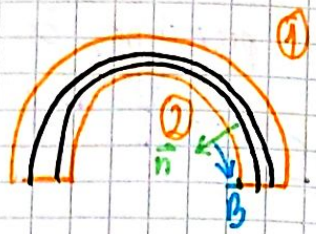
Izberemo površino:

① $\vec{F}_{m_1} = 0 \iff B = 0$



② $\vec{n} = \begin{bmatrix} -\cos\varphi \\ -\sin\varphi \end{bmatrix}$

$$\vec{F}_{m_2} = \frac{1}{\mu_0}\int_0^{\pi} \left(-\frac{1}{2}\right) B^2 \vec{n}\, la\, d\varphi = \frac{+1}{2\mu_0} la \int_0^{\pi} \begin{bmatrix} \cancel{\cos\varphi}^{\,0} \\ \sin\varphi \end{bmatrix} \left(\frac{\mu_0 I}{2\pi r}\right)^2 =$$

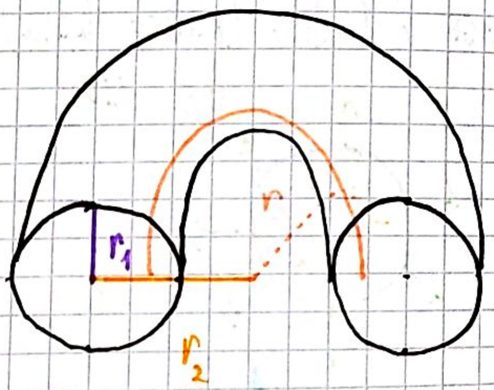
$$= \frac{\mu_0^2 I^2 la}{\mu_0\, 2 \cdot 4\pi^2 a^2} \underbrace{\int_0^{\pi} \sin\varphi\, d\varphi}_{2}\ \hat{e}_y$$

$$\Rightarrow \quad \boxed{\vec{F}_m = \hat{e}_y \frac{\mu_0 I^2 l}{4\pi^2 a}} \qquad F_1 = \frac{F_m}{2}$$

Sila kaže navzgor

$$\frac{F_1}{l} = \frac{\mu_0 I^2}{8\pi^2 a}$$

16. [Magnetna sila v toroidni tuljavi]

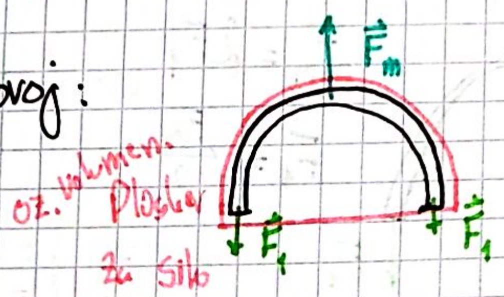


r_1, r_2, N, I, μ_0, $\quad N \gg 1 \quad r_2 \gg r_1$

$F_1 = ?$

↳ Sila napetosti posameznega ovoja

Če si pogledamo en ovoj:



$F_m = 2F_1$

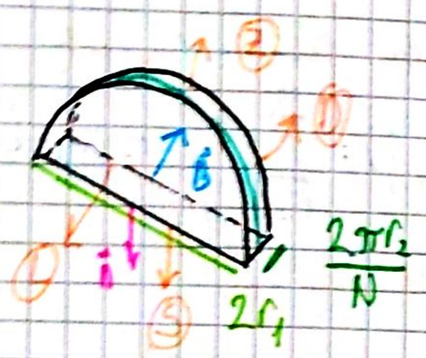
$F_1 = F_m/2$

Ampere da

$$B \cdot 2\pi r = IN$$

Znotraj $B = \frac{\mu_0 N I}{2\pi r}$ $\qquad$ Zunaj: $\vec{B} = 0$

Volumen za sib:



(Z): $\vec{F}_{m_z} = 0$ ker $B = 0$

(L)+(D): $\vec{F}_{ml} = -\vec{F}_{md}$ $\quad \vec{B} \perp \vec{n}$

(S): $\vec{n}_s = -\hat{e}_y$

$$\vec{F}_{m_s} = \frac{1}{\mu_0}\int \underbrace{\left(\frac{-1}{2}\right)\left(\frac{\mu_0 N I}{2\pi r_2}\right)^2}_{\text{Vse konstante}} (-\hat{e}_y)\, dS =$$

$$= \hat{e}_y \frac{1}{2} \frac{\mu_0^2 N^2 I^2}{4\pi^2 r_2^2} 2r_1 \cdot \frac{2\pi r_2}{N} =$$

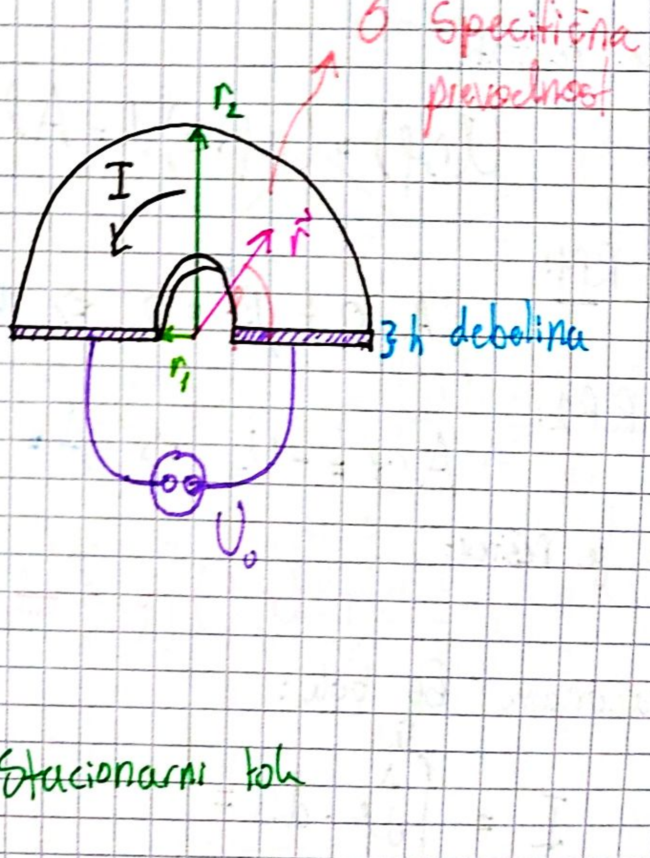
$$= \hat{e}_y \frac{\mu_0 I^2 N}{2\pi} \cdot \frac{r_1}{r_2} = F_m$$

$$F_1 = \frac{\mu_0 I^2 N r_1}{4\pi r_2}$$

17. [Upor prevodne ploščice]

r_1, r_2, h, δ

$R = ?$



Ohmov zakon:

$$\vec{j} = \delta \vec{E}$$

$\hookrightarrow -\vec{\nabla} U$

Kontinuitetna enačba:

$$\vec{\nabla} \cdot \vec{j} + \underset{\substack{\parallel \\ 0}}{\frac{\partial \rho}{\partial t}} = 0$$

0 za stacionarni tok

Vstavimo skupaj:

$$\vec{\nabla} \cdot \delta(-\vec{\nabla} U) = 0 \Rightarrow \nabla^2 U = 0$$

To smo že reševali

Splošna rešitev od prej:

$$U(r,\varphi) = \sum_{m=1}^{\infty} (A_m \cos m\varphi + B_m \sin m\varphi)(C_m r^m + D_m r^{-m}) + (a\varphi + b)(c\ln r + d)$$

Robni pogoj:

<u>RP1</u>: $U(r, 0) = 0$

<u>RP2</u>: $U(r, \pi) = -U_0$

<u>RP3,4</u>: $j_r(r_{1,2}, \varphi) = 0 \rightarrow \frac{\partial U}{\partial r}(r_{1,2}, \varphi) = 0$

$$\vec{j} = -\sigma \vec{\nabla} U \rightarrow \frac{\partial U}{\partial r}(r_{1,2}, \varphi) = 0$$

Iz <u>RP3,4</u>:

$$\frac{\partial U}{\partial r} = \sum_{m=1}^{\infty} (A_m \cos m\varphi + B_m \sin m\varphi)(\cancel{C_m} m r_{1,2}^{m-1} - \cancel{D_m} m r_{1,2}^{-m-1}) + (a\varphi + b)\frac{\cancel{c}}{r_{1,2}} = 0$$

$$\Rightarrow C_m = 0 \quad D_m = 0 \quad c = 0$$

Vmesen rezultat:

$$U(r,\varphi) = (a\varphi + b)d = A\varphi + B$$

Iz <u>RP1</u>:

$$A \cdot 0 + \cancel{B} = 0 \Rightarrow B = 0$$

Iz RP2:

$$A\pi = -U_0 \Rightarrow A = -\frac{U_0}{\pi}$$

Tako je rešitev:

$$U(r,\varphi) = -\frac{U_0}{\pi}\varphi$$

Izračunam še tok:

$$I = \int_{r_1}^{r_2} \vec{j}_\varphi \cdot dS =$$

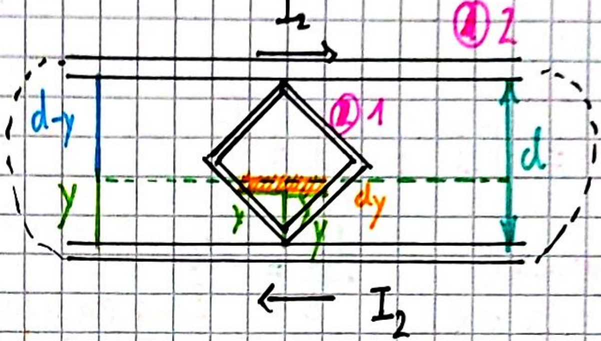
$$\vec{j}_\varphi = \sigma E_\varphi = -\sigma \frac{1}{r}\frac{\partial U}{\partial \varphi} =$$

$$= -\frac{1}{r}\sigma\left(-\frac{U_0}{\pi}\right) = \frac{U_0 \sigma}{\pi r}$$

$$I = \frac{U_0 \gamma}{\pi} \int \frac{1}{r} \cdot h \, dr = \frac{U_0 \gamma h}{\pi} \ln \frac{r_2}{r_1} = I$$

In še $R = U_0 / I$

$$R = \frac{\pi}{\gamma h \ln \frac{r_2}{r_1}}$$



18. [Indukcija v okvirju]

