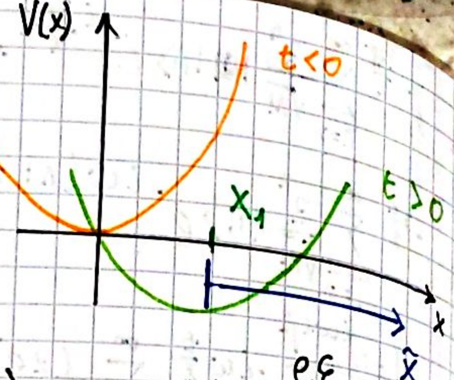
$$H(t) = \begin{cases} t < 0; & \frac{p^2}{2m} + \frac{1}{2}kx^2 = H \\ t > 0; & \frac{p^2}{2m} + \frac{1}{2}kx^2 - e\mathcal{E}_x = \tilde{H} \end{cases}$$



$$|\psi, 0\rangle = |0\rangle \rightarrow a|\psi, 0\rangle = 0$$

$$|\psi, t\rangle = ? \rightarrow \tilde{a}|\psi, 0\rangle = -\frac{x_1}{\sqrt{2}x_0}|\psi, 0\rangle \qquad x_1 = \frac{e\mathcal{E}}{k}$$

Ukvarjali smo se z koherentnimi stanji (ostanejo koherentna tudi skozi čas)

$$a|z\rangle = z|z\rangle$$

$$|z, t\rangle = e^{-i\frac{\omega}{2}t}|ze^{-i\omega t}\rangle \qquad \langle x\rangle = \sqrt{2}x_0 \mathrm{Re}\, z$$

$$\psi_z(x) = \frac{1}{\sqrt[4]{\pi x_0}} e^{-\frac{(x-\sqrt{2}\mathcal{E}_0 P_{oz})^2}{2x_0}} e^{\frac{i\sqrt{2}p_0 \ln z}{\hbar}x} \leftarrow \text{gaussov valovni paket}$$

Poglejmo si:

$$z(t)$$

$$z(0) = -\frac{x_1}{\sqrt{2}x_0}$$

$$z(t) = -\frac{x_1}{\sqrt{2}x_0}e^{-i\omega t}$$

$\langle x, t\rangle = ?$

$$\tilde{x} = x - x_1$$

$$x = \tilde{x} + x_1$$

$$\langle \tilde{x}, t\rangle = \sqrt{2}x_0 \mathrm{Re}(z(t)) =$$

$$= \sqrt{2}x_0 \mathrm{Re}\left(-\frac{x_1}{\sqrt{2}x_0}e^{-i\omega t}\right) = -x_0\cos(\omega t)$$

Upoštevamo povezavo med x in $\tilde{x}$:

$$\langle x, t\rangle = -x_1\cos(\omega t) + x_1 = x_1(1 - \cos(\omega t))$$

To pa dobimo tudi v klasičnem primeru

[2D Harmonski oscilator]

$$H = \frac{p^2}{2m} + \frac{1}{2}k_x x^2 + \frac{1}{2}k_y y^2 \qquad p^2 = (\vec{p})^2$$

$$\vec{p} = (p_x, p_y) = \left(-i\hbar\frac{\partial}{\partial y}, -i\hbar\frac{\partial}{\partial y}\right) = -i\hbar\nabla$$

$$p^2 = p_x^2 + p_y^2$$

Tako lahko Hamiltonian razstavimo na dva dela:

$$H = H_x + H_y$$

Separacija spremenljivk

$$\psi(x,y) = f(x)\chi(y)$$

$$H_x f_n(x) = E_n^x f_n(x)$$

$$H_y \chi_m(y) = E_m^y \chi_m(y)$$

Če poznamo obe rešitvi v 1D lahko rešimo v 2D

$$\Rightarrow \quad H f_n(x)\chi_m(y) = (E_n^x + E_m^y) f_n(x)\chi_m(y)$$

V Diracovem zapisu je to:

ubistvo ⊗

$$H|n\rangle_x|m\rangle_y = (E_n^x + E_m^y)|n\rangle_x|m\rangle_y$$

prostor x

oz.

