$$\vec{L}^2 = \vec{L}\cdot\vec{L} = (\vec{r}\times\vec{p})(\vec{r}\times\vec{p}) = L_x^2 + L_y^2 + L_z^2 = L^2$$

• Lastnosti

• $[L_\alpha, L_\beta] = i\hbar\,\varepsilon_{\alpha\beta\gamma} L_\gamma$

$[x_\alpha, p_\beta] = i\hbar\,\delta_{\alpha\beta}$

$[x_\alpha, x_\beta] = 0$

$[p_\alpha, p_\beta] = 0$

$$[L_x, L_y] = [yp_z - zp_y,\ zp_x - xp_z] =$$

$$= [yp_z, zp_x] - \underbrace{[p_y z, zp_x]}_{0} - \underbrace{[yp_z, xp_z]}_{0} + [p_y z, xp_z] =$$

$$= yp_x[p_z, z] + xp_y[z, p_z] = i\hbar\,(xp_y - yp_x) = i\hbar L_z$$

•• $[L_\alpha, A_\beta] = i\hbar\,\varepsilon_{\alpha\beta\gamma} A_\gamma$; $\vec{A} = \vec{r}, \vec{p}, \vec{L}, \dots$

∴ Z $\vec{L}$ komutirajoči operatorji

$[\vec{L}, \hat{A}] = 0$ $\qquad u(\vec{p})\hat{A} = \hat{A}u(\vec{p})$

neu op.

a) $\hat{A} = c \in \mathbb{C}$ konstanta ✓

b) $\hat{A} = \vec{r}\cdot\vec{r} = r^2 = |r|^2$ ✓

$\vec{p}\cdot\vec{p} = p^2$ ✓

$\vec{L}\cdot\vec{L} = L^2$ ✓ $\Rightarrow [L_\alpha, L^2] = 0$

$\Rightarrow [L^2, H] = 0$

če $\Rightarrow H = \frac{p^2}{2m} + V(|\vec{r}|)$

abs. vrednost

:: Lestvični operatorji (ladder operator)

$$L_\pm = L_x \pm i L_y = (L_\mp)^\dagger \quad \xleftrightarrow{\text{"ustreza"}}$$

$$[L^2, L_\pm] = 0$$

$L_z \sim \hat{n}$

$a \sim L_-$

$a^\dagger \sim L_+$

$[n, a^\dagger] = a^\dagger$

$[n, a] = -a$

$L_z \sim \hat{n}$

$x \to y \to z$

$\to 1 \to 2 \to 3$

1) $[L_z, L_+] = [L_z, L_x + iL_y] =$

$= [L_z, L_x] + i[L_z, L_x] =$

$= i\hbar L_y + (-i)\hbar i L_x = \hbar L_x + i\hbar L_y = \hbar L_+$

$$\Rightarrow \quad [L_z, L_\pm] = \pm \hbar L_\pm$$

2) $L_+ L_- = (L_x + iL_y)(L_x - iL_y) = L_x^2 + L_y^2 + iL_yL_x - iL_xL_y = / + L_z^2 - L_z^2$

$= L^2 + \hbar L_z - L_z^2$

$$\Rightarrow \quad L_\pm L_\mp = L^2 \pm \hbar L_z - L_z^2$$

$$[L_+, L_-] = 2\hbar L_z \quad \xleftrightarrow[\text{ustreza}]{} \quad [a, a^\dagger] = 1$$

Lastne vrednosti L_z, L^2

$$L_z |m\rangle = m\hbar |m\rangle \ ; \quad m \in \mathbb{R}$$

To bi lahko rešili običajno:

$$-i\hbar \frac{\partial}{\partial \varphi} = L_z$$

$$-i\hbar \frac{\partial}{\partial \varphi} \psi_m(\varphi) = m\hbar \psi_m(\varphi) \Rightarrow \psi_m = C_m e^{im\varphi}$$

Raje naredimo kot pri LHO z a in $a^\dagger$:

$\cdot\ L_z L_\pm |m\rangle = (L_\pm L_z \pm \hbar L_\pm)|m\rangle = (m\pm 1)\underbrace{\hbar L_\pm |m\rangle}_{\propto |(m\pm 1)\rangle}$

L_+ in L_- torej zvišujeta ali znižujeta lastno vrednost.

$\cdot\cdot\ [L^2, L_z] = 0$

$L^2|m\rangle = \lambda|m\rangle$; $\lambda \in \mathbb{R}$ ker je L^2 hermitski

$\langle m|L^2|m\rangle = \langle m|\sum_\alpha L_\alpha^2|m\rangle = \sum_\alpha \langle L_\alpha m|L_\alpha m\rangle \geq 0$

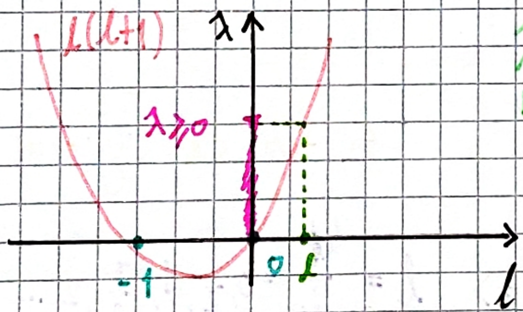
↑ Ker je to pravzaprav norma vektorja

in $\quad = \lambda\langle m|m\rangle \geq 0$

$\Rightarrow \quad \lambda \geq 0$ Operator L^2 je semipozitivno definiten

$L^2 L_\pm|m\rangle = L_\pm L^2|m\rangle = \lambda L_\pm|m\rangle$

$\lambda = l(l+1)\hbar^2$



$\lambda \leftrightarrow l \geq 0$ bijektivno

$\cancel{L^2|lm\rangle}$ $\quad L^2|lm\rangle = l(l+1)\hbar^2|lm\rangle$

$L_z|lm\rangle = m\hbar|lm\rangle$

$\langle L_+\psi_{lm}|L_+\psi_{lm}\rangle = \langle\psi_{lm}|(L_+)^\dagger L_+|\psi_{lm}\rangle = \langle\psi_{lm}|L_- L_+|\psi_{lm}\rangle \geq 0$

$\langle lm|L_\mp L_\pm|lm\rangle = \langle lm|(L^2 - L_z^2 \mp \hbar L_z)|lm\rangle =$

$= \big(\underbrace{l(l+1)\hbar^2 - m(m\pm 1)\hbar^2}_{\geq 0}\big)\langle lm|lm\rangle \geq 0$

ie $m \geq 0;\ l \geq 0:\ m(m+1) \leq l(l+1) \Rightarrow m \leq l$

$m \leq 0;\ l \geq 0: \Rightarrow -m \geq -l$

$$\Rightarrow \underline{\underline{|m| \leq l}}$$

$$L_{\pm}|lm\rangle = \hbar\sqrt{l(l+1)-m(m\pm1)}\,|l, m\pm1\rangle$$

$$|l, m\pm1\rangle = \frac{1}{\hbar\sqrt{l(l+1)-m(m\pm1)}} L_{\pm}|lm\rangle$$

Vzamemo največji m možen:

$L_-|ll\rangle = C_{l,l-1}|l, l-1\rangle$

$L_-L_-|ll\rangle = C_{l,l-2}|l, l-2\rangle$

$\vdots$

$L_-^k|ll\rangle = C_{l,l-k}|l, \underbrace{l-k}_{-l}\rangle$

Tolko lirat da je

$l-k = l \Rightarrow 2l = k \Rightarrow l = \frac{k}{2} = 0, \frac{1}{2}, 1, \frac{3}{2}, 2, \dots$

S tem smo ugotovili kakšni so l

Torej smo ugotovili:

$L_z|lm\rangle = m\hbar|lm\rangle$

$L^2|lm\rangle = l(l+1)\hbar^2|lm\rangle$

$l = 0, \frac{1}{2}, 1, \frac{3}{2}, 2, \dots$

$|m| \leq l$

Zakaj so rešitve ℓ za Vodikov atom samo celoštevilske?

Bohr:

Kvantiziral Vrtilno količino

$\ell = 0, 1, 2, \ldots$

$m \ldots$

Vodikova VF

$$\psi_{\ell m} = C e^{im\varphi} \qquad H\psi(\vec{r}) = E\psi(\vec{r})$$

Zvezna

$$L_z \psi = m\hbar\psi \qquad e^{im(\varphi+2\pi)} = e^{im\varphi}$$

$$\Downarrow$$

Če hočemo zvezno

$$\underline{\underline{m \in \mathbb{Z}}} \quad \Leftarrow \quad e^{im2\pi} = 1$$

Rešitve (funkcije)

Rešitve so sferični harmoniki. $Y_\ell^m(\theta, \varphi)$; $\langle r | \ell m \rangle = \psi(r) Y_\ell^m$

Sferični harmoniki:

$$Y_0^0 = \frac{1}{\sqrt{4\pi}}$$

$$Y_1^0 = \sqrt{\frac{3}{8\pi}} \cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{4\pi}} \sin\theta\, e^{i\varphi} \quad \longleftrightarrow \quad Y_{\ell m}$$

$\vdots$

Tudi samo realni

To so tisti "mehurčki", ki predstavljajo orbitale

$\cos\varphi$, $\cos m\varphi$ namesto $e^{i\varphi m}$

Orbitale: s, p, d, f, ... (ne j)

p_x, d_{xy}, d_{z^2}

To so trije načini

Zapis operatorja L z matriko

$$|\psi\rangle = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} c_{\ell m} |\ell m\rangle$$

Splošno:

$$L_+ = \sum_{nn'} |n\rangle\langle n| L |n'\rangle\langle n'|$$

Pri nas:

$$= \sum_{\substack{\ell\ell' \\ mm'}} |\ell' m'\rangle (L_+)_{\substack{\ell'\ell \\ m'm}} \langle \ell m|$$

$$(L_+)_{\substack{l'l\\m'm}} = \langle l'm' | L_+ | lm \rangle =$$

$\hookrightarrow$ Matrika matričnih elementov

ortogonalnost

$$= \langle l'm' | \hbar \sqrt{l(l+1) - m(m+1)} \, | l, m+1 \rangle \; \delta_{l'l} \; \delta_{m', m+1}$$

$$L_+ |\psi\rangle \to \left[\begin{array}{c|ccc|ccccc|c}
0 & & & & & & & & & \cdots \\
\hline
\underbrace{}_{l=0} & 0 & \sqrt{2} & 0 & & & & & & \\
 & 0 & 0 & \sqrt{2} & & & & & & \cdots \\
 & 0 & 0 & 0 & & & & & & \\
\hline
 & & \underbrace{}_{l=1} & & 0 & \sqrt{4} & 0 & 0 & 0 & \\
 & & & & 0 & 0 & \sqrt{6} & 0 & 0 & \cdots \\
 & & & & 0 & 0 & 0 & \sqrt{6} & 0 & \\
 & & & & 0 & 0 & 0 & 0 & \sqrt{4} & \\
 & & \vdots & & 0 & 0 & 0 & 0 & 0 & \\
 & & \vdots & & & & \underbrace{}_{l=2} & & &
\end{array}\right]
\begin{pmatrix} c_{00} \\ \hline c_{11} \\ c_{10} \\ c_{1-1} \\ \hline c_{22} \\ c_{21} \\ c_{20} \\ c_{2-1} \\ c_{2,-2} \\ \vdots \end{pmatrix}$$

Rumšal ne mum

$|\psi\rangle \neq \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

Ampak je to sicer ... večji ime

$\psi = \begin{pmatrix} \vdots \end{pmatrix}$,

da ne enačiš ket s stolpcem.