$\frac{d}{L_{1,2} = ?}$

↳ Medsebojna/Vzajemna induktivnost

Lastna indukcija (induktivnost)

$$\Phi_{m_1} = L_{11} I_1$$

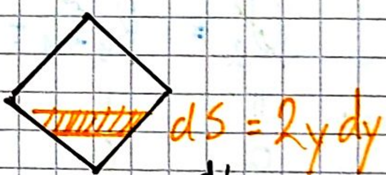
↑ stara param

Vzajemna induktivnost $L_{1,2}$

$$\Phi_{m_2} = L_{21} I_1$$

$$L_{12} = L_{21}$$ ↑ stara param

a) Dve zanki (1, 2): $\Phi_1 = L_{12} I_2$



$$B = \frac{\mu_0 I_2}{2\pi y} + \frac{\mu_0 I_2}{2\pi (d-y)}$$

$$\Phi_1 = \int B \, dS = \frac{\mu_0 I_2 \cdot 2 \cdot 2}{2\pi} \int_0^{d/2} \left(\frac{y}{y} + \frac{y}{d-y} \right) dy = C \int_0^{d/2} \frac{y(d-y) + y^2}{y(d-y)} dy =$$

$$= C \int_0^{d/2} \frac{d}{d-y} dy = \frac{2\mu_0 I_2 d}{\pi} \left[-\ln \left(\frac{d/2}{d} \right) \right] = \frac{2 \ln 2 \; I_2 \mu_0 d}{\pi}$$

L_{12}

$-d(d-y)$

Geometrijska lastnost !

$$\Rightarrow \quad L_{12} = \frac{2\mu_0 d}{\pi} \ln(2)$$

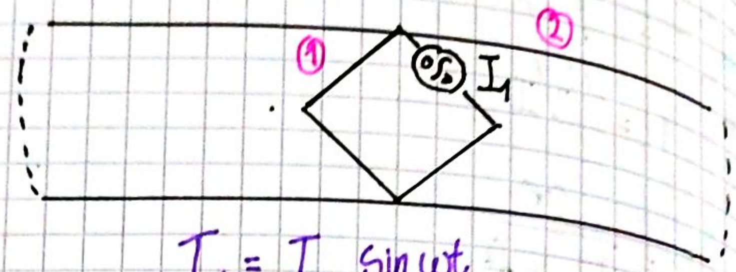
V splošnem za dve zanki

$$\boxed{L_{12} = \frac{\mu_0}{4\pi} \oint \oint \frac{d\vec{l}_1 \cdot d\vec{l}_2}{|\vec{r}_1 - \vec{r}_2|}}$$

b) V notranjo zanko damo vir izmenične napetosti

$I_1(t) \to B_2(t) \to \Phi_2(t) \to$
$\to U_{i_2}(t) \to I_2(t)$

Kolikšno je razmerje amplitud?
$I_{20}/I_{10} = ?$



$I_1 = I_{10} \sin \omega t$

Tokovni krogi:

$$U = RI + \underbrace{L\dot{I}}_{\frac{d\Phi}{dt} = -U_i} + \frac{e}{C}$$

↑ gonilna napetost

Zanki sta induktivno sklopljeni

Dve zanki:

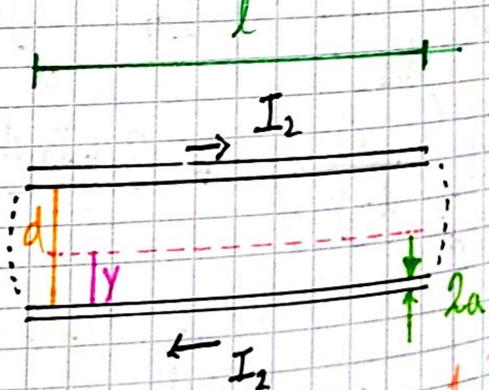
$$\underset{\substack{\text{pri}\\\text{nas}}}{U_1} = \underset{\substack{\parallel\\ 0}}{R_1} I_1 + \underbrace{L_{11}\dot{I}_1 + L_{12}\dot{I}_2}_{\frac{d\Phi_1}{dt}} + \underset{\substack{\parallel\\ 0}}{\frac{e_1}{C_1}} \quad \text{pri nas}$$

$$\cancel{U_2} = R_2 I_1 + L_{22}\dot{I}_2 + L_{21}\dot{I}_1 + \frac{e_2}{C_2}$$

pri nas: $U_2 = 0$, $R_2 = 0$, $\frac{e_2}{C_2} = 0$

Torej: $L_{22}\dot{I}_2 + L_{21}\dot{I}_1 = 0 \Rightarrow I_2 = I_{20}\sin(\omega t)$

$$I_{20} = \frac{L_{21}}{L_{22}} I_{10}$$



Podproblem: L_{22}

$$\Phi_2 = \int B_2\, dS = \frac{\mu_0 I_2}{2\pi}\int_a^{d-a}\left[\frac{1}{y} + \frac{1}{d-y}\right] l\,dy =$$

$$B_2 = \frac{\mu_0 I_2}{2\pi}\left[\frac{1}{y} + \frac{1}{d-y}\right]$$

$$= \frac{\mu_0 I_2 l}{2\pi}\left[\ln\left(\frac{d-a}{a}\right) - \ln\left(\frac{a}{d-a}\right)\right] =$$

$$= \frac{\mu_0 l}{\pi}\ln\left(\frac{d-a}{a}\right) I_2$$

$$\Rightarrow L_{22} = \frac{l\mu_0}{\pi}\ln\left(\frac{d-a}{a}\right)$$

In tako lahko poračunamo razmerje amplitud:

$$\left|\frac{I_{20}}{I_{10}}\right| = \frac{2\ln(2)\,d}{l\ln\left(\frac{d-a}{a}\right)} = \frac{2\ln(2)}{\frac{l}{d}\cdot\ln\left(\frac{d-a}{a}\right)} \overset{d\gg a}{=} \frac{2\ln(2)}{\underbrace{\frac{l}{d}}_{\gg 1}\ln\underbrace{\frac{d}{a}}_{\gg 1}} \ll 1$$

Če vzamemo tipične vrednosti za obroček:

$$\frac{l}{d} = 10 \qquad \Rightarrow \qquad \left|\frac{I_{20}}{I_{10}}\right| = 0{,}06 = 6\%$$

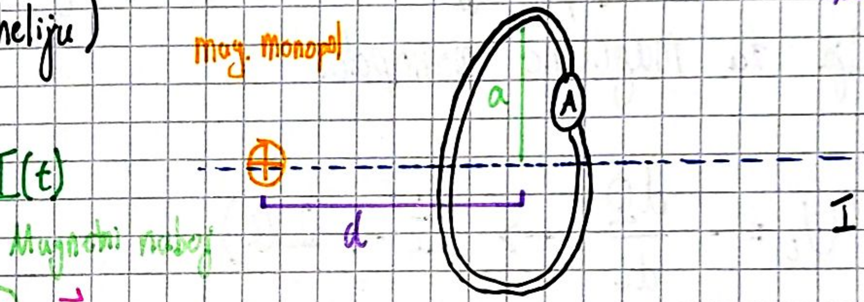
$$\frac{d}{a} = 10$$

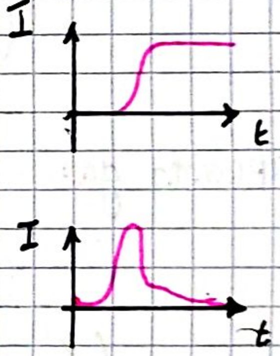
19. [Cabrerov (Blas Cabrera) eksperiment] Krožna zanka Eksperiment za detekcijo mag. monopolov

$R = 0$ (v tekočem heliju)

L

$\Phi(d) \rightarrow \Phi(t) \rightarrow I(t)$





$$\vec{B} = \frac{\mu_0 g}{4\pi r^2}\frac{\vec{r}}{r}$$

$$[B] = \frac{Vs}{m^2}$$

$$[\mu_0] = \frac{Vs}{Am}$$

$$[g] = \frac{Vs}{m^2}\,\frac{Am\,m^2}{Vs} = Am$$

Tipična pot z Faradayjevim zakonom ne deluje ker Faradayev zakon ne velja za magnetne monopole.

$$U_i = -\frac{d\Phi}{dt} \rightarrow L\dot{I} \rightarrow I(t)$$

Popravimo ga:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} - \mu_0 \vec{j}_m$$

$$\left(\text{Simetrično z } \vec{\nabla} \times \vec{B} = \mu_0 \vec{j} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$$

$$\left.\begin{aligned} [\mu_0 j_m] &= \frac{Vs}{Am}\,\frac{Am}{m^2 s} = \frac{V}{m^2} \\ [\vec{\nabla} \times \vec{E}] &= \frac{1}{m} \cdot \frac{V}{m} = \frac{V}{m^2} \end{aligned}\right\} \text{Dodaten člen je smiselni}$$

$$\int \underbrace{(\vec{\nabla} \times \vec{E})\, d\vec{S}}_{\text{Stokes}} = -\frac{\partial}{\partial t} \int \underbrace{\vec{B} \cdot d\vec{S}}_{\Phi} - \mu_0 \underbrace{\int \vec{j}_m\, d\vec{S}}_{I_m}$$

$$\oint \vec{E} \cdot d\vec{l} = U_i$$

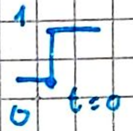
Tako dobimo posplošeni Faradayev zakon

$$U_i = -\dot{\Phi} - \underbrace{\mu_0 I_m}_{\substack{\text{Pri nas:} \\ g\delta(t)}} = RI + L\dot{I} + \frac{e}{C}$$