$$H|n\,m\rangle = (E_n^x + E_m^y)|n\,m\rangle$$

Se nanaša na x

Se nanaša na y

Oznaka:

$$|n\rangle_x = |n_x\rangle$$

Spomnimo kako se reši LHO za 1D:

$$H_x|n\rangle_x = \hbar\omega_x\left(n_x + \frac{1}{2}\right)|n\rangle_x \quad ; \quad \omega_x = \sqrt{\frac{k_x}{m}}$$

$$H_y|n_y\rangle = \hbar\omega_y\left(n_y + \frac{1}{2}\right)|n_y\rangle \quad ; \quad \omega_y = \sqrt{\frac{k_y}{m}}$$

Tako je celotno:

$$H|n_x n_y\rangle = \hbar\left(\omega_x\left(n_x+\tfrac{1}{2}\right)+\omega_y\left(n_y+\tfrac{1}{2}\right)\right)|n_x n_y\rangle$$

To vse velja $k_x > 0$ in $k_y > 0$

Poglejmo si primer $k_x > 0$ in $k_y = 0$

$$H_y = \frac{p_y^2}{2m} + 0 \,;$$ Konstanten potencial (rešitve so ravni valovi)

$$p_y e^{iq_y y} = \hbar q_y e^{iq_y y}$$

q_y ... valovni vektor ki karakterizira ravni val

$$\Rightarrow H_y e^{iq_y y} = \frac{\hbar^2 q_y^2}{2m} e^{iq_y y}$$

$$H_y|q_y\rangle = \frac{\hbar^2 q_y^2}{2m}|q_y\rangle$$

Tako je 2D problem:

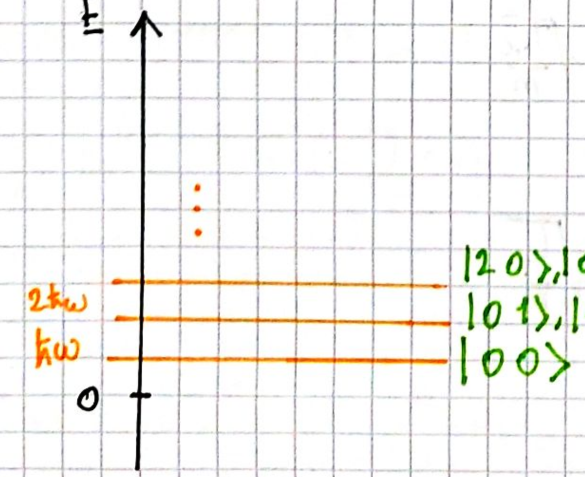
$$H|n_x q_y\rangle = \left[\hbar\omega_x\left(n_x+\tfrac{1}{2}\right)+\frac{\hbar^2 q_y^2}{2m}\right]|n_x q_y\rangle$$

Vrnimo se na splošen primer $k_x > 0$ in $k_y > 0$

Poglejmo si izotropni harmonski oscilator $\boxed{k_x = k_y = k}$ $\omega_x = \omega_y = \omega$

$$H = \frac{p^2}{2m} + \frac{1}{2}k\underbrace{(x^2+y^2)}_{r^2} = \frac{p^2}{2m} + \frac{1}{2}kr^2$$ → Odvisen le od oddaljenosti od izhodišča ampak ne od polarnega kota!

$$H|n_x n_y\rangle = \hbar\omega(n_x+n_y+1)|n_x n_y\rangle$$



→ Degenerirano vzbujeno stanje

$$\alpha|1\,0\rangle + \beta|0\,1\rangle \qquad |\alpha|^2+|\beta|^2 = 1$$

Pokažimo, da $|01\rangle$ in $|10\rangle$ res tvorita 2D bazo prostora vseh funkcij, ki imajo energijo $2\hbar\omega$

$$H(\alpha|10\rangle + \beta|01\rangle) = \alpha \underbrace{H|10\rangle}_{2\hbar\omega|10\rangle} + \beta \underbrace{H|01\rangle}_{2\hbar\omega|01\rangle} = 2\hbar\omega(\alpha|10\rangle + \beta|01\rangle)$$