<u>D.N.</u>

$l = 1$

$$L_x = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Izraženi v lastni bazi L_z

$$L_y = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

$$L_z = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$ $\to$ Diagonalna sereda

Centralni potencial $V(r)$

1) $H = \frac{p^2}{2m} + V(r)$ invariantna na rotacije v prostoru

$[H, \vec{L}] = 0$ $\qquad \vec{r} \cdot \vec{L} = 0$

$[H, L^2] = 0$ $\qquad \vec{p} \cdot \vec{L} = 0$

$\nabla^2 = \Delta = \frac{1}{r^2} \frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} + (\theta, \varphi)$ v sferičnih

$$H = -\frac{\hbar^2}{2m}\left(\frac{1}{r^2}\frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r}\right) + \frac{\vec{L}^2}{2mr^2}$$

To nam omogoči, da zapišemo: $\Psi(r, \theta, \varphi) = \psi(r) Y_\ell^m(\theta, \varphi)$

$$H\Psi = E\Psi$$

$$\Rightarrow \Big[-\frac{\hbar^2}{2m}\left(\frac{1}{r^2}\frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r}\right) + \overbrace{\underbrace{\frac{\ell(\ell+1)\hbar^2}{2mr^2}}_{V_{cf}(r)} + V(r)}^{V_{eff}(r)}\Big]\Psi(r, \theta, \varphi) = E\Psi(r, \theta, \varphi)$$

cfg

Nastavek za $\psi(r)$:

$$\psi(r) = \frac{u(r)}{r} \quad \text{v 3D} \quad !$$

$$\psi(r) = \frac{u(r)}{\sqrt{r}} \quad \text{v 2D}$$

$$\psi(r) = u(r) \quad \text{v 1D}$$

Uporabimo nastavek:

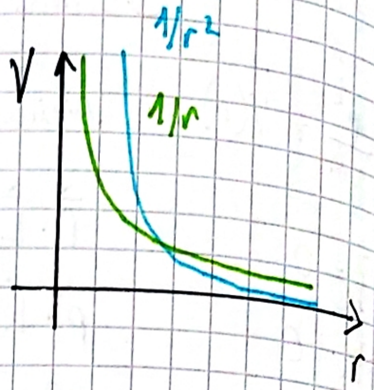
$$\frac{1}{r^2}\frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} \frac{u}{r} = \frac{1}{r^2}\frac{\partial}{\partial r} r^2 \left(\frac{u'}{r} - \frac{u}{r^2}\right) = \frac{1}{r^2}\frac{\partial}{\partial r}(ru' - u) = \frac{u''}{r} = \frac{1}{r}\frac{\partial^2 u}{\partial r^2}$$

Torej se operator poenostavi v tem primeru.

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dr^2} + V_{ef}(r)\right] u(r) = E u(r);$$

$$V_{ef} = V(r) + \frac{\ell(\ell+1)\hbar^2}{2mr^2}$$

Lastnosti rešitev



a) $r \to 0$: Omejimo na primere $\lim_{r\to 0} r^2 V(r) = 0$

$$-\frac{\hbar^2}{2m}\frac{d^2}{dr^2}u = \frac{l(l+1)\hbar^2}{2mr^2}u \quad ; \quad u = C r^{\lambda}$$

$$\lambda(\lambda-1) = l(l+1) \Rightarrow \lambda_{1,2} = l+1,\ -l$$

$$u(r) = C_l r^{l+1} + D_l \frac{1}{r^l}$$

$$l>0: \langle \Psi, \Psi \rangle = \underbrace{\int |Y_l^m|^2 d\Omega}_{=1} \int_0^\infty |\psi|^2 r^2 dr \leq C_1 < \infty$$

$\Rightarrow D_l = 0$; Sicer se ne da normirati

$l=0$: $u = C_0 r + D$

$$\int_0^\infty |\psi|^2 r^2 dr \xrightarrow[r\to 0]{} \frac{|D_0|^2}{r^2} r^2 = |D_0|^2 \ \sqrt{??}$$

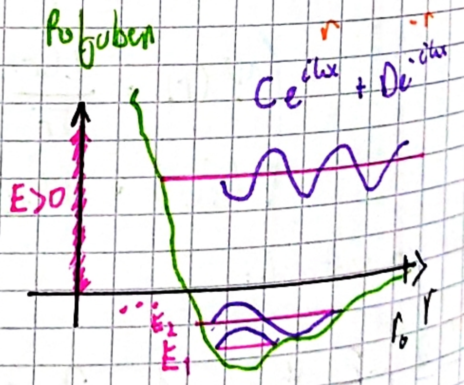
V elektrostatiki:

$$\phi = \frac{C_1}{r} = \frac{e}{4\pi\varepsilon_0 r}$$

$$\nabla^2 \phi = \lambda\,\delta(\vec r)$$

$\Rightarrow D_0 = 0$

Rešitev je: $\psi = C_l r^l = \frac{C_l r^{l+1}}{r}$



b) $r \to \infty$:

- $E > 0$:

Predpostavimo $V \to$ dovolj hitro

$V = 0$ za $r > r_0$

$$-\frac{\hbar^2}{2m}\frac{d^2u}{dr^2} = Eu$$

$$u(r) = C_+ e^{ikr} + C_- e^{-ikr}$$

$$E_k = \frac{\hbar^2 k^2}{2m}$$

.. $E<0$

$$u(r) = D_+ e^{\varkappa r} + D_- e^{-\varkappa r} \qquad E_n = -\frac{\hbar^2 \varkappa^2}{2m}$$

$\rightarrow D_+ = 0$, da jo lahko normiramo

Takeawy:

$$u(r) = r^{l+1} \, v(r) \, e^{-\varkappa r}$$

r^{l+1}: majhne razdalje

$v(r)$: kar je vmes (konst za $r \to 0$ in $r \to \infty$)

$e^{-\varkappa r}$: velike razdalje

Coulombski potencial

$$\left(-\frac{\hbar^2}{2m} \frac{d^2}{dr^2} + \frac{l(l+1)\hbar^2}{2mr^2} - \frac{e_0^2}{4\pi\varepsilon_0 r} \right) u = E u$$

$$\psi(\vec{r}) = \frac{u(r)}{r} Y_l^m(\theta, \varphi) \qquad \rho = \varkappa r, \quad |E| = \frac{\varkappa^2 \hbar^2}{2m}$$

Enačbo pomnožimo z $\frac{2m}{\varkappa^2 \hbar^2}$ in dobimo:

Wc le parameter

$$\left(-\frac{d^2}{d\rho^2} + \frac{l(l+1)}{\rho^2} + \frac{\rho_0}{\rho} - 1 \right) u = 0 \; ; \; \rho_0 = \frac{m e_0^2}{2\pi\varepsilon_0 \hbar \varkappa}$$

$$u(\rho) \leftarrow u(r(\rho))$$

Za majhne razdalje smo pokazali $u(\rho) = \rho^{l+1}$

Za velike razdalje pa $u(\rho) \to e^{-\rho}$

$$\Rightarrow u(\rho) = \rho^{l+1} v(\rho) e^{-\rho}$$

$$\vdots$$

$$u''(\rho) = \dots v'' \dots v' \dots v$$

Vstavimo in poračunamo.

Dobimo:

$$\rho v'' + 2(l+1-\rho)v' + (\rho_0 - 2(l+1))v = 0$$

To se rešuje z razvojem v vrsto (kot pri mat 4)

$$v(\rho) = \sum_{k=0}^{\infty} c_k \rho^k$$

$$v'(\rho) = \sum_{k=0}^{\infty} k c_k \rho^{k-1} \rightarrow v' = \sum_{k=0}^{\infty} (k+1) c_{k+1} \rho^k$$

$$v''(\rho) = \sum_{k=0}^{\infty} k(k+1) c_{k+1} \rho^{k-1}$$

To vstavimo:

$$\sum_{k=0}^{\infty} \rho^k \left(\left[k(k+1) + 2(l+1)(k+1) \right] c_{k+1} + \left[-2k + (\rho_0 - 2(l+1) \right] c_k \right) = 0$$

$$\Rightarrow c_{k+1} = \frac{2(k+l+1) - \rho_0}{(k+1)(k+2l+1)} c_k$$

Pogljemo $k \gg 1$:

$$c_{k+1} = \frac{2k}{k^2} c_k = \frac{2}{k} c_k$$

$$e^{2x} = \sum_{k=0}^{\infty} \frac{2^k x^k}{k!}$$

$$\frac{2^{k+1}}{(k+1)!} \frac{k!}{2^k} \rightarrow \frac{2}{k}$$

Kar smo ugotovili:

$$u(\rho) \rightarrow e^{2\rho} \quad (\rho \rightarrow \infty, k \rightarrow \infty)$$

$$\Rightarrow u(\rho) \Rightarrow \rho^{l+1} e^{2\rho} e^{-\rho} = \rho^{l+1} e^{\rho}$$

$\exists k_{max}: 2(k_{max}+l+1) - \rho_0 = 0$

$$C_{k_{maks}+1} = 0$$

S tem omejimo vrsto na polinom reda k_{maks}:

$$\sum_{k=0}^{k_{max}} C_k \rho^k = v(\rho) \; ; \; k = 0, 1, 2, \dots, k_{max} \; ; \; n = k_{maks} + l + 1 \geq 1$$

$$\rho_0 = 2n = 2, 4, 6, \dots$$

$$E = -\frac{\chi^2 \hbar^2}{2m} = -\frac{m e_0^2}{8\pi^2 \varepsilon_0^2 \hbar^2 \rho_0^2}$$

In dobimo točno to kar je Bohr dobil:

$$E_n = -\frac{m}{2\hbar^2}\left(\frac{e_0^2}{4\pi\varepsilon_0}\right)\frac{1}{n^2} = -\frac{|E_1|}{n^2}$$

$|E_1| = 1Ry = 13.6 eV$

Rydbergova konstanta

Degeneracija

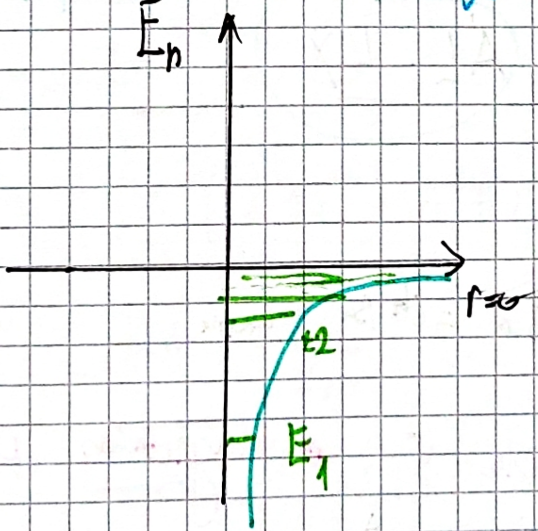
$$E_n = -\frac{1Ry}{n^2} \; ; \; n = k + l + 1$$

