_0 : 10^7 \frac{V}{m}$$

$$P = 0{,}000177 = 2 \cdot E \varepsilon_0 = P\left(27°C, 10^7 \frac{V}{m}\right)$$

$\frac{m^3}{m^3/kg}$ m^3

$\frac{m}{\rho}$ V

$V \cdot \rho = m$

Pomožni list

$$\frac{\overbrace{P\left(1-h-\frac{hc}{T}\right)}^{=f}}{\underbrace{\varepsilon_0 3\left(h+\frac{hc}{T}\right)}_{=g}} = E(P,T) \qquad \frac{f'g-fg'}{g^2}$$

$$\left(\frac{\partial E}{\partial T}\right)_p = \frac{P}{3\varepsilon_0}\,\frac{\left(\frac{hc}{T^2}\right)\left(h+\frac{hc}{T}\right)-\left(1-h-\frac{hc}{T}\right)\left(-\frac{hc}{T^2}\right)}{\left(h+\frac{hc}{T}\right)^2} =$$

$$= \frac{P}{3\varepsilon_0}\,\frac{\cancel{\frac{h^2c}{T^2}}+\cancel{\frac{h^2c^2}{T^3}}+\frac{hc}{T^2}-\cancel{\frac{h^2c}{T^2}}-\cancel{\frac{h^2c^2}{T^3}}}{-||-} =$$

$$= \frac{P}{3\varepsilon_0}\,\frac{\frac{hc}{T^2}}{\left(h+\frac{hc}{T}\right)^2} = \frac{Phc}{3\varepsilon_0}\,\frac{1}{(hT+hc)^2}$$

$$P = \chi(T)\,\varepsilon_0 E \qquad \left(\frac{\partial P}{\partial E}\right)_T$$

$$\left(\frac{\partial P}{\partial E}\right)_T = \chi(T)\,\varepsilon_0$$