Tudi lin. komb. je lastna funkcija z isto energijo ✓

Torej: $H = \frac{p^2}{2m} + V(r)$

V smeri simetrijske osi potenciala se ohranja vrtilna količina. To se zapiše s komutatorjem kot:

$$[H, L_z] = 0 \quad ; \quad L_z = xp_y - yp_x = -i\hbar\frac{\partial}{\partial\varphi}$$

Ker operatorja komutirata lahko najdemo lastne funkcije obeh hkrati. Katere linearne kombinacije lastnih stanj H so lastne funkcije L_z? Poglejmo to za 2x degenerirano 1. vzbujeno stanje.

$$L_z|\Psi\rangle = \lambda|\Psi\rangle$$

$$-i\hbar\frac{\partial}{\partial\varphi}\Psi(\varphi) = \lambda\Psi(\varphi)$$

Rešimo diferencialno enačbo

$$-i\hbar\Psi'(\varphi) - \lambda\Psi(\varphi) = 0 \quad \Rightarrow \quad \Psi(\varphi) = Ce^{i\frac{\lambda}{\hbar}\varphi}$$

Robni pogoj: $\Psi(\varphi) = \Psi(\varphi + 2\pi) \quad \Rightarrow \quad \frac{\lambda}{\hbar} = m \in \mathbb{Z}$

Tako so lastne funkcije:

$$\Psi(\varphi) = Ce^{im\varphi}$$

Iz normalizacije:

$$\int_0^{2\pi} |\Psi(\varphi)|^2 d\varphi = 1$$

$$\Psi(\varphi) = \frac{1}{\sqrt{2\pi}}e^{im\varphi}$$

Pokažimo, da $|01\rangle$ in $|10\rangle$ res tvorita 2D bazo prostora vseh funkcij, ki imajo energijo $2\hbar\omega$

$$H(\alpha|10\rangle + \beta|01\rangle) = \alpha \underbrace{H|10\rangle}_{2\hbar\omega|10\rangle} + \beta \underbrace{H|01\rangle}_{2\hbar\omega|01\rangle} = 2\hbar\omega(\alpha|10\rangle + \beta|01\rangle)$$

Tudi lin. komb. je lastna funkcija z isto energijo ✓

Torej: $H = \frac{p^2}{2m} + V(r)$

V smeri simetrijske osi potenciala se ohranja vrtilna količina. To se zapiše s komutatorjem kot:

$$[H, L_z] = 0 \quad ; L_z = x p_y - y p_x = -i\hbar \frac{\partial}{\partial \varphi}$$

Ker operatorja komutirata lahko najdemo lastne funkcije obeh hkrati. Katere linearne kombinacije lastnih stanj H so lastne funkcije L_z? Poglejmo to za $2\times$ degenerirano 1. vzbujeno stanje.

$$L_z|\psi\rangle = \lambda|\psi\rangle$$

$$-i\hbar \frac{\partial}{\partial \varphi}\psi(\varphi) = \lambda \psi(\varphi)$$

Rešimo diferencialno enačbo

$$-i\hbar\psi'(\varphi) - \lambda\psi(\varphi) = 0 \Rightarrow \psi(\varphi) = C e^{i\frac{\lambda}{\hbar}\varphi}$$

Robni pogoj: $\psi(\varphi) = \psi(\varphi + 2\pi) \Rightarrow \frac{\lambda}{\hbar} = m \in \mathbb{Z}$