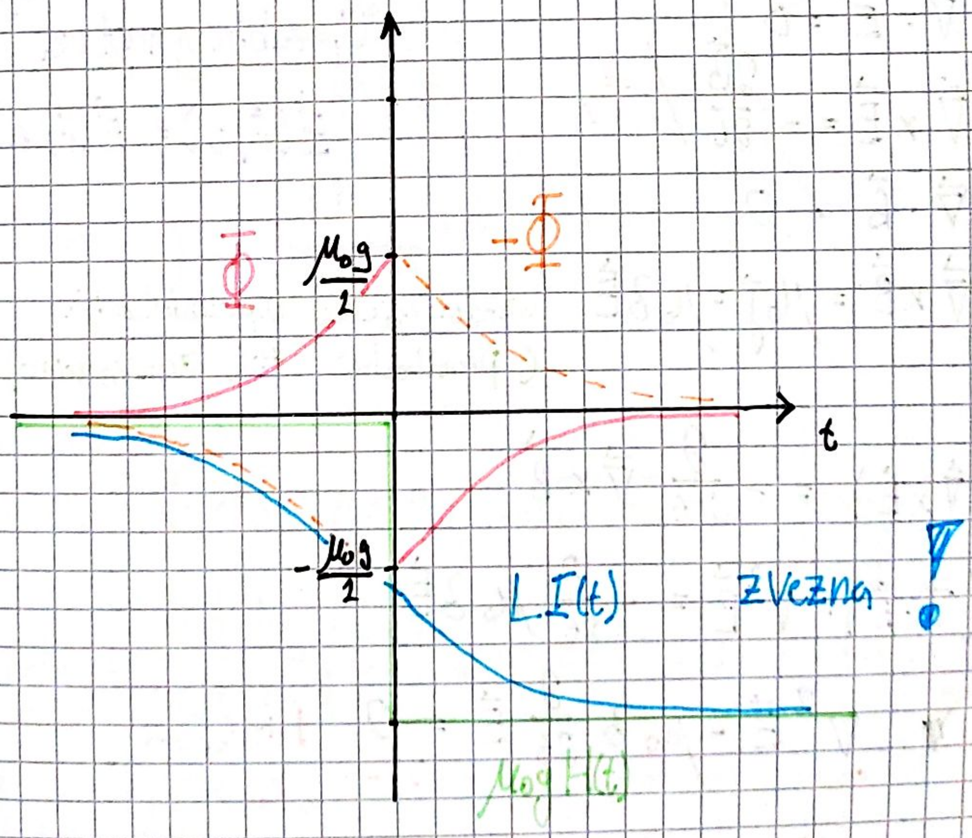
$$U_i = -\dot{\Phi} - \mu I_m = L\dot{I}$$

$$U_i = -\dot{\Phi} - \mu_0 g\delta(t) = L\dot{I} \quad \Big/ \int_{-\infty}^{t} dt$$

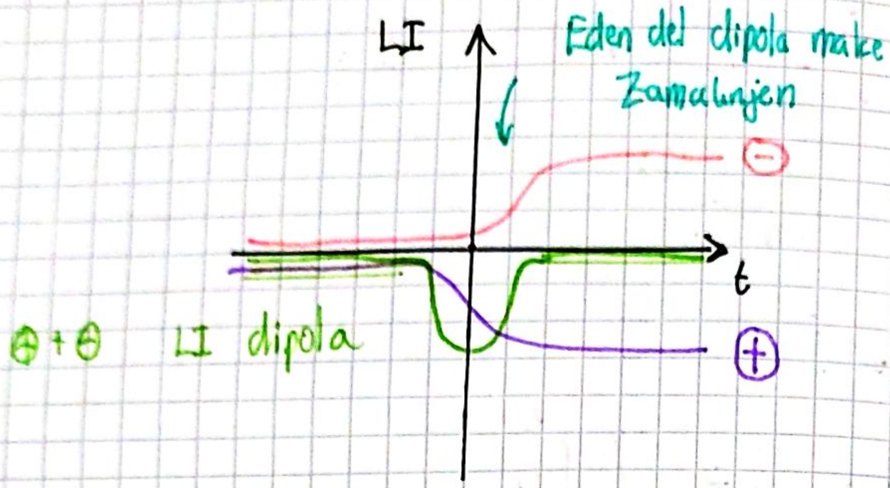
$$-[\Phi(t) - \underbrace{\Phi(-\infty)}_{0}] - \mu_0 g \underbrace{\int_{-\infty}^{t} \delta(t)}_{} = L[I(t) - \underbrace{I(-\infty)}_{0}]$$

Heaviside $H(t) =$ 

$$\Rightarrow LI(t) = -\Phi(t) - \mu_0 g H(t)$$



Zakaj pa pri dipolu pride do špice :

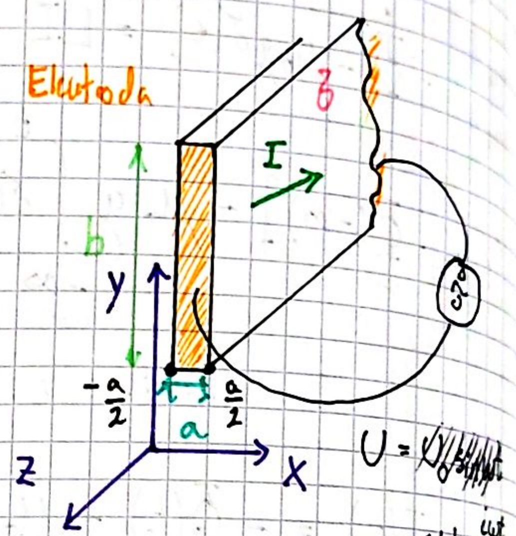


20. [Kožni pojav v prevodnem traku]

$$\frac{a, \sigma, \omega \quad\quad l, b \gg a}{\frac{Z(\omega)}{R_0} = ?}$$

σ ... specifična prevodnost

R_0 statična upornost



$U = U_0 e^{i\omega t}$

Izpeljimo enačbe:

$$\vec{\nabla} \cdot \vec{E} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad / \vec{\nabla} \times$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{j} = \mu_0 \sigma \vec{E}$$ kvazistatična aproksimacija (premikalni tok zanemarimo)

Uporabimo nastavek

$$\vec{E}(\vec{r}, t) = \vec{E}(\vec{r}) e^{i\omega t}$$

$$\Rightarrow \vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\frac{\partial}{\partial t} \vec{\nabla} \times \vec{B}$$

$$\vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \nabla^2 \vec{E} = -\frac{\partial}{\partial t} \mu_0 \sigma \vec{E}$$

$$\underline{\nabla^2 \vec{E} - \mu_0 \sigma \frac{\partial}{\partial t} \vec{E} = 0}$$ Difuzijska enačba

$$\nabla^2 \vec{E}(\vec{r}) - \underbrace{i \mu_0 \sigma \omega}_{k^2} \vec{E}(\vec{r}) = 0$$

E_x in E_y sta 0 in v smereh y in z ne bo sprememb. Prevede se na

$$\frac{\partial^2 E_z(x)}{\partial x^2} - k^2 E_z(x) = 0$$

Rešitev take enačbe poznamo: $\quad 0$ zaradi simetrije

$$E_z(x) = \cancel{A\,sh(kx)} + B\,ch(kx)$$

Uporabimo še robni pogoj

$$U\left(\frac{a}{2}\right) = U\left(-\frac{a}{2}\right) = U_0 \quad ; \quad E_z = \frac{U}{l}$$

Torej

$$E_z\left(\frac{a}{2}\right) = B\,ch\left(\frac{ka}{2}\right)$$

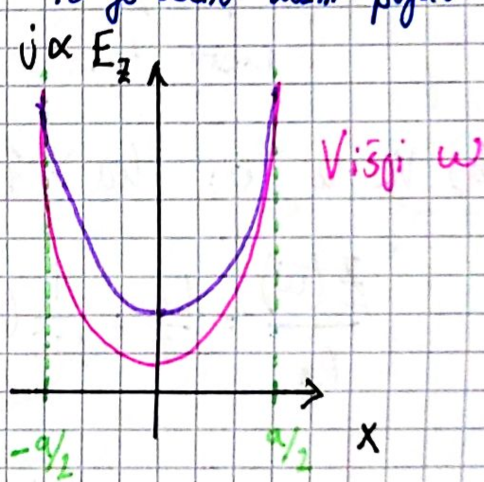
$$U\left(\frac{a}{2}\right) = l\,B\,ch\left(\frac{ka}{2}\right) = U_0$$

$$\Rightarrow B = \frac{U_0}{l\,ch\left(\frac{ka}{2}\right)}$$

in je:

$$E_z(x) = \frac{U_0}{l\,ch\left(\frac{ka}{2}\right)}\,ch(ka)$$

To je točno kožni pojav



Izračunajmo še impedanco: $Z(\omega) = \frac{U_0}{I_0}$

I_z polja dobimo lahko gostoto toka če pomnožimo z σ.

$$I_0 = b\int j(x)\,dx = \frac{b\sigma U_0}{k l\,ch\left(\frac{ka}{2}\right)}\left(sh\left(\frac{ka}{2}\right) - sh\left(-\frac{ka}{2}\right)\right) \Leftarrow \frac{b\sigma U_0}{l\,ch\left(\frac{ka}{2}\right)}\int_{-a/2}^{a/2} ch(kx)\,dx$$

$$= \frac{2b\sigma U_0}{kl}\,tgh\left(\frac{ka}{2}\right)$$

Tako je impedanca:

$$Z(\omega) = \frac{lk}{2b\sigma} \frac{1}{\tanh\left(\frac{ka}{2}\right)}$$

$$R_0 = \frac{l}{S\varrho} \ ; \ \varrho = \frac{1}{\sigma}$$

$$R_0 = \frac{l}{\sigma ab}$$

In tako je:

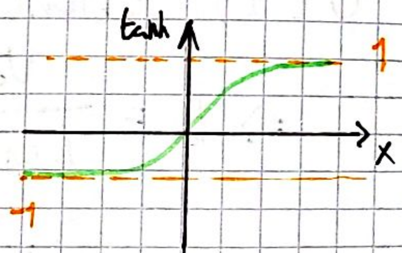
$$\frac{Z(\omega)}{R_0} = \frac{\frac{ka}{2}}{\tanh\left(\frac{ka}{2}\right)}$$

Poglejmo si še dve limiti:

a) Nizke ω: $k^2 = i\sigma\mu_0\omega$, $ka \ll 1$

$$\frac{Z(\omega)}{R_0} = \frac{\frac{ka}{2}}{\frac{ka}{2}} = 1 \Rightarrow Z(\omega) = R_0$$

razvoj tanh



$$\lim_{x \to \infty} \tanh(x) = 1$$

b) Visoke ω: $ka \gg 1$

$$\frac{Z(\omega)}{R_0} = \frac{ka}{2} = \frac{1}{2\sqrt{2}}(1+i)\, a\sqrt{\mu_0\sigma\omega}$$

$$\sqrt{i} = \frac{1}{\sqrt{2}}(1+i)$$

$$R(\omega) = \mathrm{Re}\, Z(\omega) = \frac{1}{2\sqrt{2}} R_0\, a\sqrt{\mu_0\sigma\omega} \ \propto \sqrt{\omega}$$

Večji kot je ω večja je upornost.

