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$N = 10^{10}$

$m = 4\cdot 10^{-10}\,g = 4\cdot 10^{-13}\,kg$

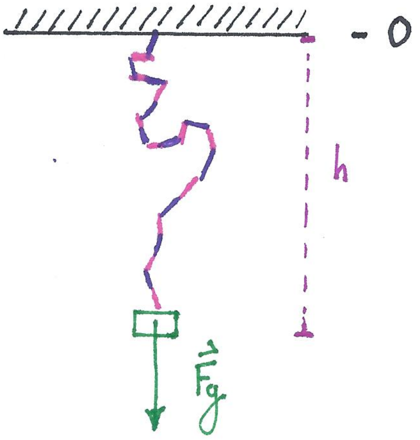
$x = 2\,nm$

$T = 300\,K$

a) $W_p = ?$

b) $\Delta Q = ?$ $\quad m \to 2m$

$V\,3D = ?$



a) $W_p = -mgh \neq -mg\langle h\rangle = \langle W_p\rangle$ Fazni prostor: Vse možne orientacije? "Vezave"

Vez nima energije, torej ni "preferenčne" smeri

$E_i = 0 - mgh_i = -\mathcal{F} x \cos\theta_i$

$d\Gamma = \sin\theta\, d\theta\, d\varphi$

Dolžina ni odvisna od φ, zato se bo pokrajšal.

$h = \sum_i^N x\cos\theta_i$ oz. $N x\langle\cos\theta\rangle = \langle h\rangle$

$$\langle\cos\theta\rangle = \frac{\int_0^{\frac{\pi}{2}}\int_0^{\pi} \exp(-\beta(-\mathcal{F}x\cos\theta))\cos\theta\cdot\sin\theta\, d\theta\, d\varphi}{\int_0^{\frac{\pi}{2}}\int_0^{\pi}\exp(-\beta(-\mathcal{F}x\cos\theta))\cdot\sin\theta\, d\theta\, d\varphi}$$

$\alpha = \beta\mathcal{F}x$

$y = \cos\theta$

$dy = -\sin\theta\, d\theta$

$$= \frac{\int_{-1}^{1} y e^{\alpha y} dy}{\int_{-1}^{1} e^{\alpha y} dy} = \frac{\cancel{\frac{1}{\alpha}} y e^{\alpha y}\Big|_{-1}^{1} - \int_{-1}^{1}\cancel{\frac{1}{\alpha}} e^{\alpha y} dy}{\cancel{\frac{1}{\alpha}} e^{\alpha y}\Big|_{-1}^{1}} =$$

$u = y \qquad dv = e^{\alpha y} dy$

$du = dy \qquad v = \frac{1}{\alpha} e^{\alpha y}$

$$= \frac{e^{\alpha} + e^{-\alpha} - \frac{1}{\alpha}(e^{\alpha} - e^{-\alpha})}{e^{\alpha} - e^{-\alpha}} = \frac{\cancel{2}\operatorname{ch}\alpha - \frac{1}{\alpha}\cancel{2}\operatorname{sh}\alpha}{\cancel{2}\operatorname{sh}\alpha} = \operatorname{cth}\alpha - \frac{1}{\alpha}$$

$$\langle h\rangle = Nx\langle\cos\theta\rangle = Nx\left(\operatorname{cth}(\beta\mathcal{F}x) - \frac{1}{\beta\mathcal{F}x}\right)$$

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$$\langle W_p \rangle = -mg\langle h \rangle$$

$$\langle W_p \rangle = -mgNx\left(\mathrm{cth}\left(mg\frac{1}{k_BT}x\right) - \frac{1}{\frac{1}{k_BT}mgx}\right) = \underline{-4.07 \cdot 10^{-11}}\ J$$

b) $\Delta Q = ?$ če $m \to 2m$ izotermno $dT = 0$, reverzibilno

Iz 2. zakona TD:

$\rightarrow Q = T\Delta S$ $\qquad \Delta S = \dfrac{\langle E \rangle - F}{T}$; $\langle E \rangle = \langle W_p \rangle$

↳ če gledamo spremembo od ko je $m = m$?

