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$c = 2800\ m/s$

$n = \frac{N}{A} = 2 \cdot 10^{20} / m^2$

$P = 2$

a) $\omega_d = ?$

b) $C(T = 3K) = ?$

c) $C(T = 1800K) = ?$

Disperzijska relacija:

$\omega = ck$

a) $2N = \sum_j 1$

$$\sum_j \rightarrow P \frac{d^2p\, d^2r}{h^2} = \frac{2A\, 2\pi p dp}{h^2} \rightarrow \frac{2A}{h^2} 2\pi \hbar k \hbar dk$$

$$= \frac{2A\hbar^2}{(2\pi)^2 \hbar^2} 2\pi k dk = \frac{A}{\pi} k dk = \frac{A}{\pi c^2} \omega d\omega$$

Enote $\left[\frac{1}{\sqrt{m^2}} \frac{m}{s} = \frac{1}{s} \checkmark \right]$

Torej:

$$2N = \frac{A}{\pi c^2} \int_0^{\omega_d} \omega d\omega = \frac{A}{2\pi c^2} \omega_d \Rightarrow \omega_d = 2\sqrt{n\pi}\, c =$$

$$= \underline{\underline{1{,}4 \cdot 10^{14}\ s^{-1}}}$$

Določitev prispevka k specifični toploti

$\langle E \rangle = \sum_j E_j \langle N^{BE} \rangle$

$E_j = \hbar \omega_j$

$\omega = ck$

$$\langle E \rangle = \sum_j E_j \langle N^{BE} \rangle = \int_{\omega_{min}}^{\omega_d} \frac{A}{\pi c^2} \frac{\hbar \omega^2 d\omega}{\exp(\beta\hbar\omega) - 1} = \frac{A\hbar}{\pi c^2} \int_{\omega_{min}}^{\omega_d} \frac{\omega^2 d\omega}{\exp(\beta\hbar\omega) - 1}$$

b) Nizkotemperaturna limita ($T = 3K$):

$$\langle E \rangle = \int_0^{\infty} \frac{A\hbar}{\pi c^2} \left(\frac{1}{\beta\hbar} \right)^3 \frac{u^2}{e^u - 1} du = \frac{A\hbar}{\pi c^2} \left(\frac{1}{\beta\hbar} \right)^3 \cdot \underbrace{(2.404)}_{\alpha}$$

$u = \beta\hbar\omega$ $\quad$ $\beta\hbar\omega_{max} \rightarrow \infty$

$$= \frac{A\hbar}{\pi c^2} \frac{1}{\hbar^3} k_B^3 T^3 \alpha = \alpha \frac{A (k_B T)^3}{\pi c^2 \hbar^2}$$

$$C = \frac{d\langle E \rangle}{dT} = 3\alpha \frac{A k_B^3 T^2}{\pi c^2 \hbar^2}$$

$$\frac{C}{A} = \underline{\underline{6{,}23 \cdot 10^{-7}}}\ \frac{J}{m^2 K}$$

c)

$$\langle E \rangle = \frac{A\hbar}{\pi c^2} \int_{\omega_{min}}^{\omega_d} \frac{\omega^2 d\omega}{\exp(\beta\hbar\omega)-1}$$

$u \ll 1$?

Recimo

$e^u \approx 1 + u + \frac{u^2}{2} + \dots$

$$= \frac{A\hbar}{\pi c^2} \int_0^{\omega_d} \frac{\omega^2 \, d\omega}{\exp(\beta\hbar\omega)-1} = \frac{A\hbar}{\pi c^2}\left(\frac{1}{\beta\hbar}\right)^3 \int_0^{u_{max}} \frac{u^2}{e^u - 1} du =$$

$U = \beta\hbar\omega$

$\omega = \left(\frac{1}{\beta\hbar}\right) u$

$$= \frac{A\hbar}{\pi c^2 (\beta\hbar)^3} \int_0^{u_{max}} \frac{u^2}{u + \frac{u^2}{2}} du = \frac{A\hbar}{\pi c^2 \beta^3 \hbar^3}\left[2u_{max} - 4\left(\ln|u_{max}+2| - \ln(2)\right)\right]$$