Kontinuitetna enačba za energijo EMP

$$\vec{\nabla}\cdot\vec{P} + \frac{\partial w}{\partial t} + \vec{J}\cdot\vec{E} = 0$$

$$w = \frac{dW}{dV} = \frac{dW_e}{dV} + \frac{dW_m}{dV}$$ Volumska gostota energije

$$\vec{P} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$$ Poyntingov vektor (Gostota energijskega toka)

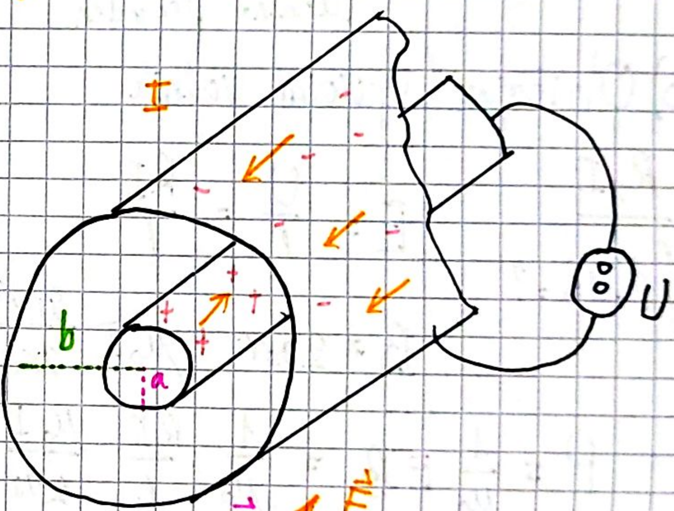
$$\int \vec{\nabla}\cdot\vec{P}\, dV = \oint \vec{P}\cdot d\vec{S}$$

Energijski zakon

$$\Rightarrow \boxed{\underbrace{\oint \vec{P}\cdot d\vec{S}}_{\text{Energijski tok}} + \frac{\partial W}{\partial t} + \underbrace{\int \vec{J}\cdot\vec{E}\, dV}_{\text{Ohmske izgube}} = 0}$$

21. [Poyntingov vektor v dveh vodnikih]

a) Koaksialni kabel



$\underline{U, I}$

$\int \vec{P}\cdot d\vec{S} = ?$ $\qquad B(r) = \frac{\mu_0 I}{2\pi r}$

Električno polje pa po gaussu:

$$E\, 2\pi r l = \frac{e}{\varepsilon_0}$$

$$\Rightarrow E = \frac{e}{\varepsilon_0}\frac{1}{2\pi r l}$$

$$\frac{\partial U}{\partial r} = E \Rightarrow U = \int E\, dr = \int_a^b \frac{e}{\varepsilon_0}\frac{1}{2\pi r l} = \frac{e}{\varepsilon_0}\frac{1}{2\pi l}\ln\left(\frac{b}{a}\right)$$

$$\Rightarrow e = 2\pi l \varepsilon_0 \frac{U}{\ln\left(\frac{b}{a}\right)}$$

Tako je potem polje:

$$\vec{E} = \frac{e}{\varepsilon_0}\frac{1}{2\pi r l} = \frac{U}{\ln\frac{b}{a}\, r}$$

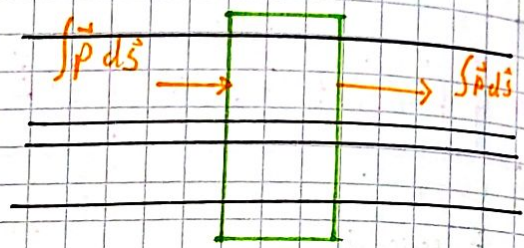
$$P = \frac{1}{\mu_0}\frac{U}{\ln\frac{b}{a}\, r}\cdot\frac{\mu_0 I}{2\pi r} = \frac{IU}{2\pi r^2 \ln(b/a)}$$

$$\int P\, dS = \int_a^b \frac{IU}{2\pi r^2 \ln(b/a)} 2\pi r\, dr = \frac{IU}{\ln b/a}\ln b/a = \underline{UI}$$

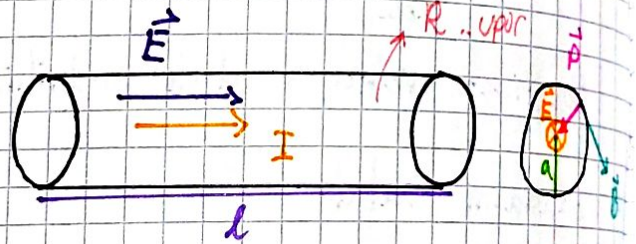
Poglejmo še energijski zakon:

$$\underbrace{\oint \vec{P}\, d\vec{S}}_{0} + \underbrace{\frac{\partial W}{\partial t}}_{0} + \underbrace{\int dV \vec{j}\cdot\vec{E}}_{0} = 0$$

idealno prevoden



b) Običajen uporovni vodnik



R, I

$\oint \vec{P}\cdot d\vec{S} = ?$

$$E = \frac{U}{l} = \frac{RI}{l}$$

$$B \cdot 2\pi r = \mu_0 I \frac{\pi r^2}{\pi a^2}$$

$$P = \frac{1}{\mu_0} E B = \frac{1}{\mu_0}\frac{RI}{l}\frac{\mu_0 I}{2\pi}\frac{r}{a^2}$$

$$\Rightarrow \quad P(r) = RI^2 \frac{r}{2\pi l a^2}$$

Za cel vodnik

$$\oint \vec{P}\, d\vec{S} = -P(a)\cdot 2\pi a l = -RI^2 \frac{a}{2\pi l a^2} 2\pi a l = \underline{\underline{-RI^2}}$$

Potem je energijski zakon:

$$\underbrace{\oint \vec{P}\cdot d\vec{S}}_{-RI^2} + \frac{\partial W}{\partial t} + \underbrace{\int dV \vec{j}\cdot\vec{E}}_{UI \to RI^2} = 0$$

22. [Prekinjeni vodnik]

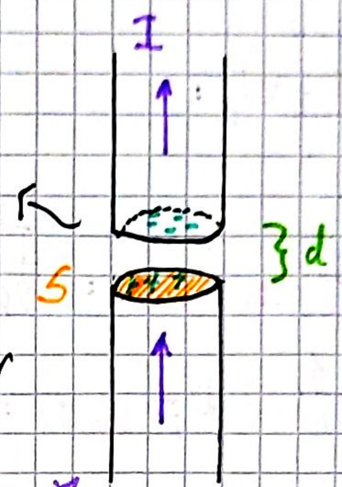
$\underline{d, S, I}$ $\quad (d \ll \sqrt{S})$

$\int \vec{P} d\vec{S} = ?$ (špranja)

Špranja

$\Downarrow$

Ploščati kondenzator

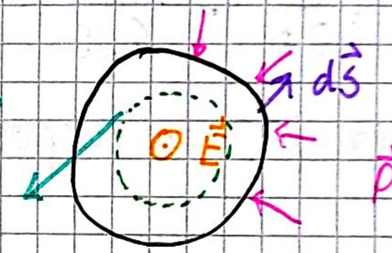


$$E = \frac{\sigma}{\varepsilon_0} = \frac{e}{\varepsilon_0 S} = \frac{It}{\varepsilon_0 S} \to E(t)$$

$\Downarrow$ Povzroča!

$\vec{B}$

$$\vec{\nabla}\times\vec{B} = \mu_0 \underbrace{\vec{j}}_{0} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \quad / \cdot \int d\vec{S}$$



$$\oint \vec{B} d\vec{l} = \mu_0 \varepsilon_0 \frac{\partial}{\partial t} \int \vec{E} \cdot d\vec{S}$$

$$B 2\pi r = \mu_0 \varepsilon_0 \frac{\partial}{\partial t}\left(\vec{E} \cdot \pi r^2\right) = \mu_0 \varepsilon_0 \frac{I \pi r^2}{\varepsilon_0 S}$$

$$\Rightarrow B = \frac{\mu_0 I}{2S} r$$

$$\vec{P} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

$$P = \frac{1}{\mu_0} EB = \frac{1}{\mu_0} \frac{I}{\varepsilon_0 S} t \frac{\mu_0 I}{2S} r = \frac{I^2}{2\varepsilon_0 S^2} rt$$

Zanima nas $\int \vec{P} d\vec{S}$ za celo špranjo:

$$\int \vec{P} \cdot d\vec{S} = -2\pi a d\, P(a) = -\frac{I^2}{2\varepsilon_0 S^2} a t 2\pi a d = -\frac{I^2 d}{\varepsilon_0 S} t =$$

$$= -I \underbrace{dE}_{U} = -I U(t)$$

Poglejmo če velja energijski zakon:

$$\oint \vec{P} \cdot d\vec{S} + \frac{\partial W}{\partial t} + \underbrace{\int \underset{\overset{\parallel}{0}}{\vec{j}} \cdot \vec{E}\, dV}_{0} = 0$$

$$W = W_e + W_m$$

$$W_e = \frac{1}{2} \varepsilon_0 E^2 \cdot Sd = \frac{I^2 d}{2\varepsilon_0 S} t^2$$

$$W_m = \int \frac{1}{2\mu_0} B^2 dV \neq W_m(t) \Rightarrow \frac{\partial W_m}{\partial t} = 0$$

Torej je:

$$\frac{\partial W}{\partial t} = \frac{\partial W_e}{\partial t} = \frac{I^2 d}{\varepsilon_0 S} t$$

Ta izraz je ravno nasproten $\int \vec{P} d\vec{S}$ in torej energijski zakon velja.