$\Rightarrow Q = \langle W_p \rangle - F$

$$\underline{\Delta Q = \Delta\langle W_p \rangle - \Delta F}$$

$$e^{-\beta F} = \int e^{-\beta E}\, d\Gamma = 2\pi \int_0^{\pi} e^{\beta \mathcal{F} x \cos\theta} \sin\theta\, d\theta$$

podobno kot pred $\Rightarrow = \dots = \dfrac{2\pi}{\alpha}\left(e^{\alpha} - e^{-\alpha}\right) = \dfrac{4\pi}{\alpha}\,\mathrm{sh}(\alpha)$

$$\Rightarrow \underline{F = -k_BT \ln\left[\frac{4\pi}{\alpha}\,\mathrm{sh}(\alpha)\right]}$$

// Pozabil N potenco (Fazna vsota na bonus listu)

$$\Delta\langle W_p \rangle = \langle W_p \rangle_k - \langle W_p \rangle_z$$

$$= -2mgNx\left(\mathrm{cth}\left(2mgx\frac{1}{k_BT}\right) - \frac{1}{\frac{1}{k_BT}2mgx}\right) - \left(-mgNx\left(\mathrm{cth}\left(mgx\frac{1}{k_BT}\right) - \frac{1}{\frac{1}{k_BT}mgx}\right)\right)$$

$$= \underline{-7.50 \cdot 10^{-11}\ J}$$

$$\Delta F = F_k - F_z =$$

$$= -k_BT\ln\left[\frac{4\pi}{2mgx\frac{1}{k_BT}}\,\mathrm{sh}\left(2mgx\frac{1}{k_BT}\right)\right] - \left(-k_BT\ln\left[\frac{4\pi}{mgx\frac{1}{k_BT}}\,\mathrm{sh}\left(mgx\frac{1}{k_BT}\right)\right]\right)$$

$= //$ vbistvu $F = -k_BTN\ln\left[\dfrac{4\pi k_BT}{mgx}\,\mathrm{sh}\left(\dfrac{mgx}{k_BT}\right)\right]$

$= N \cdot (\text{-}\,||\,\text{-})$

$= -5.07 \cdot 10^{-11}\ J$

$$\Rightarrow \underline{\Delta Q = -2.43 \cdot 10^{-11}\ J}$$

Bonus List

$$e^{-\beta F} = \int e^{-\beta E} d\Gamma$$

$$= (2\pi)^N \int_0^{\pi} e^{-\beta F_x x \cos\theta_1} \dots e^{\beta \mathcal{F} x \cos\theta_N} d\theta_1 \dots d\theta_N$$

? ↗

$$= (2\pi)^N \left(\int_0^{\pi} e^{-\beta \mathcal{F} x \cos\theta} d\theta \right)^N$$

$$e^{-\beta F} = \left(2\pi \frac{1}{\alpha} (e^{\alpha} - e^{-\alpha}) \right)^N$$

$$-\beta F = N \ln\left[\frac{2\pi}{\alpha} (e^{\alpha} - e^{-\alpha}) \right] = N \ln\left[\frac{2\pi}{\beta \mathcal{F} x} (e^{\beta \mathcal{F} x} - e^{-\beta \mathcal{F} x}) \right]$$

$$\beta F = -N \ln\left(\frac{2\pi}{\beta \mathcal{F} x} \right) - N \ln\left[e^{\beta \mathcal{F} x} - e^{-\beta F x} \right] \Big/ \frac{d}{d\beta} \Rightarrow \quad \langle W_p \rangle = \frac{d(\beta F)}{d\beta}$$

$$\langle W_p \rangle = -N \frac{1}{\frac{2\pi}{\beta \mathcal{F} x}} \cdot \left(-\frac{2\pi}{\beta^2 \mathcal{F} x} \right) - N \frac{1}{e^{\beta \mathcal{F} x} - e^{-\beta F x}} \left(x\mathcal{F} e^{\beta \mathcal{F} x} - (-x\mathcal{F}) e^{-\beta \mathcal{F} x} \right)$$

$$= N \frac{2\pi \beta \mathcal{F} x}{2\pi \beta^2 \mathcal{F} x^2} - N \frac{x\mathcal{F} (e^{\beta \mathcal{F} x} + e^{-\beta \mathcal{F} x})}{e^{\beta \mathcal{F} x} - e^{-\beta \mathcal{F} x}}$$

$$= N \cdot \frac{1}{\beta} - N x \tilde{\mathcal{F}} \frac{ch(\beta \mathcal{F} x)}{sh(\beta \tilde{\mathcal{F}} x)} = N \left[\frac{1}{\beta} - x \tilde{\mathcal{F}} \, cth(\beta \mathcal{F} x) \right]$$

$$= -N x m g \left[cth(x m g \beta) - \frac{1}{x m g \beta} \right]$$

Kot alternativni način
in preverba
formul