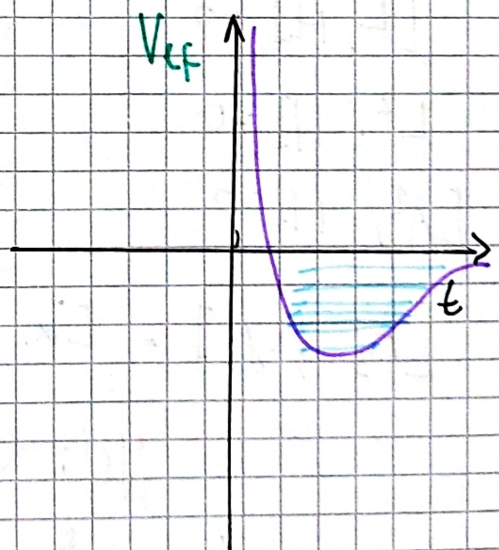
$k = 0, 1, 2, \dots$

$l = 0, 1, 2, \dots$

n	l	k
1	0	0
2	1	0
2	0	1
3	2	0
3	1	1
3	0	2

$\vdots$



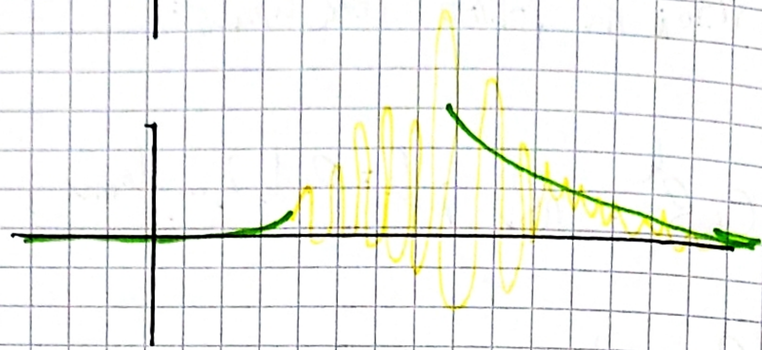


$$l = 0, 1, \dots, n-1$$

$$\underbrace{m_l = -l, -l+1, \dots, l}_{2l+1}$$

Klasična rešitev

$n \gg 1$



Primer:

$\sigma = \frac{n(n+1)}{\beta^2}$

$r = 1\,\mathrm{cm}$

$F = 100\,000\ V/$

$n \sim 10^{36}$

Kvantno: Laplace - Runge Lenz vektor

$$\hat{A} = \frac{1}{2}\left[\vec{p} \times \vec{L} + (\vec{p} \times \vec{L})^{+}\right] - \frac{m e^2}{4\pi\varepsilon_0}\frac{\vec{r}}{r}$$

$-(\vec{L} \times \vec{p})$

$[L, H] = 0$

$A L \cdot L Y$

$[L^2, H] = 0$

$[A^2, H] = 0$

$[A_\beta, A_\alpha] = [\alpha, \beta\, a = i\hbar\, \beta a$ grupa

Nabit delec v magnetnem polju ($\vec{B}$ = konst)

$$m\vec{a} = e\vec{E} + e\vec{v} \times \vec{B}$$

$$H = \frac{(\vec{p} - e\vec{A})^2}{2m} + \phi e$$

Velju

$\vec{B} = \nabla \times \vec{A}$

$\vec{E} = -\nabla\phi - \frac{\partial}{\partial t}\vec{A}$

$$i\hbar\frac{\partial\psi}{\partial t} = \frac{1}{2m}(-i\hbar\nabla - e\vec{A})^2\psi + e\phi\psi$$

$$(\nabla\vec{A})\psi + (\vec{A}\cdot\nabla)\psi = \nabla A\psi \quad \hat{A}\nabla\psi =$$

$$= 2\hat{A}\nabla\psi + \psi\nabla A.$$

$$=$$

$$i\frac{\hbar\partial\psi}{\partial t} = \frac{-\hbar^2\nabla^2}{2m}\psi + \underbrace{i\frac{\hbar e}{m}\vec{A}\nabla\psi}_{\text{Zeeman}} + \left(\frac{i\hbar e}{m}(\nabla\cdot\vec{A}) + \frac{e^2A^2}{2m}e\phi\right)\psi$$

Coulombova potenčna vrsta

Zamenjiva skalopriter

$\nabla = 0$, $\vec{A} = \frac{1}{2}(\vec{r}\times\vec{B})$ ∇f

$$\frac{ie\hbar}{m}\vec{A}\cdot\nabla\psi = -\frac{i\hbar}{2m}(\vec{r}\times\vec{B}) =$$

$$= \frac{I\hbar}{2m} \dots = i\frac{\hbar e}{2m}\left((\vec{r}\times\vec{B})\nabla\psi\right) =$$

$$= \frac{g}{2m}(\vec{r}\times\vec{p}) - B\psi = -\frac{e}{2m}\vec{B}\cdot\vec{L}\,m$$

$H_{zemann} = -\vec{\mu}\cdot B \rightarrow \vec{\mu} = \frac{e}{2m}\vec{L}$

2.) $H_{zeeman} = -\mu\vec{B}$; $\vec{\mu} = \frac{e}{2m}\vec{L}$

$\dots \frac{e^2}{2m}\vec{A}\vec{A}\frac{d\vec{B}^2}{8g}(x^2+y^2)_{q}$; $q \Rightarrow A, \frac{B}{2}(-y\,x, 0)$

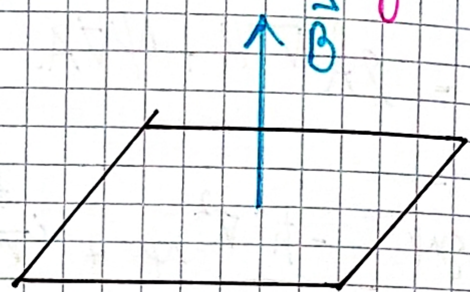
ana $(L_a = \frac{e}{2m}\vec{L})$

~~$B \sim 10\,T$~~

~~$x_2 \sim$ [illegible] nm~~

~~$\frac{H_2}{H} \sim$ [illegible] $\sim 10^{-6}$~~

Homogeno magnetno polje: Landaovi nivoji



$\vec{B} = B e_z$

$\vec{A} = -\frac{1}{2}(\vec{r} \times \vec{B}) = \frac{B}{2}(-y, x, 0)$

oz.

$\vec{A} = B(-y, 0, 0)$ } Landauova umeritev

ali

$\vec{A} = B(0, x, 0)$

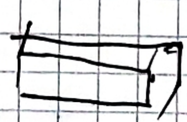
$\vec{B} = \nabla \times \vec{A} = (0, 0, B)$ ne $\exists \vec{A}$, ki ima simetrije problema

$\frac{1}{2m}\Big[(\overbrace{-i\hbar \frac{\partial}{\partial x} + eBy}^{\text{Pirates}})^2 - \frac{\hbar^2 \partial^2}{\partial y^2} - \hbar^2 \frac{\partial^2}{\partial z^2}\Big]\Psi + e\phi\Psi = E\Psi$

$\Psi(\vec{r}) = e^{i\left(\frac{p_x}{\hbar} x + \frac{p_z z^2}{\hbar z^2}\right)} \chi(y)$; ϕ le $\phi(z)$

$= 0$

$\hat{A} = (1, 2, 5, \text{pog.})$



$\hat{A}$; $f(\hat{A})\Psi_a = f(a)\Psi_a$

$\hat{A}\Psi$, [illegible] U_{out}

Ostane
$f(-i\hbar \frac{\partial}{\partial x}) e^{i\frac{p_x x}{\hbar}} = f(p_x) \exp \frac{\chi}{\hbar}(\hat{p}_x + eB_y)^2$

$\frac{1}{2m}[p_x + eB_0 y] - \left[\frac{2uh^2}{\text{Tmonth} \quad \text{off}}\right] \quad v(y) - E\lambda.$

$V(y) = e\phi(y) + \frac{p_z^2}{2m} \Rightarrow V_y = e\phi_y + \frac{p_z^2}{2M}$ → Lastna funkcija LHO.

$V=0 \quad \omega = \frac{eB}{m}; \quad \frac{\varepsilon}{5} = \sqrt{\frac{\hbar}{eB}}; \quad p_x = \hbar k$

$y_k = -\beta^2 k = -\frac{\hbar}{eB} k$

$\Rightarrow \left[-\frac{\hbar^2}{2m}\frac{d^2}{dy^2} + \frac{1}{2}m\omega^2(y - y_k)^2\right]\chi_m = E_\lambda \chi_n;$

$E_n = \frac{1}{2}(\omega + \frac{1}{2})$

$\Rightarrow E_{nk}(x, y)(X_i) = \frac{1}{\sqrt{2\pi}} e^{ikx} \frac{1}{\sqrt{\text{index}}},$

$\rho_m(y - y_k)$

$\Psi(x,t) = \int \tilde{\Psi} \frac{e}{\sqrt{2\pi}} dk \quad \frac{e}{A}$

Tipično bi:

$\psi(x,t) = \int_{-\infty}^{\infty} \tilde{\Psi}$

Landauov nivo

$\boxed{E_n = \hbar\omega(n + \frac{1}{2}) \neq E_m}$

$\Psi(-\infty, \infty) = 0:$

$\psi(x, y) = \psi(x, 0) = \int_{-\infty}^{\infty} \tilde{\Psi}(k) \frac{e^{ikx}}{\sqrt{2\pi}} \quad E_b$

$\psi(x,t) = \int \tilde{\Psi}(k) \frac{e^{ikx - \varphi i}}{\sqrt{2\pi}} dk \quad E(b)$

$E = \hbar(k + \frac{1}{2}) \neq$ konstanta

$\psi(x,t) = \int \tilde{\psi}(l) \frac{e^{l}}{\sqrt{(t\hbar A)}} dk$

$H = \hbar\omega(a^\dagger a + \frac{1}{2})$; $H = \frac{(\vec{p} - e\vec{A})^2}{2m}$ kjer je $\vec{B} = \vec{B}_0$

↑ To je pa splošno

$E_n = \hbar\omega(n + 1/2)$

$\vec{A}_1 = B_0(-y, 0, 0)$

$\vec{A}_2 = B_0(0, x, 0)$

$\vec{A}_3 = \frac{1}{2}B_0(-y, x, 0) = \frac{1}{2}\vec{r} \times \vec{B}_0$

Lokalne umeritvene transformacije

$\vec{B} = \nabla \times \vec{A}$

$\vec{E} = -\nabla\phi$

$\vec{A}' = \vec{A} + \nabla\Lambda$; $\vec{B}' = \vec{B}$

$\phi' = \phi - \frac{\partial}{\partial t}\Lambda$; $\vec{E}' = \vec{E}$

Klasično:

$\vec{F} = e\vec{E} + e\vec{v} \times \vec{B}$

Rešitev je neodvisna od $\vec{A}$.