Snov v električnem polju

Polarizacija

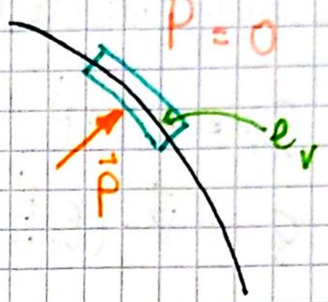
Vezani naboj:

$$\rho_v = -\vec{\nabla}\cdot\vec{P}$$

Vsi naboji (vezani + prosti):

$$\rho = \vec{\nabla}\cdot(\varepsilon_0\vec{E})$$

Rob snovi:



$$\underbrace{\int \rho_v \, dV}_{e_v} = -\int \vec{\nabla}\cdot\vec{P}\, dV \quad \text{Gauss}$$

$$e_v = -\oint \vec{P}\cdot d\vec{S} = -\vec{P}\cdot\vec{S}\Big|_{Not}^{Zun} = 0-(-\vec{P}\vec{n}S) = \vec{P}\cdot\vec{n}S$$

$$\Rightarrow \quad \frac{e_v}{S} = \sigma_v = \vec{P}\cdot\vec{n}$$

23. [Radialno polarizirana krogla]

a, k

$\rho_v = ?$

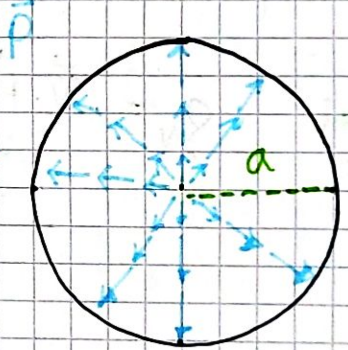
$\sigma_v = ?$

$e_v = ?$

$\vec{E}(\vec{r}) = ?$

$$\vec{P} = k\vec{r} \quad ; \quad k > 0$$

Spontana polarizacija



$$\rho_v = -\vec{\nabla}\cdot(k\vec{r}) = -3k$$

$$\sigma_v = k\vec{r}\cdot\vec{n}\Big|_{r=a} = ka$$

$$e_v = \rho_v V + \sigma_v S = -3k\frac{4\pi a^3}{3} + ka4\pi a^2 = -4\pi k a^3 + 4\pi k a^3 = \underline{0}$$

Volumen + rob volumna

$\vec{P}$ je sestavljena iz dipolov (enako + in −)

$$\cancel{e(\vec r) = \int \rho_v \, dV}$$

$$e(r) = \int_0^r \rho_v \, dV = \frac{4\pi r^3}{3}(-3k) = \varepsilon_0 E(r) 4\pi r^2$$

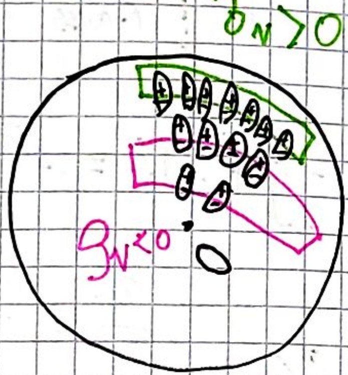
$$\Rightarrow \quad \vec E(r) = -\frac{k\vec r}{\varepsilon_0} = -\frac{\vec P}{\varepsilon_0}$$

To bi lahko dobili tudi iz Maxwellovih enačb:

$$\rho_v = -\vec\nabla\cdot\vec P \quad \Rightarrow \quad -\vec P = \varepsilon_0 \vec E$$

$$\rho = \vec\nabla\cdot(\varepsilon_0 \vec E)$$

$$\rho = \rho_v + \cancelto{0}{\rho_{zun}}$$



24. [Homogeno polarizirana krogla]

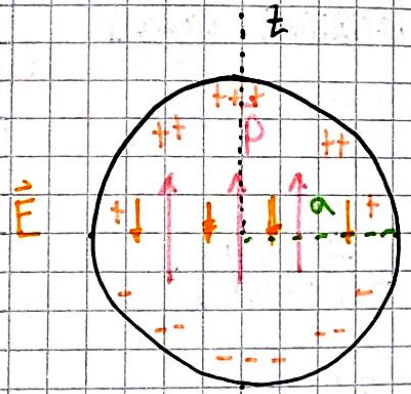
P, a — povsod

$U(r,\theta) = ?$

$\vec E(\vec r) = ?$

znotraj

$$\rho_v = -\vec\nabla\cdot\vec P = 0 \qquad (\vec P \text{ konst.})$$



$$\sigma_v = \vec P\cdot\vec n = P\cos\theta$$

Ker $\cos\theta = P_1(\cos\theta)$ imamo nastavek samo z $l = 1$:

$$U(r,\theta) = A_1 r\cos\theta + \frac{B_1}{r^2}\cos\theta$$

$$\Rightarrow \quad U(r,\theta) = \begin{cases} A_1 r\cos\theta \ ; & r < a \\ \frac{B_1}{r^2}\cos\theta \ ; & r > a \end{cases}$$

Robni pogoji:

RP1: U zvezen $\Rightarrow A_1 a\cos\theta = \frac{B_1}{a^2}\cos\theta \Rightarrow A_1 = \frac{B_1}{a^3}$

RP2: $E^{\perp}_{zun} - E^{\perp}_{not} = \frac{\sigma_v}{\varepsilon_0}$ Gaussov

$$E_N^{\perp}\Big|_{r=a} = -\frac{\partial}{\partial r}\left(A_1 r\cos\theta\right)\Big|_{r=a} = -A_1\cos\theta$$

$$E_Z^{\perp}\Big|_{r=a} = \frac{2B_1}{r^3}\cos\theta\Big|_{r=a} = \frac{2B_1}{a^3}\cos\theta$$

$$\Rightarrow \frac{2B_1}{a^3} + A_1 = \frac{P}{\varepsilon_0} \Rightarrow B_1 = \frac{Pa^3}{3\varepsilon_0}$$

Tako je potencial:

$$U(r,\theta) = \begin{cases} \frac{P}{3\varepsilon_0}\underbrace{r\cos\theta}_{z}; & r<a \\ \frac{Pa^3}{3\varepsilon_0 r^2}\cos\theta; & r>a \end{cases}$$

In še polje:

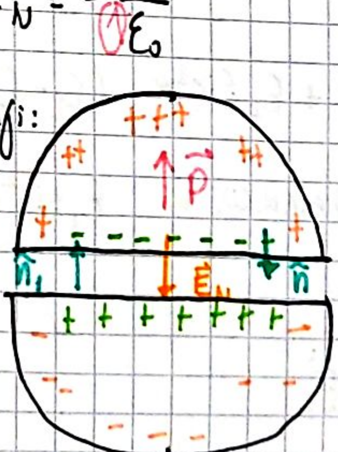
$$\vec{E}_N(\vec{r}) = -\vec{\nabla}U = -\left(\frac{\partial}{\partial z}U\right) = -\frac{\overbrace{P\,\hat{e}_z}^{\vec{P}}}{3\varepsilon_0} = -\frac{\vec{P}}{3\varepsilon_0}$$

Če bi naredili enako nalogo za valj: $\vec{E}_N = -\frac{\vec{P}}{2\varepsilon_0}$ Odvisno od dimenzionalnosti

Če bi naredili enako nalogo za ploščico: $\vec{E}_N = -\frac{\vec{P}}{\varepsilon_0}$

Razrežemo kroglo na pol in pogledamo polje v špranji:

$$\vec{E}_1 = \vec{E}_N + \frac{\sigma_v'}{\varepsilon_0}\hat{e}_z = \frac{2}{3}\frac{\vec{P}}{\varepsilon_0}$$



V špranji polje E_1

$$\sigma_v' = \vec{P}\,\hat{n}_1 = P$$

Dielektriki:

Nimajo spontane polarizacije.

$$\rho_v = -\vec{\nabla}\cdot\vec{P} \quad \rightarrow \rho_z = \rho - \rho_v = \vec{\nabla}\cdot\underbrace{(\varepsilon_0\vec{E}+\vec{P})}_{\vec{D}}$$

$$\rho = \vec{\nabla}\cdot(\varepsilon_0\vec{E})$$

$$\rho = \rho_v + \rho_z$$

$$\vec{D} = \varepsilon_0\vec{E} + \vec{P}$$

$$\rho_z = \vec{\nabla}\cdot\vec{D}$$

Za dielektrike velja linearna zveza (konstitutivna relacija):

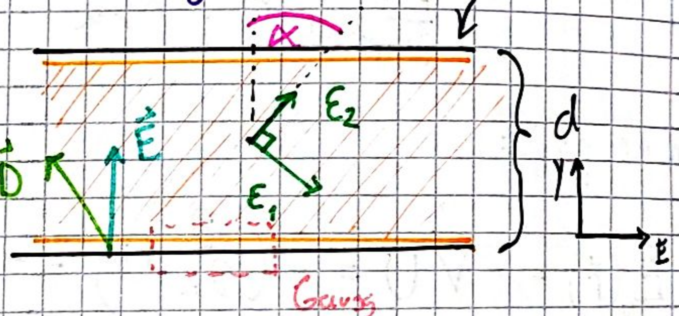
$$\vec{D} = \varepsilon_0 \underline{\varepsilon}\, \vec{E} \qquad \vec{D} \not\parallel \vec{E}$$

V splošnem tenzor

25. [Anizotropni dielektrik v ploščatem kondenzatorju]

$\alpha, \varepsilon_1, \varepsilon_2, d$

$C = ?$



$$C = \frac{e_z}{U} \qquad e_z \to D, \quad U \to E$$

$$\underline{\varepsilon} = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \varepsilon_1 & 0 \\ 0 & \varepsilon_2 \end{bmatrix} \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} =$$

$$= \begin{bmatrix} \varepsilon_1\cos^2\alpha + \varepsilon_2\sin^2\alpha & (\varepsilon_2-\varepsilon_1)\sin\alpha\cos\alpha \\ (\varepsilon_2-\varepsilon_1)\sin\alpha\cos\alpha & \varepsilon_1\sin^2\alpha + \varepsilon_2\cos^2\alpha \end{bmatrix}$$

$$\rho_z = \nabla\cdot\vec{D} \overset{\text{Gauss}}{\Rightarrow} e_z = \vec{D}\cdot\vec{S} = D_y S$$

$$D_y = \frac{e_z}{S}$$

$$\vec{D} = \varepsilon_0\,\underline{\varepsilon}\,\vec{E}$$

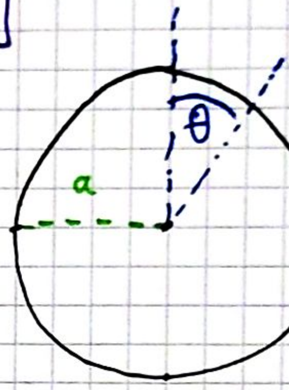
$$\begin{bmatrix} D_x \\ D_y \end{bmatrix} = \varepsilon_0 \begin{bmatrix} \quad \underline{\varepsilon} \quad \end{bmatrix} \begin{bmatrix} 0 \\ E \end{bmatrix} \qquad E = \frac{U}{d}$$

$$\Rightarrow D_y = \varepsilon_0 \left(\varepsilon_1 \sin^2\alpha + \varepsilon_2 \cos^2\alpha \right) \frac{U}{d} = \frac{e_z}{S}$$

$$\Rightarrow C = \frac{e_z}{U} = \frac{\varepsilon_0 S}{d} \left(\varepsilon_1 \sin^2\alpha + \varepsilon_2 \cos^2\alpha \right)$$

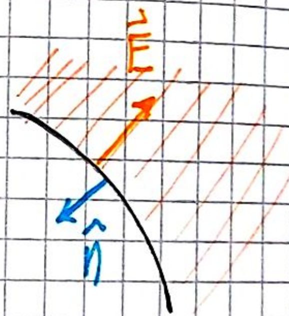
C_0 prazen kondenzator

26. [Točkasti dipol v krogelni votlini dielektrika]



c) $\sigma_v = ?$

Gauss : $\frac{\sigma_N}{\varepsilon_0} = E_\perp^{zun} - E_\perp^{not}$ Pravzaprav robni pogoj za $\vec{E}$.