Tako so lastne funkcije:

$$\psi(\varphi) = C e^{im\varphi}$$

Iz normalizacije:

$$\int_0^{2\pi} |\psi(\varphi)|^2 d\varphi = 1$$

$$\psi(\varphi) = \frac{1}{\sqrt{2\pi}} e^{im\varphi}$$

$$\psi(x,y) = \alpha \psi_{10}(x,y) + \beta \psi_{01}(x,y)$$

$$\psi_{10}(x,y) = \psi_1(x)\cdot\psi_0(y)$$

$$\psi_{01}(x,y) = \psi_0(x)\cdot\psi_1(y)$$

Zanima nas:

$$\psi_0(x) = \frac{1}{\sqrt[4]{\pi x_0^2}} e^{-\frac{x^2}{2x_0^2}}$$

$a|0\rangle = 0|0\rangle$

$\Leftarrow$ $\hookrightarrow$ Poseben primer koherentnega stanja. Koherentna stanja so Gaussovski valovni paketi

$$\psi_1(x) = ?$$

$$a^\dagger|0\rangle = |1\rangle$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}}\left(\frac{x}{x_0} - i\frac{p}{p_0}\right)\psi_0(x) = \psi_1(x) \ ; \quad p_x = -i\hbar\frac{d}{dx}\ , \quad p_0 = \frac{\hbar}{x_0}$$

Torej je:

$$\psi_1(x) = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{\pi x_0^2}}\left[\frac{x}{x_0}e^{-\frac{x^2}{2x_0^2}} - i\frac{1}{p}\left(-i\hbar\left(\frac{-2x}{2x_0^2}\right)e^{-\frac{x^2}{2x_0^2}}\right)\right] =$$

$$= \frac{1}{\sqrt[4]{4\pi x_0^2}} e^{-\frac{x^2}{2x_0^2}}\left[\frac{x}{x_0} + \frac{x_0}{\hbar}\frac{x}{x_0^2}\right] =$$

$$x_0 = \sqrt{\frac{\hbar}{m\omega}} =: y_0$$

$$= \sqrt{2}\frac{x}{x_0}\psi_0(x)$$

Tako lahko zapišemo ψ_{10} in ψ_{01} kot:

$$\psi_{10}(x,y) = \frac{1}{\sqrt[4]{\pi x_0^2}}\left[\sqrt{2}\frac{x}{x_0}e^{-\frac{x^2}{2x_0^2}}\right]\frac{1}{\sqrt[4]{\pi x_0^2}} e^{-\frac{y^2}{2x_0^2}} =$$

$$= \frac{\sqrt{2}}{\sqrt{\pi x_0^2}}\frac{x}{x_0} e^{-\frac{1}{2x_0^2}(x^2+y^2)}$$

V polarnih koordinatah torej:

$$\psi_{10}(r,\varphi) = \sqrt{\frac{2}{\pi x_0^2}}\frac{r\cos\varphi}{x_0}e^{-\frac{1}{2x_0^2}r^2} = \cos\varphi\, F(r)$$

Rabimo dobiti $e^{im\varphi}$

$$\psi_{10}(r,\varphi) = \quad \dots \quad = \sin\varphi\, F(r)$$

$1 \cdot \cos\varphi + i\sin\varphi = e^{i\varphi}$ $(m=1)$

$1 \cdot \cos\varphi - i\sin\varphi = e^{-i\varphi}$ $(m=-1)$

$|m=1\rangle = 1 \cdot |10\rangle + i|01\rangle$

$|m=-1\rangle = 1 \cdot |10\rangle - i|01\rangle$

Te dve VF nista normirani $\left|\frac{1}{\sqrt{2}}\right|^2 + \left|\frac{i}{\sqrt{2}}\right|^2 = 1$