Vemo že, da valovnim funkcijam lahko naredimo:

$\psi_\ell \to e^{i\delta_\ell}\psi_\ell$; $\delta_\ell \in \mathbb{R}$

Še bo ravno pokrajšal $\to \langle \psi_i | \hat{O} | \psi_j \rangle = \hat{O}_{ij}$

Kako se bo spremenil ψ pri taki zamenjavi $\vec{A}'$? ($\psi \to \psi'$)

$\Psi'(\vec{r}, t) = e^{i\delta(\vec{r}, t)}\Psi(\vec{r}, t)$; $\delta \in \mathbb{R}$

↑ Lokalna transf.
(na vsakem $\vec{r}$ je lahko δ drugačen)

To damo v SE:

$$i\hbar\frac{\partial}{\partial t}\Psi' = \frac{1}{2m}\left(-i\hbar\nabla - \frac{e\vec{A}'}{\hbar}\right)^2\Psi' + e\phi'\Psi'$$

$\hbar^2$

f_{A_x}

(Priprava / stranski račun)

$$\left(i\frac{\partial}{\partial x} + f\right)e^{i\sigma}\psi = -\left(\frac{\partial \sigma}{\partial x}\right)e^{i\sigma}\psi + e^{i\sigma}\, i\left(\frac{\partial \psi}{\partial x}\right) + f e^{i\sigma}\psi =$$

$$= e^{i\sigma}\left(i\frac{\partial}{\partial x} + \left(f - \frac{\partial \sigma}{\partial x}\right)\right)\psi$$

$$\left(i\frac{\partial}{\partial x} + f\right)^2 e^{i\sigma}\psi = \left(i\frac{\partial}{\partial t} + f\right)e^{i\sigma}\left(i\frac{\partial}{\partial x} + \left(f - \frac{\partial \sigma}{\partial x}\right)\right)\psi =$$

$$= e^{i\sigma}\left(i\frac{\partial}{\partial x} + \left(f - \frac{\partial \sigma}{\partial x}\right)\right)^2\psi \qquad e^{i\sigma} \neq 0$$

$$i\hbar\frac{\partial}{\partial t}\Psi = \frac{1}{2m}\sum_{\substack{\alpha \\ \uparrow \\ x,y,z}}\left(-i\hbar\frac{\partial}{\partial x_\alpha} - eA'_\alpha + \hbar\frac{\partial \sigma}{\partial x_\alpha}\right)^2\Psi + \left(e\phi' + \hbar\frac{\partial \sigma}{\partial t}\right)\Psi$$

$$i\hbar\frac{\partial}{\partial t} = \left[\frac{(\vec{p} - e\vec{A})^2}{2m} + e\phi\right] i \qquad \Lambda = \frac{\hbar}{e}\sigma$$

Torej se ψ' po transformaciji $\vec{A} \to \vec{A}'$ izrazi kot:

$$\Psi'(\vec{r},t) = e^{i\frac{e}{\hbar}\Lambda(\vec{r},t)}\,\Psi(\vec{r},t)$$

$$\rho' = |\Psi'|^2 = |\Psi|^2 = \rho(\vec{r},t)$$

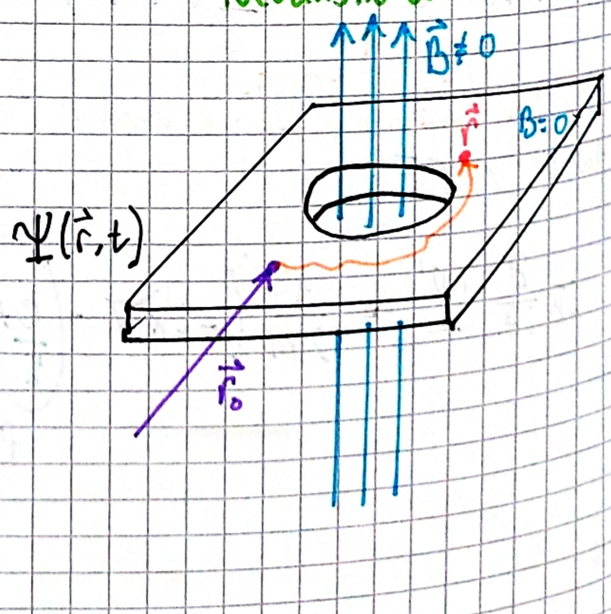
Neodvisno od umeritve

Aharonov - Bohmov pojav

$$\vec{B} = \nabla \times \vec{A} = 0$$

$$\vec{A} = \nabla \Lambda$$

$$\Lambda(\vec{r},t) = \Lambda(\vec{r}_0,t) - \int_{\vec{r}_0}^{\vec{r}} \vec{A}(\vec{r}\,')\cdot d\vec{r}\,'$$



$$\Psi_A: \quad i\hbar \frac{\partial \Psi_A}{\partial t} = \frac{(\vec{p} - e\vec{A})^2}{2m}\Psi_A + V\Psi_A \quad ; \quad \vec{B} = 0$$

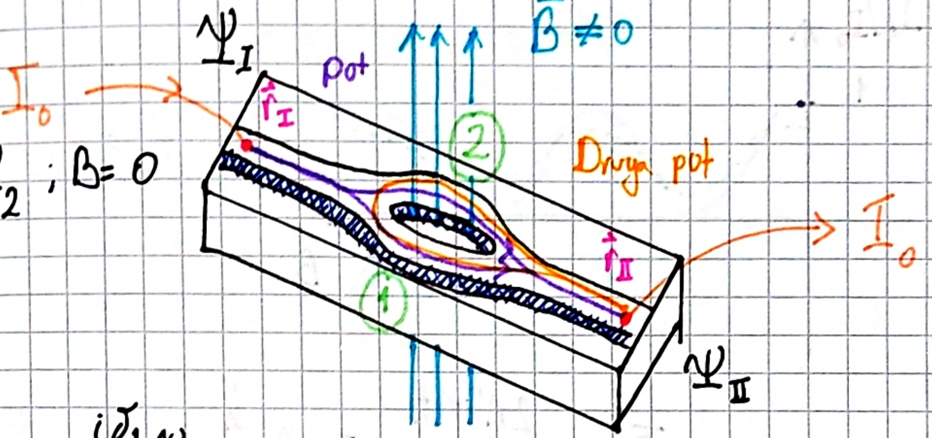
$$\Psi_0: \quad i\hbar \frac{\partial \Psi_0}{\partial t} = \frac{\vec{p}^2}{2m}\Psi_0 + V\Psi_0$$

$$\vec{A}' = \vec{A} + \nabla(-\Lambda) = 0$$

$$\Psi_A(\vec{r},t) = e^{-i\frac{e}{\hbar}\Lambda(\vec{r},t)}\Psi_0 = $$

$$= e^{i\frac{e}{\hbar}\int_{\vec{r}_0}^{\vec{r}} \vec{A}(\vec{r}')\cdot d\vec{r}'}\Psi_0(\vec{r},t)$$

$$\Psi_{II} = \Psi_1 + \Psi_2 \; ; B = 0$$



$$\Psi_{II_A} = e^{i\delta_1}\Psi_1 + e^{i\delta_2}\Psi_2$$

$$\delta_{1,2} = \frac{e}{\hbar}\int_{\vec{r}_I}^{\vec{r}_{II}} \vec{A}(\vec{r})\cdot d\vec{r}$$

pot 1
pot 2

$$\Psi_1 \approx \Psi_2$$

$$\Psi_{II_A} = e^{i\delta_1}\Psi_1 + e^{i\delta_2}\Psi_2 = e^{i\delta_2}\left(e^{i(\delta_1-\delta_2)}\Psi_1 + \Psi_2\right) =$$

$$= e^{i\delta_2}\left(1 + e^{i(\delta_1-\delta_2)}\right)\Psi_1$$

$$\delta_1 - \delta_2 = \frac{e}{\hbar}\oint_{pot} \vec{A}(\vec{r})\cdot d\vec{r} = \frac{e}{\hbar}\left(\int_{I\,pot_1}^{II} \vec{A}(\vec{r})\cdot d\vec{r} - \int_{I\,pot_2}^{II} \vec{A}(\vec{r})\cdot d\vec{r}\right) =$$

Stokes

$$= \frac{e}{\hbar}\iint_S (\nabla\times\vec{A})\,d\vec{S} = \frac{e}{\hbar}\iint_S \vec{B}\cdot d\vec{S} = \frac{e}{\hbar}\Phi_B$$

Magnetni pretok (lahko merimo)

$$\frac{I_A}{I_0}^{\nearrow B} = \frac{|\Psi_{IIA}|^2}{|\Psi_{II0}|^2} = |1 + e^{i\frac{e}{\hbar}\Phi_B}|^2 \cdot \boxed{\frac{1}{4}} =$$

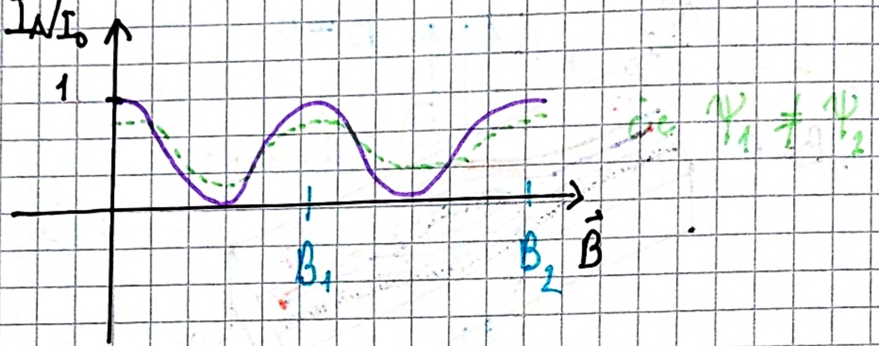
Od tega ko ni polja $(1+1)^2$

$$= \cos^2\left(\frac{e}{2\hbar}\Phi_B\right)$$

Torej je tok:

$$I_A = I_0 \cos^2\left(\frac{e}{2\hbar}\Phi_B\right)$$

Aharonov-Bohmov pojav



Za maksimum:

$$\frac{e}{2\hbar}\Phi_B = \pi n \quad ; \quad \Phi_B = \frac{2n\pi\hbar}{e} = n\frac{h}{e}$$

Iz tega lahko naredijo sonde za merjenje magnetnega polja zelo natančno (SQUID)

Spin

Pri poglavju o vrtilni količini smo videli:

$\vec{L}$: $l = 0, \cancel{\frac{1}{2}}, 1, \cancel{\frac{3}{2}}, \dots$ $\quad [L_\alpha, L_\beta] = i\hbar\varepsilon_{\alpha\beta\gamma}L_\gamma$

Nezvezne $\quad Y_l^{m_l} \quad |l m_l\rangle$

Pri spinih bomo pa polovične pustili in bomo videli, kam pridemo.

$$[S_\alpha, S_\beta] = i\hbar\varepsilon_{\alpha\beta\gamma}S_\gamma$$

$$S = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \dots \qquad S_\pm = S_x \pm iS_y$$

$$|s m_s\rangle = |s m\rangle = |\tfrac{1}{2} m\rangle = |m\rangle = \begin{matrix} |\uparrow\rangle \ \tfrac{1}{2} \\ |\downarrow\rangle \ -\tfrac{1}{2} \end{matrix}$$ Oznake

$$\begin{pmatrix}1\\0\end{pmatrix};\begin{pmatrix}0\\1\end{pmatrix};\begin{pmatrix}c_1\\c_2\end{pmatrix}$$ Spinor

Analogno kot prej:

$$S_\pm |sm\rangle = \hbar\sqrt{(s+1)s - m(m\pm 1)}$$

$$S_z |sm\rangle = m\hbar |s m\rangle$$

$$\vec{S} = (S_x, S_y, S_z)$$

$$\vec{S}^2 |sm\rangle = s(s+1)\hbar^2 |sm\rangle$$

Od zdej gledamo $s = 1/2$

$$S_+|\downarrow\rangle = \hbar|\uparrow\rangle$$

$$S_-|\uparrow\rangle = \hbar|\downarrow\rangle$$

(*) kot prej

$$L_+ Y_l^l = 0$$

$$L_- Y_l^{-l} = 0$$

Poglejmo matrične elemente:

$$\langle\uparrow|S_+|\uparrow\rangle = 0 = \langle\downarrow|S_+|\downarrow\rangle$$

$$\langle\downarrow|\underbrace{S_+|\uparrow\rangle}_{(*)=0} = 0 \quad \langle\uparrow|S_+|\downarrow\rangle\ \hbar$$

$$S_+ = \hbar\begin{pmatrix}0&1\\0&0\end{pmatrix} \quad (S_+)^\dagger = S_-$$

$$l=1: \ L_+ = \hbar\begin{pmatrix}0&\sqrt{2}&0\\0&0&\sqrt{2}\\0&0&0\end{pmatrix}$$

$$S_- = \hbar\begin{pmatrix}0&0\\1&0\end{pmatrix}$$

$$S_x = \tfrac{1}{2}(S_+ + S_-) = \frac{\hbar}{2}\begin{pmatrix}0&1\\1&0\end{pmatrix} = S_x^\dagger$$

$$S_y = \frac{1}{2i}(S_+ - S_-) = \frac{\hbar}{2}\begin{pmatrix}0&-i\\i&0\end{pmatrix} = S_y^\dagger$$

$$S_x = \frac{\hbar}{2}\begin{pmatrix}0&1\\1&0\end{pmatrix}$$

$$S_y = \frac{\hbar}{2}\begin{pmatrix}0&-i\\i&0\end{pmatrix}$$!