Alternativno:

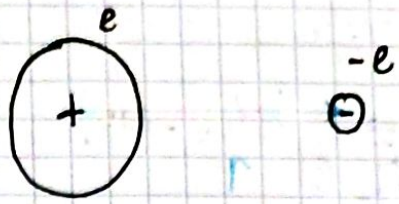
$$\sigma_v = \vec{P}\cdot\vec{n} = (\varepsilon - 1)\,\varepsilon_0\,\underbrace{\vec{E}\cdot\vec{n}}_{-E_r(r=a)}\Big|_{r=a} \;!$$

Še:

$$\rho_v = -\vec{\nabla}\cdot\vec{P} = -(\varepsilon-1)\,\varepsilon_0\,\vec{\nabla}\cdot\vec{E} = \emptyset$$

27. [Dielektričnost plazme]

m_e, n: Plazma je plin nabitih delcev.



$\vec{E}(t) = \vec{E}_0 e^{-i\omega t}$

$\downarrow$

$\vec{r}(t)$

$\downarrow$

$\vec{p}_e(t)$

$\downarrow$

$\vec{P}(t) = n\,\vec{p}_e(t) = (\varepsilon - 1)\varepsilon_0 \vec{E}(t)$

↳ številska gostota elektronov

Zanima nas $\varepsilon(\omega)$!

$$\vec{F} = e\vec{E} = m\ddot{\vec{r}} \Rightarrow \vec{r} = r_0 e^{-i\omega t}$$

Torej:

$$m(\mp\omega^2)\vec{r}_0 \cancel{e^{-i\omega t}} = \mp e\vec{E} \cancel{e^{-i\omega t}}$$

$$\Rightarrow \vec{r}_0 = \frac{e}{m\omega^2}\vec{E}_0$$

$$p_e = r_0 \cdot e$$

$$\vec{p}_{e_0} = -e\vec{r}_0 = -\frac{e^2}{m\omega^2}\vec{E}_0$$

$$\vec{P} = n\vec{p}_{e_0} = \underbrace{\frac{-ne^2}{m\omega^2}}_{(\varepsilon-1)\varepsilon_0}\vec{E}_0$$

Plazemska frekvenca ω_p^2

$$\hookrightarrow \varepsilon(\omega) = 1 - \frac{ne^2}{\varepsilon_0 m\omega^2} = 1 - \frac{\omega_p^2}{\omega^2}$$

b) Disperzijska relacija $\omega(\vec{k}) = ?$

$$\vec{E} = \vec{E}_0 e^{i(kz - \omega t)}$$ } ravni val, ki potuje po plazmi

Za vakuum bi veljalo: $\omega = c_0 k$

Ker imamo snov bo pa drugače:

Širjenje ravnega vala:

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad / \vec{\nabla}\times$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \varepsilon_0 \varepsilon \frac{\partial \vec{E}}{\partial t}$$

$$\Rightarrow \quad \vec{\nabla}(\nabla \times \vec{E}) = -\frac{\partial}{\partial t}\left(\mu_0 \varepsilon_0 \varepsilon \frac{\partial \vec{E}}{\partial t}\right)$$

$$0 \quad \parallel$$

$$\vec{\nabla}(\cancel{\vec{\nabla} \cdot \vec{E}}) - \nabla^2 \vec{E}$$

Tako dobimo valovno enačbo

$$\nabla^2 \vec{E} - \mu_0 \varepsilon_0 \varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

Če vstavimo nastavek za ravni val:

$$-k^2 \vec{E} - \mu_0 \varepsilon_0 \varepsilon (-\omega^2) \vec{E} = 0$$

$$1/c_0^2$$

$$\vec{E}\underbrace{\left(k^2 - \mu_0 \varepsilon_0 \varepsilon \omega^2\right)}_{0} = 0$$

$$\Rightarrow \quad \omega = \frac{c_0}{\sqrt{\varepsilon}} k$$

Vstavimo še naš izraz za $\varepsilon(\omega)$:

$$\omega = \frac{c_0 k}{\sqrt{1-\frac{\omega_p^2}{\omega^2}}}$$

Mali k: $\omega = \omega_p \sqrt{1+\frac{c_0^2 k^2}{\omega_p^2}} = \omega_p\left(1+\frac{1}{2}\frac{c_0^2}{\omega_p^2}k^2+\dots\right)$

$$\omega\sqrt{1-\frac{\omega_p^2}{\omega^2}} = c_0 k$$

Veliki k: $\omega \approx c_0 k$

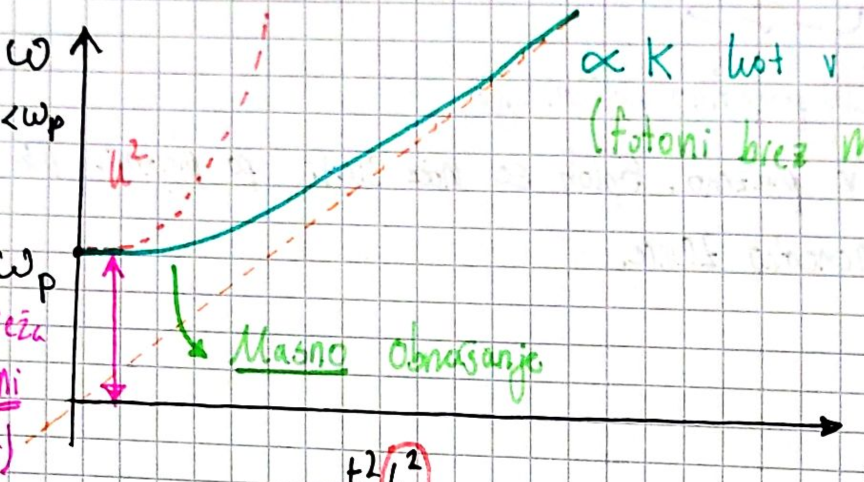
$$\omega^2\left(1-\frac{\omega_p^2}{\omega^2}\right) = c_0^2 k^2 \Rightarrow \omega = \sqrt{c_0^2k^2+\omega_p^2}$$

(A)

$\varepsilon = 1-\frac{\omega_p^2}{\omega^2}$ $\quad \omega<\omega_p$

$\varepsilon \leq 0$

Frekvenčna reža (pri teh frekvencah ni ravnih valov v plazmi)



$$\hbar\omega = W_k = \frac{\hbar^2 k^2}{2m}$$

c) Fazna in grupna hitrost:

$$v_F = \frac{\omega}{k} \qquad v_G = \frac{\partial\omega}{\partial k}$$

Širjenje faze

↳ širjenje gruč

$$v_F = \frac{\omega}{k} = \frac{\sqrt{c_0^2k^2+\omega_p^2}}{k} = \sqrt{c_0^2+\frac{\omega_p^2}{k^2}} > c_0$$

S fazami ne prenašamo informacij. Torej to je lahko večje od c_0

$$v_G = \frac{\partial\omega}{\partial k} = \frac{2c_0^2k}{2\sqrt{\omega_p^2+c_0^2k^2}} = \frac{c_0\, c_0 k}{\sqrt{\omega_p^2+c_0^2k^2}} = \frac{c_0}{\sqrt{1+\frac{\omega_p^2}{c_0^2k^2}}} < c_0$$

To je pa ok!

$$v_F v_G = c_0\sqrt{1+\frac{\omega_p^2}{c_0^2k^2}}\;\frac{c_0}{\sqrt{1+\frac{\omega_p^2}{c_0^2k^2}}} = c_0^2$$

$(\star) \Rightarrow \varepsilon = \varepsilon'$

$$\omega = \frac{c_0}{i\sqrt{\varepsilon'}} k$$

$$;\vec{E} = E_0 \exp(i(kz-\omega t)) = \vec{E}_0 \exp\left(i\left[\frac{i\sqrt{\varepsilon'}\omega}{c_0} z\right] - i\omega t\right) =$$

$$= \vec{E}_0 \exp\Big(\underbrace{-\frac{\sqrt{\varepsilon}\,\omega}{c_0} z}_{\text{Eksponentno pojemanje}} - i\omega t\Big)$$

To noče noter v plazmo. Sploh se noče širiti po plazmi. Na robu valovanje eksponentno zamre.

$$\left[\nabla_\perp^2 + \left(\frac{\omega^2}{c_0^2} - k^2\right)\right]\begin{Bmatrix}\vec{E}(\rho)\\ \vec{H}(\rho)\end{Bmatrix} = 0$$

$$H_x = i\,\frac{\omega\varepsilon_0 \frac{\partial E_z}{\partial y} - k\frac{\partial H_z}{\partial x}}{k^2 - \frac{\omega^2}{c_0^2}}$$

$$H_y = i\,\frac{-\omega\varepsilon_0 \frac{\partial E_z}{\partial x} - k\frac{\partial H_z}{\partial y}}{k^2 - \frac{\omega^2}{c_0^2}}$$

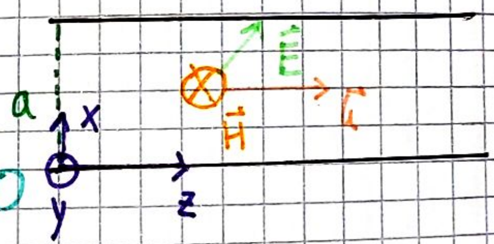
$$E_y = i\,\frac{\omega\mu_0 \frac{\partial H_z}{\partial x} - k\frac{\partial E_z}{\partial y}}{k^2 - \omega^2/c_0^2}$$

$$E_x = i\,\frac{-\omega\mu_0 \frac{\partial H_z}{\partial y} - k\frac{\partial E_z}{\partial x}}{k^2 - \omega^2/c_0^2}$$

28. [Valovni vodnik iz dveh vzporednih plošč]

a) $\frac{\partial}{\partial y} = 0$

Iščemo



Valovanje v TM načinu: $H_z = 0$; $E_z \neq 0$

Hitro lahko vidimo: $H_x = 0 \quad E_y = 0$

$E_x \neq \quad H_y \neq 0$

$$\left(\frac{\partial^2}{\partial x^2} + \varkappa^2\right) E_z(x) = 0 \qquad \text{RP: } E_z(a) = 0 = E_z(0)$$