Torej:

$$|m=1\rangle = \frac{1}{\sqrt{2}}(|10\rangle + i|01\rangle)$$

$$|m=-1\rangle = \frac{1}{\sqrt{2}}(|10\rangle - i|01\rangle)$$

Drug postopek za to, ki je preprostejši in ni treba nič "videti"

$$|\psi\rangle = \alpha|10\rangle + \beta|01\rangle$$

$$\langle n_x n_y | n_x' n_y' \rangle = \delta_{n_x n_x'} \delta_{n_y n_y'}$$

$$L_z|\psi\rangle = \lambda|\psi\rangle$$

$$L_z(\alpha|10\rangle + \beta|01\rangle) = \lambda(\alpha|10\rangle + \beta|01\rangle)$$

$$\alpha L_z|10\rangle + \beta L_z|01\rangle = \lambda(\alpha|10\rangle + \beta|01\rangle)$$

$\cdot \langle 10|$, $\langle 01|$

Projeciramo to stanje na $|10\rangle$ in $|01\rangle$

a) $\langle 10|\alpha L_z|10\rangle + \langle 10|\beta L_z|01\rangle = \lambda\alpha$

b) $\langle 01|\alpha L_z|10\rangle + \langle 01|\beta L_z|01\rangle = \lambda\beta$

Sistem linearnih enačb

$$\begin{pmatrix} \langle 10|L_z|10\rangle & \langle 10|L_z|01\rangle \\ \langle 01|L_z|10\rangle & \langle 01|L_z|01\rangle \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \lambda \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

Problem lastnih vrednosti in lastnih vektorjev

Matrični elementi operatorja L_z

$L_z = xp_y - yp_x = -i\hbar \frac{\partial}{\partial \varphi}$

1D HO

$x = \frac{x_0}{\sqrt{2}}(a+a^\dagger)$ $x_0 = \sqrt{\frac{\hbar}{m\omega}}$

$p = \frac{p_0}{\sqrt{2}i}(a-a^\dagger)$ $p_0 = \frac{\hbar}{x_0}$

Problem lahko rešimo tudi, ko zapišemo z a in $a^\dagger$

$$H = \hbar\omega(a_x^\dagger a_x + \tfrac{1}{2}) + \hbar\omega(a_y^\dagger a_y + \tfrac{1}{2})$$

Torej lahko zapišemo:

$$x = \frac{x_0}{\sqrt{2}}(a_x + a_x^\dagger) \qquad y = \frac{x_0}{\sqrt{2}}(a_y + a_y^\dagger)$$

$$p_x = \frac{p_0}{\sqrt{2}i}(a_x - a_x^\dagger) \qquad p_y = \frac{p_0}{\sqrt{2}i}(a_y - a_y^\dagger)$$

in sestavimo L_z:

$$L_z = \frac{x_0}{\sqrt{2}}\frac{p_0}{\sqrt{2}i}(a_x + a_x^\dagger)(a_y - a_y^\dagger) - \frac{x_0}{\sqrt{2}}\frac{p_0}{\sqrt{2}i}(a_y + a_y^\dagger)(a_x - a_x^\dagger) =$$

$$= \frac{\hbar}{2i}\left[a_x a_y - a_x a_y^\dagger + a_x^\dagger a_y - \cancel{a_x^\dagger a_y^\dagger} - a_y a_x + a_y a_x^\dagger - a_y^\dagger a_x + \cancel{a_y^\dagger a_x^\dagger}\right] =$$

$$\frac{\partial^2}{\partial x \partial y} = \frac{\partial^2}{\partial y \partial x} \Rightarrow [a_x, a_y] = [a_x, a_y^\dagger] = \dots = 0 \text{ Komutirajo}$$

$$= \frac{\hbar}{2i}\left[-2a_x a_y^\dagger + 2a_x^\dagger a_y\right] = \frac{\hbar}{i}(a_x^\dagger a_y - a_x a_y^\dagger)$$

Sedaj lahko izračunamo matrične elemente:

$$\langle 10|L_z|10\rangle = \langle 10|\frac{\hbar}{i}(a_x^\dagger a_y - a_x a_y^\dagger)|10\rangle =$$