$$S_z = \frac{\hbar}{2}\begin{pmatrix}1&0\\0&-1\end{pmatrix}$$

$$\begin{array}{c|cc} & |\uparrow\rangle & |\downarrow\rangle \\ \hline |\uparrow\rangle & \alpha & \beta \\ |\downarrow\rangle & \gamma & \delta \end{array}$$

$$S_x^2 = \frac{\hbar^2}{4}\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}0&1\\1&0\end{pmatrix} = \frac{\hbar^2}{4}I \; ; \quad S_\alpha^2 = \frac{\hbar^2}{4}I = \frac{\hbar^2}{4}\begin{pmatrix}1&0\\0&1\end{pmatrix}$$

$$\Rightarrow \quad [S_\alpha, S_\beta^2] = 0$$

$$\vec{S}^2 = \sum_\beta S_\beta^2$$

$$[S_\alpha, \vec{S}^2] = 0$$

$$S_- \leftrightarrow \begin{pmatrix} 0 & 1 & & & & & & & & \\ 0 & 0 & & & & & & & & \\ & & 0 & \sqrt{3} & 0 & 0 & & & & \\ & & 0 & 0 & 2 & 0 & & & & \\ & & 0 & 0 & 0 & \sqrt{3} & & & & \\ & & 0 & 0 & 0 & 0 & & & & \\ & & & & & & 0 & \sqrt{5} & 0 & 0\;0\;0 \\ & & & & & & 0 & 0 & 2\sqrt{2} & \cdots \\ & & & & & & 0 & 0 & 0 & \\ & & & & & & & \vdots & & \ddots \end{pmatrix}$$

$$\frac{1}{2} \qquad \frac{3}{2} \qquad \frac{5}{2} \qquad \dots$$

Paulijeve matrike

$$\vec{S} = \frac{\hbar}{2}\vec{\sigma} \; ; \quad \vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$$

$$\sigma_x = \begin{pmatrix}0&1\\1&0\end{pmatrix} \qquad H = E_0 I + \sum_\alpha E_\alpha \sigma_\alpha = H_\alpha^\dagger \; ; \; E_0, E_\alpha \in \mathbb{R}$$

$$\sigma_y = \begin{pmatrix}0&-i\\i&0\end{pmatrix}$$

$$\sigma_z = \begin{pmatrix}1&0\\0&-1\end{pmatrix}$$

$$I = \begin{pmatrix}1&0\\0&1\end{pmatrix}$$

Lastnosti:

- $\det \sigma_\alpha = -1$
- $\operatorname{tr} \sigma_\alpha = 0$
- $\sigma_x^2 = \sigma_y^2 = \sigma_z^2 = I = 1$

- $\sigma_x \sigma_y \sigma_z = i I$
- $\sigma_x \sigma_y = i\sigma_z = -\sigma_y \sigma_x \qquad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = i\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
- $\sigma_\alpha \sigma_\beta = \delta_{\alpha\beta} I + i\,\varepsilon_{\alpha\beta\gamma}\sigma_\gamma$
- $[\sigma_\alpha, \sigma_\beta] = 2i\,\varepsilon_{\alpha\beta\gamma}\sigma_\gamma$
- $\{\sigma_\alpha, \sigma_\beta\} = \sigma_\alpha \sigma_\beta + \sigma_\beta \sigma_\alpha = 2\delta_{\alpha\beta} I$
- $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$

$\vec{a} = (a_x, a_y, a_z)$

$\vec{a}\cdot\vec{\sigma} = \sum_\alpha a_\alpha \sigma_\alpha$

- $(\vec{a}\cdot\vec{\sigma})(\vec{b}\cdot\vec{\sigma}) = \sum_{\alpha\beta} a_\alpha \sigma_\alpha b_\beta \sigma_\beta = \vec{a}\cdot\vec{b}\, I + i(\vec{a}\times\vec{b})\cdot\vec{\sigma}$

$\vec{a} = \vec{b} = \vec{n} \; ; \; |\vec{n}| = 1$

$(\vec{n}\cdot\vec{\sigma})(\vec{n}\cdot\vec{\sigma}) = I$

Seštevanje vrtilnih količin (cont.)

$$|\Psi\rangle = |\tfrac{1}{2} m_e\rangle |\tfrac{1}{2} m_p\rangle$$

Spin elektrona: $\vec{S}_e = \vec{S}_1$

spin protona: $\vec{S}_p = \vec{S}_2$

$$\vec{S}_{\frac{1}{2}} = \frac{\hbar}{2}\vec{\sigma}_{\frac{1}{2}} = \frac{\hbar}{2}(\sigma_x, \sigma_y, \sigma_z)_{\frac{1}{2}}$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}_{\frac{1}{2}}$$

Vpeljemo še tenzorski produkt $\otimes$.

Baza za 2 dela

- $$\underline{|l_1, m_1\rangle \otimes |l_2, m_2\rangle} = |\Psi\rangle$$

$$\vec{S}_1 |\Psi\rangle = |\tilde{\Psi}\rangle$$

Hočemo da deluje le na prvi del

$$\vec{S}_1 \rightarrow \vec{S}_1 \otimes I_2$$

deluje na prvi vektor; nič ne naredi na drugem vektorju

Poseben primer:

$\phi(x,y)$; $A = \frac{\partial}{\partial x}$ $B = \frac{\partial}{\partial y}$

$C = A + B$

$C\phi = \frac{\partial \phi}{\partial x} + \frac{\partial \phi}{\partial y}$

$\phi(x,y) = \phi_1(x)\phi_2(y)$

$C\phi = \phi_2 \frac{\partial \phi_1}{\partial x} + \frac{\partial \phi_2}{\partial y}\phi_1$

$C = A \otimes I_y + I_x \otimes B$

$\phi = f_1(x) \otimes f_2(y)$

V tem duhu je operator celotne vrtilne količine:

$$\vec{S} = \vec{S}_1 \otimes I_2 + I_1 \otimes \vec{S}_2$$

$l_1 = \frac{1}{2} = l_2$; baza

$m \rightarrow \uparrow$ ali $\downarrow$

$|\uparrow\rangle \otimes |\uparrow\rangle$

$|\uparrow\rangle \otimes |\downarrow\rangle$

$|\downarrow\rangle \otimes |\uparrow\rangle$

$|\downarrow\rangle \otimes |\downarrow\rangle$

Poglejmo bolj konkreten primer:

$$S_z = S_{1z} \otimes I_2 + I_1 \otimes S_{2z}$$

$$S_z |m_1\rangle \otimes |m_2\rangle = S_{1z} \otimes I_2 \, |m_1\rangle \otimes |m_2\rangle + I_1 \otimes S_{2z} \, |m_1\rangle \otimes |m_2\rangle =$$

deluje na   deluje na

$$= m_1 |m_1\rangle \otimes |m_2\rangle + m_2 |m_1\rangle \otimes |m_2\rangle = \underbrace{(m_1 + m_2)}_{} |m_1\rangle \otimes |m_2\rangle$$

Smiselno!

$m = m_1 + m_2$

$m = -1, 0, 1$

Poglejmo si komutator:

$$[S_\alpha, S_\beta] = [S_{1\alpha} \otimes I_2 + I_1 \otimes S_{2\alpha}, S_{1\beta} \otimes I_2 + I_1 \otimes S_{2\beta}] =$$

$$= [S_{1\alpha}, S_{1\beta}] \otimes I_2 + I_1 \otimes [S_{2\alpha}, S_{2\beta}] + 0 + 0 =$$

$[I_1, S_1]$ čisto druga prostora

$$= i\hbar \varepsilon_{\alpha\beta\gamma} S_{1\gamma} \otimes I_2 + I_1 \otimes i\hbar \varepsilon_{\alpha\beta\gamma} S_{2\gamma}$$

$$\Rightarrow [S_\alpha, S_\beta] = i\hbar \varepsilon_{\alpha\beta\gamma} S_\gamma$$

Spet standardna komutacijska zveza za VK. Kot pričakovano.

To pomeni, da lahko kot pri posameznih VK vpeljemo:

$$S_\pm = S_x \pm i S_y = S_{1\pm} I_2 + I_1 \otimes S_{2\pm}$$

$\therefore \quad \vec{S}^2 |\ell m\rangle$

stanje dveh delcev

$$\vec{S}^2 |\ell m\rangle = \hbar^2 \ell(\ell+1) |\ell m\rangle$$

$$S_z |\ell m\rangle = \hbar m |\ell m\rangle$$

$$m = m_1 + m_2 = \{-1, 0, 1\} \rightarrow \ell = 0 \text{ ali } 1$$

Zanimajo nas lastna stanja $|\ell m\rangle$...

$$|\ell m\rangle = \sum_{m_1 m_2} C_{m_1 m_2} \underbrace{|m_1\rangle \otimes |m_2\rangle}_{\text{navada}}$$

$$m = m_1 + m_2 = \{-1, 0, 1\} \qquad \underbrace{|m_1\rangle |m_2\rangle}$$

$$\ell = \{0, 1\} \qquad |m_1 m_2\rangle \Rightarrow |\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle$$

Nikoli ju oboh zamenjati

a) $\ell = 1$

$m=1$

$$S^2 |1 m\rangle = 1(1+1)\hbar^2 |1 m\rangle = 2\hbar^2 |1 m\rangle$$

$$S_z |1 1\rangle = \hbar |1 1\rangle = \hbar |\uparrow\uparrow\rangle \text{ da bo } \ell = 1$$

Torej: $|\ell m\rangle = |1 1\rangle = |\uparrow\rangle \otimes |\uparrow\rangle = 1 \cdot |\uparrow\uparrow\rangle$

Spin-flip

$S_{1+} |\downarrow\rangle = \hbar |\uparrow\rangle$

$S_{1+} |\uparrow\rangle = 0$

$S_{1-} |\downarrow\rangle = 0$

$S_{1-} |\uparrow\rangle_1 = \hbar |\downarrow\rangle_1$

$m=0$

$$S_- |\underset{m}{1 1}\rangle = \sqrt{\ell(\ell+1) - m(m-1)}\, \hbar\, |1, \underset{m-1}{0}\rangle$$

$$S_- |1 1\rangle = \sqrt{2}\hbar |1 0\rangle$$

Naredimo še isto operacijo na drug način

$$(S_{1-} \otimes I_2 + I_1 \otimes S_{2-}) |\uparrow\rangle \otimes |\uparrow\rangle = S_{1-} \otimes I_2 |\uparrow\uparrow\rangle + I_1 \otimes S_{2-} |\uparrow\uparrow\rangle =$$

$$= \hbar(|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle)$$

To dvoje enačimo:

$$\hbar\left(|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle\right) = \sqrt{2}\,\hbar\,|1\,0\rangle$$

$$|1\,0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right)$$

$m = -1$

$$|1,-1\rangle = |\ell\, m\rangle = |\downarrow\downarrow\rangle$$

$$S_-|1\,0\rangle = \dots \propto |1,-1\rangle$$

$\ell = 1$:

$$\left.\begin{aligned} m=1 \quad & |1\,1\rangle = |\uparrow\uparrow\rangle \\ m=0 \quad & |1\,0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right) \\ m=-1 \quad & |1,-1\rangle = |\downarrow\downarrow\rangle \end{aligned}\right\} \text{Tripletna stanja}$$

$\ell = 0$: $m = 0$ $\quad |0\,0\rangle = c_1|\uparrow\downarrow\rangle + c_2|\downarrow\uparrow\rangle$