$U_{max} = \beta\hbar\omega_d \approx 0.5955$

Samo en člen razvoja?

$$= \frac{A\hbar}{\pi c^2}\left(\frac{1}{\beta\hbar}\right)^3 \frac{\beta^2\hbar^2\omega_d^2}{2} = \frac{\cancel{A\hbar}}{\cancel{\pi c^2}} \frac{1}{\beta\cancel{\hbar}} \frac{1}{\cancel{2}}^{2} \frac{4N\cancel{\pi c^2}}{\cancel{A}} =$$

$$= 2Nk_BT \Rightarrow C = 2Nk_B$$ Dulong-Petit

Mogoče samo dva?

$$= \frac{A}{\pi c^2 \hbar^2} k_B^3 T^3 \left[2\beta\hbar(2\sqrt{n\pi}c) - 4\left(\ln(2\beta\hbar\sqrt{n\pi}c + 2) - \ln(2)\right)\right]$$

$$= \frac{Ak_B^3}{\pi c^2 \hbar^2} T^3 \left[\frac{2\hbar}{k_BT}(2\sqrt{n\pi}c) - 4\left(\ln\left(\frac{2\hbar}{k_BT}\sqrt{n\pi}c + 2\right) - \ln(2)\right)\right]$$

$$= \frac{Ak_B^3}{\pi c^2 \hbar^2} T^3 \left[\frac{2\hbar}{k_BT}\omega_d - 4\ln\left[\frac{2\beta\hbar\omega_d + 2}{2}\right]\right]$$

$B = \frac{Ak_B^3}{\pi c^2 \hbar^2}$

$c = \frac{\hbar}{k_B}\omega_d$

$$= \frac{Ak_B^3}{\pi c^2 \hbar^2} T^3 \left[\frac{2\hbar}{k_BT}\omega_d - 4\ln\left(\frac{\hbar}{k_BT}\omega_d + 1\right)\right] \, // \quad \text{Naprej}$$

$$= \frac{Ak_B^3}{\pi c^2 \hbar^2} T^3 \left[\frac{2\hbar}{k_BT}\omega_d - 4\ln\left(\frac{T + \frac{\hbar\omega_d}{k_B}}{T}\right)\right]$$

$\Rightarrow$ Nekaj listov matematike $\Rightarrow$

Definiram $\gamma = \frac{k_B^3}{\pi c^2 \hbar^2} = 9.6 \cdot 10^{-9} \frac{kg}{s^2 K^3}$

$$\varepsilon = \frac{\hbar \omega_D}{k_B} = 1068.9\ K$$

Iz prejšnje zmešnjave (ko vzamemo $e^x \approx 1 + x + x^2/2 \ldots$) sledi:

Tu vmes je bilo par listov, ker sem smotan in sem narobe poenostavil te ln()

$$\langle E \rangle = A \gamma T^3 \left[\frac{2\varepsilon}{T} - 4\left(\ln\left(\frac{\varepsilon}{T} + 2\right) - \ln(2) \right) \right]$$

$$\frac{d\langle E \rangle}{dT}$$

Pomagalo je it spat vmes ker sem sredi noči ugotovil zmoto

$$\star\ \frac{C}{A} = \gamma \left(\frac{1}{2T+\varepsilon} \left(4\varepsilon \left(\varepsilon T + 3T^2 \right) + 12\left(2T^3 + \varepsilon T^2\right) \ln\left(\frac{2T}{\varepsilon + 2T} \right) \right) \right)$$

Zdaj lahko končno naredim primerjavo. (vse pri $T = 1800K$)

Dulong-Petit (oz. $e^x \approx 1 + x + \ldots$):

$$\frac{C}{A} = 2 n k_B = \underline{\underline{0.00552}}\ \frac{J}{m^2 K}$$

Visoko temp limita (oz. $e^x \approx 1 + x + x^2/2 \ldots$):

$$\Rightarrow \star\ \frac{C}{A} = \underline{\underline{0.06532}}\ \frac{J}{m^2 K}$$

Lahko za hec še nizko. temp limito (izraz od b)):

$\frac{C}{A} = 0.225 \frac{J}{m^2 K}$, ker vidimo da je popolnoma narobe