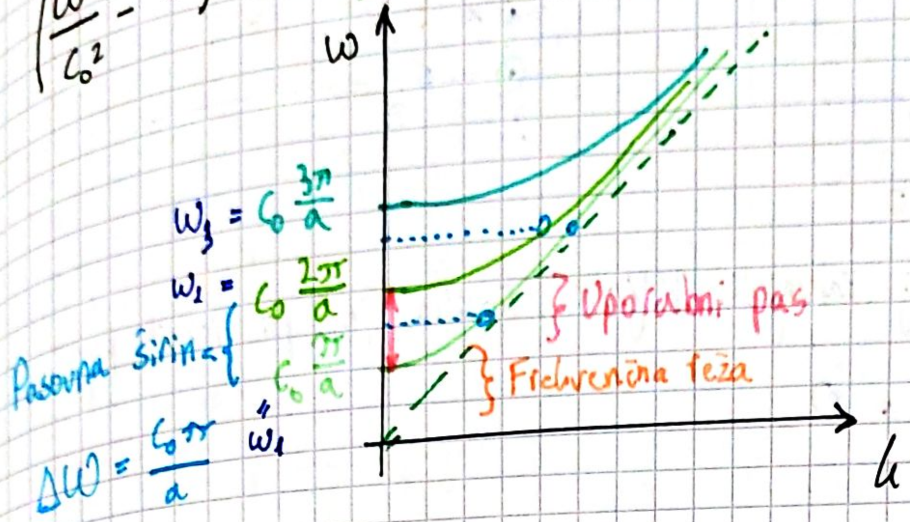
$$\Rightarrow E_z(x) = A\cos(\varkappa x) + B\sin(\varkappa x)$$

$$E_z(0) = 0 \Rightarrow A = 0$$

$$E_z(a) = 0 \Rightarrow B\sin(\varkappa a) = 0 \Rightarrow \varkappa = \frac{\pi n}{a};\ n = 1, 2, \dots$$

$$\Rightarrow E_z(x) = \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{a}x\right)$$

$$\left(\frac{\omega^2}{c_0^2} - k^2\right) = \left(\frac{n\pi}{a}\right)^2 \Rightarrow \omega = c_0\sqrt{\left(\frac{n\pi}{a}\right)^2 + k^2}$$



b) Impedanca valovnega vodnika v TM načinu

Transverzalna impedanca

$$Z \equiv \frac{E_\perp}{H_\parallel}$$

V našem primeru je to:

$$Z = \frac{E_x}{H_y} = \frac{k}{\omega \varepsilon_0}$$

$$\downarrow$$

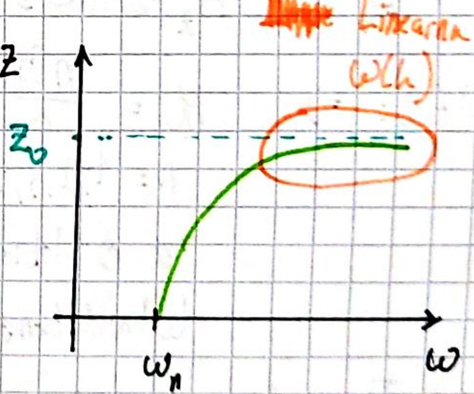
$$Z(\omega)$$

Torej izrazimo k z ω:

$$\omega = c_0\sqrt{\left(\frac{n\pi}{a}\right)^2 + k^2}, \quad \frac{n\pi}{a} = \frac{\omega_n}{c_0}$$

$$\frac{\omega^2}{c_0^2} = \left(\frac{\omega_n}{c_0}\right)^2 + k^2$$

$$k = \frac{1}{c_0}\sqrt{\omega^2 - \omega_n^2}$$

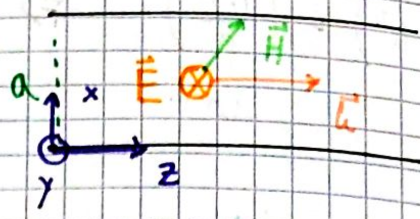


$$\Rightarrow Z(\omega) = \frac{k}{\omega\varepsilon_0} = \frac{\sqrt{\omega^2 - \omega_n^2}}{\omega\varepsilon_0 \frac{\sqrt{\varepsilon_0}}{\sqrt{\mu_0 \varepsilon_0}}} = \sqrt{\frac{\mu_0}{\varepsilon_0}}\sqrt{1 - \left(\frac{\omega_n}{\omega}\right)^2}$$

Impedanca valovna

$$Z_0 = 376\ \Omega$$

TE način (razlike glede na TM)



Iščemo

$$E_z = 0 \quad , \quad H_z \neq 0$$

$$E_x = 0 \quad , \quad H_y = 0$$

$$H_x \neq 0 \quad , \quad E_y \neq 0$$

Tu sta se vlogi polj zamenjali. Ključna razlika je pri robnih pogojih.

$$\left(\frac{\partial^2}{\partial x^2} + \varkappa^2\right) H_z(x) = 0$$

RP: $H_x(0) = H_x(a) = 0$ → Pogoj za H_x

$$\frac{\partial H_z}{\partial x}(0) = \frac{\partial H_z}{\partial x}(a) = 0 \Leftarrow$$

Pogledamo izraz za H_x in ugotovimo $\frac{\partial H_z}{\partial x}$ mora biti 0.

Normalni odvod = 0 (splošno)

$$H_z(x) = A\cos(\varkappa x) + B\sin(\varkappa x)$$

$$\frac{\partial H_z}{\partial x}(0) = -A\varkappa\sin(\varkappa x) + B\varkappa\cos(\varkappa x)\Big|_{x=0} \Rightarrow \underline{B = 0}$$

$$\Rightarrow \quad H_z = A\cos(\varkappa x)$$

RP2: $\Rightarrow \quad \varkappa = \frac{n\pi}{a}$ $\quad \Rightarrow \quad H_z(x) = \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi}{a}x\right)$

$n = 1, 2, 3, \ldots$

Za 0 dobimo statično polje

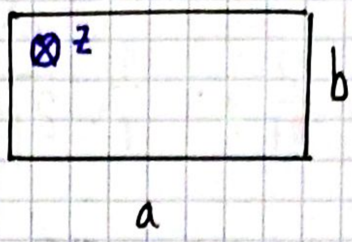
(odvod konst je 0 in ne valovanje)

b) Impedanca: $\quad Z \equiv \frac{E_{\parallel}}{H_{\perp}}$

Pri nas:

$$Z = \frac{E_y}{H_x} = \ldots = \frac{Z_0}{\sqrt{1 - \frac{\omega_n^2}{\omega^2}}}$$

Valovni vodnik s pravokotnim presekom



TM način:

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\varkappa^2\right)\underbrace{E_z(x,y)}_{X(x)\cdot Y(y)}=0$$

$$\Rightarrow X''Y + XY'' + \varkappa^2 XY = 0$$

$$\Rightarrow \underbrace{\frac{X''}{X}}_{-\varkappa_x^2} + \underbrace{\frac{Y''}{Y}}_{-\varkappa_y^2} + \varkappa^2 = 0 \quad \Rightarrow$$

$$X'' + \varkappa_x^2 X = 0 \rightarrow \varkappa_x = \frac{n\pi}{a}$$

$$Y'' + \varkappa_y^2 Y = 0 \rightarrow \varkappa_y = \frac{m\pi}{b}$$

$n, m = 1, 2, \dots$

$$\varkappa^2 = \varkappa_x^2 + \varkappa_y^2$$

Pride:

$$\varkappa^2 = \frac{\omega^2}{c_0^2} - k^2 = \varkappa_x^2 + \varkappa_y^2$$

$$\omega = c_0\sqrt{\left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2 + k^2}$$

TE način:

Vse je enako <u>razen</u> eden iz med m ali n je lahko tudi 0, ampak samo eden, da bo potem pri drugem krajevno odvisnost.

29. [Valjasti valovni vodnik]

a

TM, TE

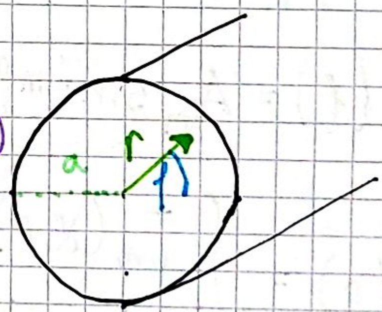
↓ ↓

$E_z = ?$ $H_z = ?$

$\omega(k) = ?$

$$\left[\nabla_\perp^2 + \underbrace{\left(\frac{\omega^2}{c_0^2} - k^2\right)}_{\varkappa^2}\right]\left\{\begin{matrix}\vec{E}(\vec{\rho})\\ \vec{H}(\vec{\rho})\end{matrix}\right\} = 0$$

RP: $E_\parallel\big|_\partial = 0, \quad H_\perp\big|_\partial = 0$



a) TM: $H_z = 0$ $E_z \neq 0$

Iščemo

$$\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2} + \varkappa^2\right] \underbrace{E_z(r,\varphi)}_{R(r)\Phi(\varphi)} = 0$$

$$\frac{1}{r}(rR')'\Phi + \frac{1}{r^2}R\Phi'' + \varkappa^2 R\Phi = 0$$

$$\left(\frac{1}{r}R' + R''\right)\Phi + \frac{1}{r^2}R\Phi'' + \varkappa^2 R\Phi = 0 \quad / \cdot \frac{r^2}{R\Phi}$$

$$\frac{rR' + r^2R''}{R} + \frac{\Phi''}{\Phi} + \varkappa^2 r^2 = 0$$

Velja le če konst.

$$\underbrace{\frac{r^2R'' + rR'}{R} + \varkappa^2 r^2}_{\text{Funkcija samo } r} = \underbrace{-\frac{\Phi''}{\Phi}}_{\text{Funkcija samo } \varphi} = m^2$$

Dobimo dve enačbe:

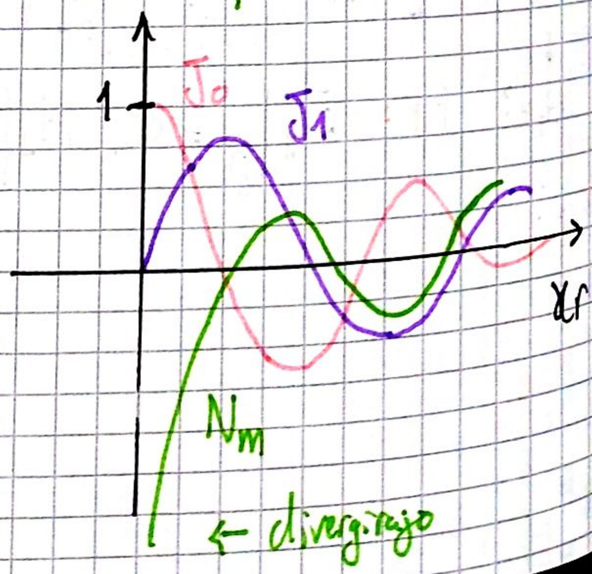
$$\Phi'' + m^2\Phi = 0$$

$$r^2R'' + rR' + (\varkappa^2r^2 - m^2)R = 0$$ → Besselova diferencialna enačba

$$\Rightarrow \quad \Phi(\varphi) = A_m \sin(m\varphi - \varphi_m)$$ sin $m\varphi$, cos $m\varphi$

$$R(r) = \begin{cases} J_m(\varkappa r) \\ N_m(\varkappa r) \end{cases}$$

Divergirajo



Tako imamo splošno rešitev:

$$E_z(r,\varphi) = \sum_{m=0,\ n=1}^{\infty} A_m J_m(\chi_{m_n} r)\sin(m\varphi - \varphi_m)$$