Mora veljati $\langle 1\,0|0\,0\rangle = 0$

$$\Rightarrow \quad |0\,0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right) \Big\} \text{ Singletno stanje}$$

primer: Heisenbergova sklopitev

Klasično: $\uparrow \nearrow \quad H \propto \vec{p}_{m_1} \cdot \vec{p}_{m_2}$

Kvantno:

$$H = J_0\, \vec{S}_1 \cdot \vec{S}_2$$

Smo se navadili na tenzorske:

$$\vec{S} = \vec{S}_1 + \vec{S}_2$$

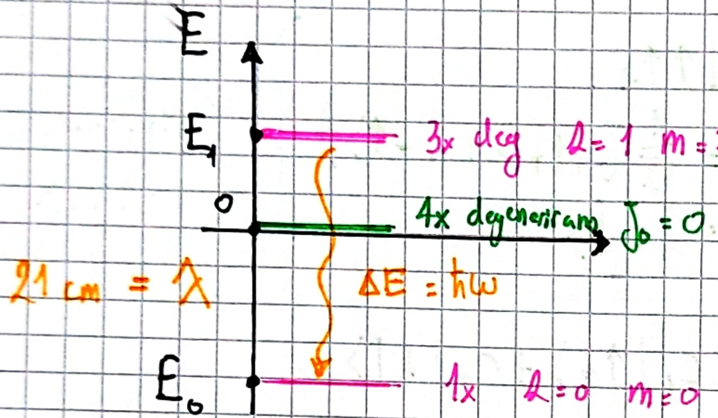
Iščemo znova $H|\Psi\rangle E|\Psi\rangle$

$$\vec{S}\cdot\vec{S} = (\vec{S}_1 + \vec{S}_2)\cdot(\vec{S}_1 + \vec{S}_2) = \underbrace{S_1^2}_{\frac{3}{4}\hbar^2 = \frac{1}{2}\left(\frac{1}{2}+1\right)\hbar^2} + \overbrace{S_2^2}^{3/4\,\hbar^2} + 2\,\vec{S}_1\cdot\vec{S}_2$$

$$\Rightarrow \quad H = \frac{J_0}{2}\left(S^2 - 2\,\frac{3}{4}\hbar^2\right)$$

$$H|lm\rangle = \underbrace{\frac{J_0\hbar^2}{2}\left(l(l+1)-\frac{3}{2}\right)}_{E_l}|lm\rangle$$

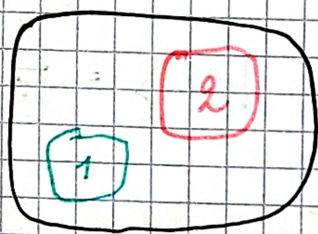
$$E_l = J_0\hbar^2 \begin{cases} \frac{1}{4} \ ; \ l=1 \\ -\frac{3}{4} \ ; \ l=0 \end{cases}$$



Clebsch-Gordanovi koeficienti

$\vec{J}_1 \ , \ J_{1z}$

$\vec{J}_2 \ , \ J_{2z}$



Kvantni sistem dveh stvari/delcev

$$[J_{n\alpha}, J_{\tilde{n}\beta}] = i\hbar\,\varepsilon_{\alpha\beta\gamma}\,\delta_{n\tilde{n}}\,J_{n\gamma}$$

$$\vec{J} = \vec{J}_1 \otimes I_2 + I_1 \otimes \vec{J}_2 = \vec{J}_1 + \vec{J}_2$$

$$\vec{J}_1 = \vec{L}_1, \vec{S}_1, \vec{\tilde{J}}_1, \ldots$$

baza: $|j_1 m_1\rangle \otimes |j_2 m_2\rangle = |j_1 m_1 j_2 m_2\rangle$

št. baznih vekt. $(2j_1+1)(2j_2+1)$

$\vec{J}_1 = \vec{L}$ $\vec{J}_2 = \vec{S}$ $2(2l+1)$ → sicer to je za en delec

$|jm\rangle = |j_1 j_2 jm\rangle$

Recimo $l=1,\ m=0$ stanje vodila: $|\frac{1}{2}\ \frac{1}{2}\ 1\ 0\rangle$

$$|j_1 j_2 jm\rangle = \sum_{m_1=-j_1}^{j_1} \sum_{m_2=-j_2}^{j_2} |j_1 m_1\rangle |j_2 m_2\rangle \underbrace{\langle j_1 m_1, j_2 m_2 | jm\rangle}_{C^{jm}_{j_1m_1 j_2m_2} \in \mathbb{C}}$$

Clebsch-gordanov koeficient!

$c \neq 0$: $m = m_1 + m_2$

$|j_1 - j_2| \leq j \leq j_1 + j_2$

a) $m_l = \frac{1}{2}$, $s = -1/2$ $\}$ $+ = 1/2$

b) $m_l = 0$, $s = 1/2$ $\}$ $+ = 1/2$

$Y_l^{m_l}$ stopnji

Primer uporabe tabel:

$l = 1$

$s = 1/2$

$|j, m\rangle = ?$

$$\Psi = \sqrt{\frac{2}{3}} \begin{pmatrix} 0 \\ Y_{1=l}^{(1)} \end{pmatrix}^{a)} - \sqrt{\frac{1}{3}} \begin{pmatrix} Y_{1=l}^{(0)\,b)} \\ 0 \end{pmatrix} \quad ; j = \frac{1}{2} \quad m = \frac{1}{2}$$

$|j_1 j_2 jm\rangle = |1\ \frac{1}{2}\ \frac{1}{2}\ \frac{1}{2}\rangle$ ($j_1 = l$, $j_2 = s$)

$|\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

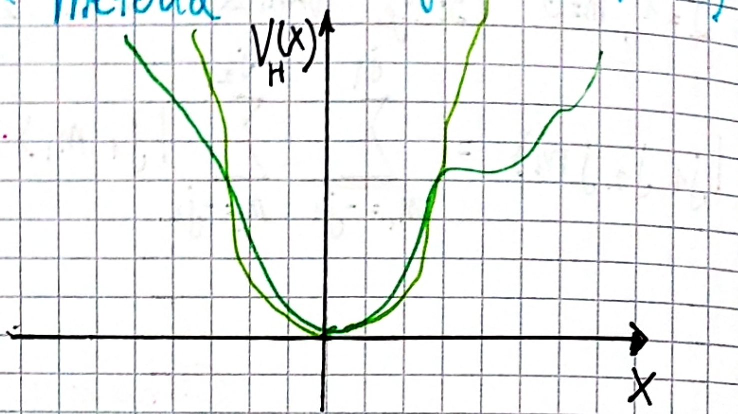
$|\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$$\Rightarrow \underset{l\ s\ j\ m}{|1\ \tfrac{1}{2}\ \tfrac{1}{2}\ \tfrac{1}{2}\rangle} = \sqrt{\frac{2}{3}}\, \underset{l\ m_l}{|1\,1\rangle} \underset{s\ m_s}{|\tfrac{1}{2}, -\tfrac{1}{2}\rangle} - \sqrt{\frac{1}{3}}\, |1\,0\rangle |\tfrac{1}{2}, \tfrac{1}{2}\rangle$$

Teorija motenj (perturbacij)

1. Rayleigh-Schrödingerjeva metoda (za nedegeneriran spekter)

$$H = \underbrace{\frac{p^2}{2m} + \frac{1}{2}kx^2}_{H_0} + \alpha x^3 + \beta x^4 + \dots$$



popravek

$$H = H_0 + H_1$$

ničti približek

$$H_0 |n^0\rangle = E_n^{(0)} |n^0\rangle \quad ; \quad \langle m^0 | n^0 \rangle = \delta_{m,n}$$

nedegenerirana baza

Parametrizirajmo motnjo:

λ brezdimenzijski parameter

$$H_1 = \lambda V \quad ; \quad \hat{V} = V$$

(lahko tudi kaj zelo kompliciranega)

$$H|n\rangle = E_n |n\rangle$$

$\lambda \to 1$

$H_1 \to \hat{V}$

Hočemo:

$$|n\rangle = |n^0\rangle + \lambda |n^1\rangle + \lambda^2 |n^2\rangle + \dots$$

$$E_n = E_n^{(0)} + \lambda E_n^{(1)} + \lambda^2 E_n^{(2)} + \dots$$

$$\langle n^0 | n \rangle \neq 0$$

$$\langle n^0 | n \rangle = 1 + \lambda \underbrace{\langle n^0 | n^1 \rangle}_{0} + \lambda^2 \underbrace{\langle n^0 | n^2 \rangle}_{0} + \dots \quad \forall \lambda$$

Vsi popravki ortogonalni

Na koncu pa še renormiramo $\langle n | n \rangle = 1$.

$$(H_0+\lambda V)(|n^0\rangle + \lambda|n^1\rangle + \dots) = (E_n^{(0)} + \lambda E_n^{(1)} + \dots)(|n^0\rangle + \lambda|n^1\rangle + \dots)$$

Pogledamo člene pri istih potencah λ:

$\lambda^0: \quad H_0|n^0\rangle = E_n^{(0)}|n^0\rangle$ $\quad / \cdot \langle n_0|$

$\lambda^1: \quad H_0|n^1\rangle + V|n^0\rangle = E_n^{(0)}|n^1\rangle + E_n^{(1)}|n^0\rangle$

$\lambda^2: \quad H_0|n^2\rangle + V|n^1\rangle = E_n^{(0)}|n^2\rangle + E_n^{(1)}|n^1\rangle + E_0^{(2)}|n^0\rangle$ $\quad / \langle n_0|$

$\vdots$

$$\Rightarrow \underbrace{\langle n^0|H_0|n^1\rangle}_{E_n^{(0)}\underbrace{\langle n^0|n^1\rangle}_{0}} + \underbrace{\langle n^0|V|n^0\rangle}_{V_{nn}} = E_n^{(0)}\underbrace{\langle n^0|n^1\rangle}_{0} + E_n^{(1)}\underbrace{\langle n^0|n^0\rangle}_{1} =$$

$$\Rightarrow E_n^{(1)} = V_{nn}$$

$$I = \sum_m |m^0\rangle\langle m^0|$$

$$|n^1\rangle = I|n^1\rangle = \sum_{m\neq n} |m^0\rangle \underbrace{\langle m^0|n^1\rangle}_{\text{koeficienti}}$$

Tako razvito funkcijo damo nazaj in pomnožimo $\langle m^0|$

$$\lambda^1: \underbrace{\langle m^0|H_0|n^1\rangle}_{E_m^{(0)}\langle m^0|n^1\rangle} + \underbrace{\langle m^0|V|n^0\rangle}_{V_{mn}} = \underbrace{\langle m^0|E_n^{(0)}|n^1\rangle}_{E_n^{(0)}\langle m^0|n^1\rangle} + \underbrace{\langle m^0|E_n^{(1)}|n^0\rangle}_{0}$$

0 stanja med sabo ortogonalna

$$|n^1\rangle = \sum_{m\neq n} \frac{V_{mn}}{E_n^{(0)} - E_m^{(0)}} |m^0\rangle$$

To je popravek reda 1. !

$$E_n^{(2)} = \langle n^0 | V | n^1 \rangle$$

$$\Rightarrow \quad E_n^{(2)} = \sum_{m \neq n} \frac{|V_{nm}|^2}{E_n^{(0)} - E_m^{(0)}}$$

$$E_n^{(2)} = \langle n^0 | V | n^1 \rangle = \sum_{m \neq n} \underbrace{\langle n^0 | V | m^0 \rangle}_{V_{nm}} \frac{V_{mn}}{E_n^{(0)} - E_m^{(0)}} =$$

$$= \sum_{m \neq n} \frac{|V_{mn}|^2}{E_n^{(0)} - E_m^{(0)}}$$

Torej je potenčnih vrsta:

- $$E_n = E_n^{(0)} + \lambda V_{nn} + \lambda^2 \sum_{m \neq n} \frac{|V_{mn}|^2}{E_n^{(0)} - E_m^{(0)}} + \mathcal{O}(\lambda^3) \; ; \; \lambda \to 1$$

- $$|n\rangle = |n^0\rangle + \lambda \sum_{m \neq n} \frac{V_{mn}}{E_n^{(0)} - E_m^{(0)}} |m^0\rangle + \mathcal{O}(\lambda^2)$$

Renormiranje: $\langle n^0 | n^k \rangle = \delta_{k0}$

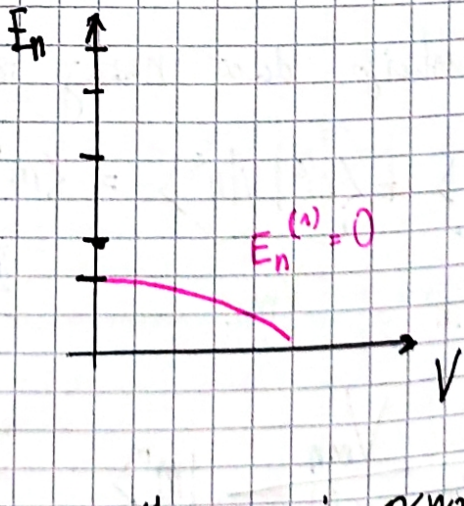
$$\langle n | n \rangle = \left(\langle n^0| + \lambda \langle n^1| + \mathcal{O}(\lambda^2) \right) \left(|n^0\rangle + \lambda |n^1\rangle + \mathcal{O}(\lambda^2) \right)$$

$$= 1 + \mathcal{O}(\lambda^2)$$ $\to$ že normirano do drugega reda

Naj bo $E_n^{(0)}$ osnovno stanje n;

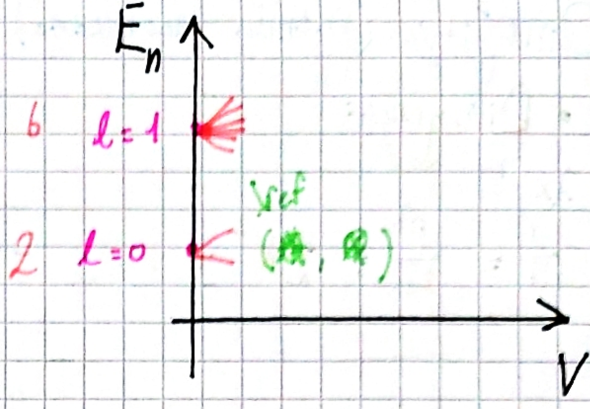
$$E_n^{(0)} < E_m^{(0)} \quad \forall m$$

$$\Rightarrow E_n^{(2)} \leq 0$$



- V drugem redu perturbacije se vedno zniža energija osnovnega stanja.

2. Degeneriran spekter



$$H = H_0 + \lambda V$$

$$E_n = E_n^{(0)} + \lambda E_n^{(1)} + \lambda^2 E_n^{(2)} + \dots$$

$$|n\rangle = \underbrace{c_1|n_1^0\rangle + c_2|n_2^0\rangle}_{\text{Predpostavimo 2x degeneracijo}} + \lambda |n^1\rangle + \dots$$

$$H|n\rangle = E_n|n\rangle$$ Na vsaki strani spet vrsti

$$\lambda^0: \quad H_0|n_1^0\rangle = E_n^{(0)}|n_1^0\rangle$$

$$H_0|n_2^0\rangle = E_n^{(0)}|n_2^0\rangle \qquad \left. + \overset{\langle n_1^1|}{E_n^{(1)}}|n^1\rangle \right\} 0$$

$$\lambda^1: \quad H_0|n^1\rangle + c_1 V|n_1^0\rangle + c_2 V|n_2^0\rangle = E_n^{(1)}\left(c_1|n_1^0\rangle + c_2|n_2^0\rangle\right) \; / \cdot \langle n_1^0|$$

$$\underbrace{\langle n_1^0| H_0|n^1\rangle}_{0} + c_1 \langle n_1^0| V|n_1^0\rangle + c_2 \langle n_1^0| V|n_2^0\rangle = E_n^{(1)}\left(c_1\underbrace{\langle n_1^0|n_1^0\rangle}_{1} + c_2\underbrace{\langle n_1^0|n_2^0\rangle}_{1}\right)$$

Torej:

$$E_n^{(1)} c_1 = V_{11} c_1 + V_{12} c_2$$

Če pa $/ \cdot \langle n_2^0|$ na začetku, dobimo:

$$E_n^{(1)} c_2 = V_{21} c_1 + V_{22} c_2$$

Dobimo problem lastnih vrednosti:

$$\begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = E_n^{(1)} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

Dve rešitve (●, ●)

Omejitve (menda neobvezno)

- Divergence:

- λx^m

Premaknjeni LHO

1) $H = \frac{p^2}{2m} + \frac{k}{2}x^2 + \lambda x \dashrightarrow (x-x_0)^2 + C$

2) $E_n = E_n^0 + \sum_k c_k \lambda^k \quad ; \quad |\lambda| \leq \lambda_R$

$$H = \frac{p^2}{2m} + \underbrace{\frac{1}{2}kx^2 + \lambda x^2}_{\frac{1}{2}kx^2 + \frac{1}{2}k\left(\frac{\lambda}{k}2\right)x^2 = \frac{1}{2}k\left(1+2\frac{\lambda}{k}\right)x^2}$$

$k = m\omega^2$

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2\left(1+2\frac{\lambda}{k}\right)x^2$$

$$E_n^{(0)} = \hbar\omega\left(n+\frac{1}{2}\right)$$

$$E_n = \hbar\omega\sqrt{1+2\frac{\lambda}{k}}\left(n+\frac{1}{2}\right)$$

< 1

3) $H = \frac{p^2}{2m} + \frac{1}{2}kx^2 + \lambda x^4 \Rightarrow$ Asimptotski razvoj

- fazni prehodi

1) Superprevodnost $\rightsquigarrow T_c \propto \underbrace{e^{-\frac{c}{\lambda}}}$

Se ne da razviti za male λ

Torej potenčna vrsta nima smisla in ne gre.

Brillouin Wignergera perturbazija (zanimivost/neobvezno)

$$H = H_0 + \lambda V$$

Velja za degeneriran in nedegeneriran primer.

$$E_n = E_n^{(0)} + \lambda V_{nn} + \lambda^2 \sum_m \frac{|V_{mn}|^2}{E_n^0 - E_m^{(0)}} + \dots +$$

$$+ \lambda^j \sum_{m_1, m_2, \dots, m_{j-1}} \frac{V_{m_1 n} V_{m_1 m_2} V_{m_2 m_3} \dots V_{m_{j-1} n}}{(E_n^0 - E_{m_1}^{(0)})(E_n^0 - E_{m_2}^{(0)}) \dots (E_n^0 - E_{m_{j-1}}^{(0)})}$$

Tu ni (0)

Od časa odvisna motnja

$$H(t) = \underbrace{H_0}_{\text{Neodvisen od } t} + \lambda \hat{V}(t)$$

$\hookrightarrow -\vec{j} \cdot \vec{B}(t)$ recimo

$$H_0 |n\rangle = E_n |n\rangle \ ; \quad \{|n\rangle\} \text{ baza}; \quad \langle m|n\rangle = \delta_{mn}$$

$$t = 0: \quad |\Psi(0)\rangle = \sum_n c_n(0) |n\rangle$$

$$t > 0: \quad |\Psi(t)\rangle = \sum_n c_n(t) e^{-i \frac{E_n t}{\hbar}} |n\rangle$$

To je naš nastavek za:

$$i\hbar \frac{\partial |\Psi(t)\rangle}{\partial t} = H(t) |\Psi(t)\rangle$$

$$i\hbar \sum_n \left(\frac{\partial c_n(t)}{\partial t} e^{-i \frac{E_n}{\hbar} t} - i \cancel{\frac{E_n}{\hbar}} c_n(t) e^{-i \frac{E_n}{\hbar} t} \right) |n\rangle = \sum_n (\cancel{E_n} + \lambda V(t)) e^{-i \frac{E_n}{\hbar} t} c_n |n\rangle$$

Celo enačbo množimo z $\langle m|$

$$i\hbar \frac{\partial c_m(t)}{\partial t} e^{-i\frac{E_m}{\hbar}t} = \lambda \sum_n \langle m|V(t)|n\rangle e^{-i\frac{E_n}{\hbar}t} c_n(t)$$

$$i\hbar \frac{\partial c_m(t)}{\partial t} = \lambda \sum_n \underbrace{\langle m|V(t)|n\rangle e^{-i\frac{E_n - E_m}{\hbar}t}}_{V_{mn}(t)}$$

$$\Rightarrow i\hbar \frac{\partial c_m(t)}{\partial t} = \lambda \sum_n V_{mn}(t)\, c_n(t) \qquad \lambda = 1$$

$$\Downarrow$$

$$i\hbar \frac{\partial}{\partial t}\begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_m \\ \vdots \end{pmatrix} = \begin{pmatrix} V_{11}(t) & V_{12}(t) & \cdots \\ \vdots & \ddots & \vdots \\ \cdots & \cdots & V_{mn}(t) \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_m \\ \vdots \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} \vdots \end{pmatrix}$$

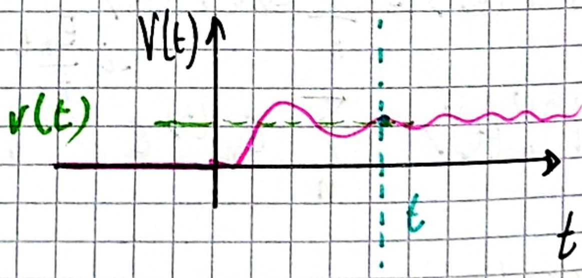
To je tocno! $\forall i \; c_i^1$

$$i\hbar \dot{\vec{c}} = \underline{V}\, \vec{c}$$

Temu se reče Diracova slika

a) Šibka motnja

$$V(t) = \begin{cases} 0; & t \le 0 \\ V(t); & t > 0 \end{cases}$$

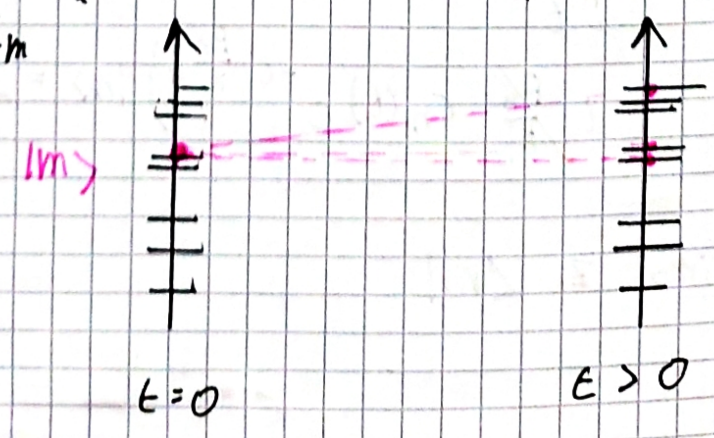


$$|\Psi(0)\rangle = |m\rangle = \sum_k c_k(0)|k\rangle \; ; \qquad c_k(0) = \delta_{km}$$