Uporabimo še RP

$$E_z(r=a,\varphi) = 0 \quad \Rightarrow \quad J_m(\chi_{m_n} a) = 0$$

Ničle Besselovih funkcij (so tabelirane)

m →

χ_{m_n}	J_0	J_1	J_2	...
1	2.40	3.83	5.14	
2	5.52	7.02	...	
3	⋮	⋮	⋮	
⋮				

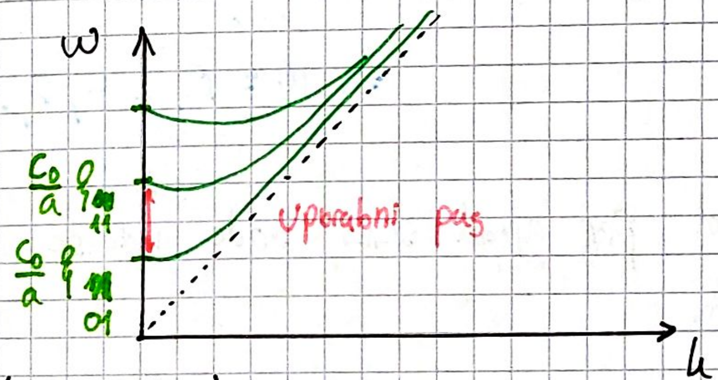
Indeks ničle ↓

$$\Rightarrow \quad \chi a = \chi_{mn}$$

$$\chi = \frac{\chi_{mn}}{a}$$

$$\frac{\omega^2}{c_0^2} - k^2 = \omega^2 \quad \Rightarrow \quad \omega = c_0\sqrt{k^2 + \frac{\chi_{mn}^2}{a^2}}$$

Disperzijska relacija



$$\Delta\omega = \frac{c_0}{a}(\chi_{11} - \chi_{10}) \quad \text{pasovna širina}$$

b) TE: $H_z \neq 0$

Iščemo

Separacija spremenljivk poteka enako:

$$H_z(r,\varphi) = \sum_m A_m J_m(\chi r)\sin(m\varphi - \varphi_m)$$

RP: $H_r(r=a,\varphi) = 0 \Rightarrow \frac{\partial H_z}{\partial r}(r=a,\varphi) = 0$

$$\Rightarrow J_m'(\chi a) = 0$$

Ničle odvodov Besselovih funkcij (tudi tabelirane)

ξ'_{mn}	J'_0	J'_1	J'_2	...
1	3.83	1.84	3.05	
2	7.02	5.33	...	
3	⋮	⋮	⋮	
⋮				

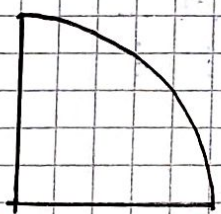
Ničle odvodov Besselovih funkcij

Imamo spet: $\omega = c_0\sqrt{k^2 + \frac{\xi'^2_{mn}}{a^2}}$

Najnižji ničli ξ'_{11}, ξ'_{21} $\Rightarrow \Delta\omega = \frac{c_0}{a}\left(\xi'_{21} - \xi'_{11}\right)$

Valovni vodniki s presekom dela kroga

a) TM:



Enaka splošna rešitev, dodatni RP

$E_z(\varphi = 0, r) = 0 \Rightarrow \sin(m\varphi)$ kosinusni del mora biti 0

$E_z(\varphi\,\frac{\pi}{2}, r) = 0 \Rightarrow \sin\left(m\frac{\pi}{2}\right) = 0$

$\Rightarrow m = 0, 2, 4, 6, \ldots$

Trivialna rešitev

Radialno isto kot prej ampak samo sode indekse m. Najnižji veji pri $m=2$ in $m=4$.

b) TE:

Po istem kopitu, robni pogoji drugačni:

$H_\varphi(\varphi=0, r) = 0 \Rightarrow \frac{\partial H_z}{\partial\varphi}(\varphi=0, r) = 0 \rightarrow \cos(m\varphi)$

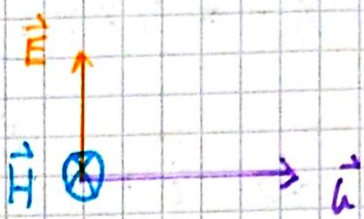
$H_\varphi(\varphi=\frac{\pi}{2}, r) = 0 \Rightarrow \frac{\partial H_z}{\partial\varphi}(\varphi=\frac{\pi}{2}, r) = 0 \rightarrow \sin\left(m\frac{\pi}{2}\right) = 0$

Spet $m = 0, 2, 4, 6, \ldots$

ni trivialna rešitev

Najnižji ~~veji~~ veji 0 in 2.

30. [TEM načini v Valovnih Vodnikih]



Torej imamo hkrati TE in TM

$E_z = 0 \quad H_z = 0$

a) Pokaži, da velja:

$$\left.\begin{aligned}\vec\nabla\times\vec E &= i\vec k\times\vec E\\ \vec\nabla\times\vec H &= i\vec k\times\vec H\end{aligned}\right\} \Rightarrow \omega = c_0 k$$

Valovni vodnik: $\vec E(\vec r,t) = \vec E(\vec\rho)\, e^{i(kz-\omega t)}$

$$\nabla\times\vec E = \vec\nabla\times\vec E(\vec\rho)\cdot e^{i(kz-\omega t)} + \underbrace{\vec E(\vec\rho)\times\underbrace{\vec\nabla e^{i(kz-\omega t)}}_{\underbrace{\hat e_z ik}_{i\vec k}\, e^{i(kz-\omega t)}}}_{i\vec k\times\vec E(\vec r,t)}$$

Še:

$$\begin{bmatrix}\frac{\partial}{\partial x}\\ \frac{\partial}{\partial y}\\ \frac{\partial}{\partial z}\end{bmatrix}\times\begin{bmatrix}E_x(\vec\rho)\\ E_y(\vec\rho)\\ 0\end{bmatrix} = \begin{bmatrix}\cancel{-\frac{\partial}{\partial z}E_y(\vec\rho)}\\ \cancel{-\frac{\partial}{\partial z}E_x(\vec\rho)}\\ \frac{\partial}{\partial x}E_y - \frac{\partial}{\partial y}E_x\end{bmatrix} = (\nabla\times\vec E)_z = -\mu_0\frac{\partial H_z}{\partial t} = 0$$

Za $\vec H$ se naredi čisto enako.

$$i\vec k\times\vec E = \vec\nabla\times\vec E = -\mu_0\frac{\partial H}{\partial t} = +i\mu_0\omega\vec H$$

$$\Rightarrow \vec H = \frac{\vec k}{\mu_0\omega}\times\vec E$$

Enako kot v praznem prostoru.

$$i\vec{k} \times \vec{H} = \vec{\nabla}\times\vec{H} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t} = -i\omega\varepsilon_0\vec{E}$$

$$\Rightarrow \quad \vec{E} = -\frac{\vec{k}}{\varepsilon_0\omega}\times\vec{H}$$

Iz tega lahko dobimo disperzijsko relacijo

$$\vec{k}\times\vec{H} = \vec{k}\times\left(\frac{\vec{k}}{\mu_0\omega}\times\vec{E}\right) = -\varepsilon_0\omega\vec{E}$$

Torej:

$$\frac{\vec{k}}{\mu_0\omega}\overset{0 \ \text{ker}\ \perp}{(\vec{k}\cdot\vec{E})} - \vec{E}\,\frac{k^2}{\mu_0\omega} = -\varepsilon_0\omega\vec{E}$$

$$\Rightarrow \quad k^2 = \underbrace{\mu_0\varepsilon_0}_{1/c_0^2}\,\omega^2 \Rightarrow \omega = c_0 k$$

Poglejmo valovanje:

$$\left[\nabla_\perp^2 + \underbrace{\left(\frac{\omega^2}{c_0^2} - k^2\right)}_{0}\right]\left\{\begin{matrix}\vec{E}(\vec{\rho})\\ \vec{H}(\vec{\rho})\end{matrix}\right\} = 0$$

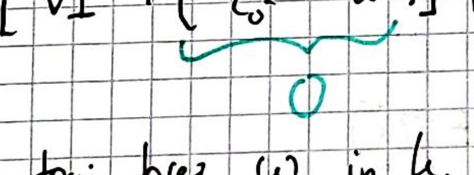
Valovna enačba je torej brez ω in k. Rešujemo statični problem.

$$\nabla_\perp^2 E_\parallel = 0 \qquad \text{RP: } E_\parallel\big|_\partial = 0$$

} Dirichletova naloga rešitev je samo 0 če je presek vodnika enostavna ploskev

$E_\parallel = 0$ Ni TEM!

Koaksialni vodnik recimo pa ni enostavna ploskev in dve plošči



Lahko imata TEM!

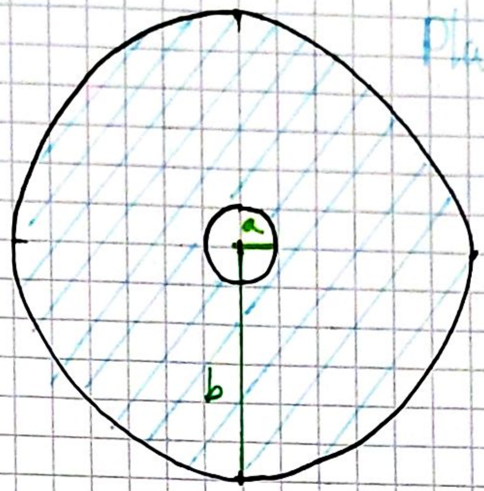
31. [TEM način v koaksialnem vodniku]

a, b, ω_p $\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$

a) $\omega(k) = ?$

b) $Z(\omega) = ?$

$= \frac{U}{I}$



Plazma