H_0, E_m

$P_{km} = |c_k|^2$

$\lambda \ll 1$



$$|\psi(0)\rangle = |m\rangle = \sum_n 1\cdot c_n(0)|n\rangle \qquad c_n(0) = \delta_{nm}$$

$$|\psi(t)\rangle = \sum_n c_n(t)\, e^{-\frac{E_n}{\hbar}t}|n\rangle$$

$$\Downarrow$$

$$\lambda$$

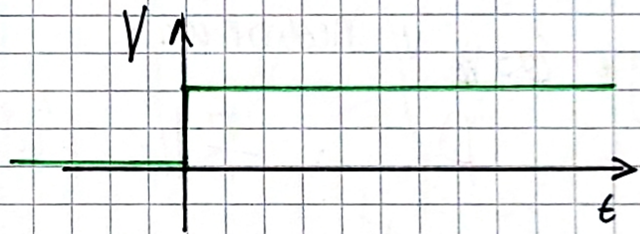
$$i\hbar \frac{\partial c_n}{\partial t} = \sum_n V_{nm}\, c_n(t) = \lambda V_{lm}(t)$$

Rezultat:
$$c_l(t) = \frac{\lambda}{i\hbar}\int_0^{t/2} V_{lm}(t)\,dt' \;; \quad \text{za } l \neq m$$

$$c_m \approx 1 \;; \qquad \text{če } |c_l|^2 \ll 1.$$

Fermijevo zlato pravilo

$$V(t) = \begin{cases} 0 & ; t \leq 0 \\ V & ; t > 0 \end{cases}$$



Tokrat vzamimo $\langle m|V|n\rangle = V_{mn}$

$$c_n(t) = \frac{1}{i\hbar}\int_0^T V_{mn}\, e^{-i\frac{E_n - E_m}{\hbar}t'}\,dt' = \qquad \omega_{nm}$$

$n \neq m$

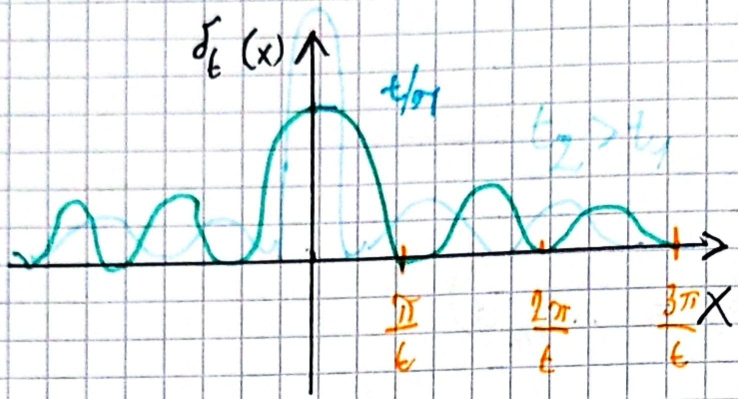
$n = m \Rightarrow |c_m|^2 \approx 1$

$$= \frac{V_{nm}}{i\hbar}\,\frac{e^{-i\omega_{nm}t} - 1}{-i\omega_{nm}}$$

$$X = \frac{1}{2}\frac{E_n - E_m}{\hbar} = \frac{1}{2}\omega_{nm}$$

$$\Rightarrow P_{nm} = \frac{|V_{nm}|^2}{\hbar^2}\,\frac{\sin^2\left(\frac{1}{2}\omega_{nm}t\right)}{\left(\frac{1}{2}\omega_{nm}\right)^2}$$

$$\delta_t(x) = \frac{1}{\pi}\frac{\sin^2 xt}{x^2 t} \quad ; \quad \int \delta_t(x)\,dx = 1$$



$$\delta_t(x) \xrightarrow[t\to\infty]{} \delta(x)$$ To je lot neka parametrizacija delte.

Velja:

$$\delta(ax) = \frac{1}{|a|}\delta(x)$$

$$P_{km} = \frac{2\pi}{\hbar} |V_{km}|^2 \delta_x(E_t - E_m)t$$

Če gre čas v neskončen:

$$P_{km} = \frac{2\pi}{\hbar} ||V_{km}|^2 \delta(E_k - E_n)(t)$$

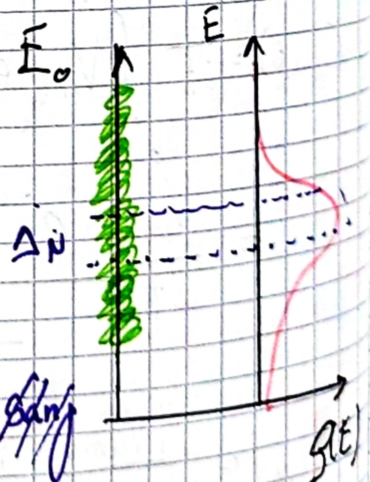
Približki:

- $P_{km} \ll 1$

[Primer: gostota vezanih stanj]

$$g(E) = \frac{\Delta N}{\Delta E} \to \frac{dN}{dE}$$

$$P_{\text{korkoli}} : \sum_{k\neq m} P_{kone} \to \int P_m(t)\, \underbrace{g(E_k)}_{\text{gostota stanj}}\, dE_k$$



$$\Rightarrow P - \frac{2\pi}{\hbar}[V_{km}]^2, g(E_m)\,t \propto t$$

~~$\frac{dP}{dt} = W = \frac{2\pi}{\hbar} |$~~

$$\boxed{\frac{dP}{dt} = W = \frac{2\pi}{\hbar} |V_{nm}|^2 \rho(E_n)}$$

Fermijevo Zlato pravilo

$\Rightarrow$

[Primer:] Radioaktivnost

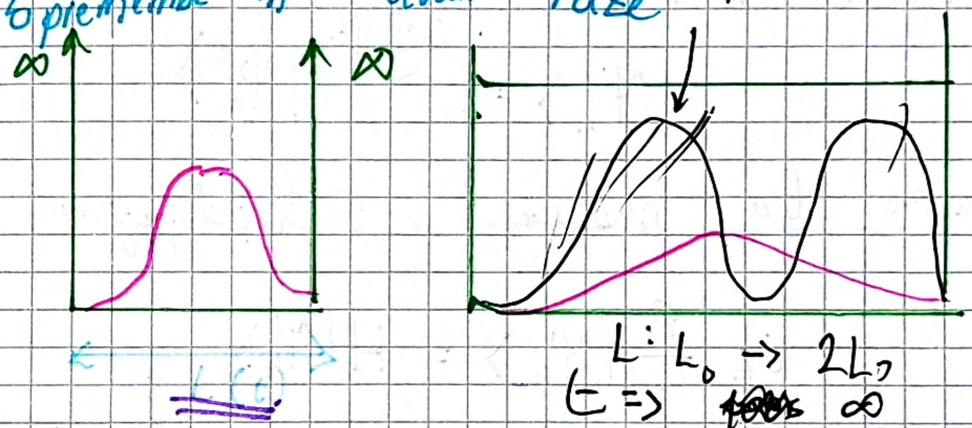
$t=0 : |\psi(0)\rangle$

Torej za ei prehode: $-dN = Npd = Nw\,dt$

Ni odvisnosti od časa/denarja:

Adiabatne ~~sp~~ spremembe in kvantne faze

H(t)

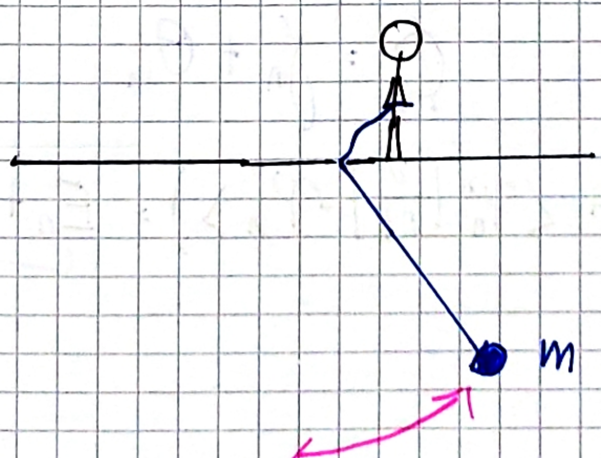


Zanima nas rešitev problema

~~$t=0$ i $L=2L_0$~~

$$\frac{dL}{dt} \ll \left(\frac{\Delta}{\hbar}\right) L \; ; \quad \Delta = E_n - E_m$$

Klasično je to lot:



$$H(\vec{Q}(t)) \quad ; \quad \vec{Q} = (L, V_0, B, \dots ?) = \vec{Q}(t)$$

$$\vec{Q} = (q_1, \dots, q_n)$$

$$t: \quad H(\vec{Q})|\Psi_n(\vec{Q})\rangle = E_n(\vec{Q})|\Psi_n(\vec{Q})\rangle$$

$$\psi^0_n(\vec{r}, t) = \langle \vec{r} | \Psi(\vec{Q}(t)) \rangle$$

Vstavimo v enačbo:

$$i\hbar \frac{\partial}{\partial t} |\Psi_n^0(\vec{r}, t)\rangle \neq H(\vec{Q}(t))|\Psi_n^0\rangle$$

Adiabatni režim

Kvantna faza

$$|\Psi_n\rangle = e^{i\phi_n(t)} |\Psi_n^0\rangle$$

Nastavimo ta nastavek v Schrödingerjevo enačbo:

$$i\hbar \frac{\partial}{\partial t}|\Psi_n\rangle = H|\Psi_n\rangle$$

$$i\hbar\left(i \frac{d\phi_n}{dt} \cancel{e^{i\phi_n}} |\Psi_n^0\rangle + \cancel{e^{i\phi_n}} |\dot{\Psi}_n^0\rangle\right) = H \cancel{e^{i\phi_n}} |\Psi_n^0\rangle \cancel{e^{i\phi_n}} |\Psi_n^0\rangle$$

$$= e^{i\phi_m} E_n |\Psi_n^0\rangle \quad / \langle \Psi_n^0|$$

$$i\hbar\left(i \frac{d\phi_n}{dt} \langle \Psi_n^0 | \Psi_n^0 \rangle + \langle \Psi_n^0 | \frac{\partial}{\partial t} | \Psi_n^0 \rangle\right) =$$

$$= E \langle \Psi_n^0 | \Psi_n^0 \rangle$$

$$\phi_n = \gamma_n + \theta_m$$

$$i\hbar\left(i \frac{d\gamma_n}{dt} + \langle \Psi_n^0 | \frac{\partial}{\partial t} | \Psi_n^0 \rangle\right) = \underbrace{E_n + \hbar \frac{d\theta_n}{dt}}_{0} = 0$$

$$\Rightarrow \frac{d\theta_n}{dt} = -\frac{E_n(\vec{Q}(t))}{\hbar} \; ; \; \theta_n = -\frac{1}{\hbar}\int_0^t E_n(t')dt'$$

$$\frac{d\gamma_m}{dt} = i\langle \Psi_n^0 | \frac{\partial}{\partial t} | \Psi_n^0 \rangle$$

$$\frac{\partial}{\partial t}\Psi_m = \sum_i \frac{\partial \Psi_n^0}{\partial q_i}\dot{q}_i = (\nabla_{\vec{Q}}\Psi_n^0)\cdot\dot{\vec{Q}}$$

$$\gamma = \int_0^t i\langle \Psi_n^0 | \vec{\nabla}_Q \Psi_n^0 \rangle \cdot \dot{Q}\, dt$$

$$\gamma(t) = \gamma_n \int_0^t i\langle \Psi_n^0 | \vec{\nabla}_Q \Psi_n^0 \rangle \cdot \underbrace{\dot{\vec{Q}}\, dt}_{d\vec{Q}} = i\int_{Q(0)}^{\vec{Q}(t)} \langle \Psi_n | \vec{\nabla}_Q \Psi_m^0 \rangle \cdot d\vec{Q}$$

Berryjeva faza:

$$\gamma_m = i\oint \langle \Psi_n^0 | \vec{\nabla}_Q \Psi_n^0 \rangle\, d\vec{Q}$$