Deluje na

$$= \frac{\hbar}{i}\langle 10|\left(a_x^\dagger a_y|10\rangle - a_x a_y^\dagger|10\rangle\right) =$$

$$a_x^\dagger a_y|10\rangle = a_x^\dagger a_y|1\rangle_x|0\rangle_y =$$

$$= (a_x^\dagger|1\rangle_x)(a_y|0\rangle_y) = \sqrt{2}|2\rangle \cdot 0 = \underline{\underline{0}}$$

Zaradi ort.

$$= \frac{\hbar}{i}\langle 10|\left(0 - |01\rangle\right) = -\frac{i}{\hbar}\langle 10|01\rangle = \underline{\underline{0}}$$

$$\langle 10|01\rangle = {}_x\langle 1|0\rangle_x\, {}_y\langle 0|1\rangle_y = 0 \cdot 0 = \underline{\underline{0}}$$

Poglejmo si še:

$$L_z|10\rangle = \frac{\hbar}{i}(-|01\rangle)$$

$$\Rightarrow \langle 01|L_z|10\rangle = -\frac{\hbar}{i}\langle 01|01\rangle = i\hbar$$

$$L_z|01\rangle = \frac{\hbar}{i}(a_x^\dagger a_y - a_x a_y^\dagger)|01\rangle = \frac{\hbar}{i}(|10\rangle - 0) = \frac{\hbar}{i}|10\rangle$$

Še zadnja dva elementa data torej:

$$\langle 10|L_z|01\rangle = \frac{\hbar}{i}\langle 10|10\rangle = -i\hbar$$

$$\langle 01|L_z|01\rangle = \frac{\hbar}{i}\langle 01|10\rangle = 0$$

Torej imamo problem diagonalizacije matrike:

$$\begin{bmatrix} 0 & -i\hbar \\ i\hbar & 0 \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \lambda\begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

Diagonalizirajmo to matriko

$$\det\begin{bmatrix} 0-\lambda & -i\hbar \\ i\hbar & 0-\lambda \end{bmatrix} = 0 = \lambda^2 + (-\hbar)^2 = \lambda^2 - \hbar^2 \Rightarrow \underline{\lambda_{1,2} = \pm\hbar}$$

Poiščimo še lastne vektorje

$$|\alpha|^2 + |\beta|^2 = 0$$
$$|-i\beta|^2 + |\beta|^2 = 1$$

$$\begin{bmatrix} -\hbar & -i\hbar \\ i\hbar & -\hbar \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = 0 \qquad \begin{matrix} -\alpha - i\beta = 0 \\ i\alpha - \beta = 0 \end{matrix} \Rightarrow \begin{matrix} \alpha = i\beta \\ \underline{\beta - \beta = 0} \end{matrix} \Rightarrow \begin{matrix} 2|\beta|^2 = 1 \\ \beta = \pm\frac{1}{\sqrt{2}} \end{matrix}$$

Da dobimo enolično rešitev
uporabimo še normalizacijo
$|\alpha|^2 + |\beta|^2 = 1$

$$\text{Torej je } N_1 = \begin{bmatrix} -i\beta \\ \beta \end{bmatrix} = \begin{bmatrix} -\frac{i}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

Ponovimo še za λ_2:

$$\begin{bmatrix} \hbar & -i\hbar \\ i\hbar & \hbar \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = 0 \rightarrow \underset{\text{norm.}}{\underset{+}{\alpha = -i\beta}} \rightarrow \begin{bmatrix} i\beta \\ \beta \end{bmatrix}$$

$$v_2 = \begin{bmatrix} \frac{i}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

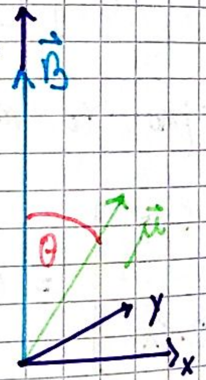
Za L_z vemo: $L_z |m\rangle = m\hbar |m\rangle$

Lastne vrednosti operatorja VK
$$|1\rangle = \frac{1}{\sqrt{2}}\left(-i|10\rangle + |01\rangle\right)$$

$$|-1\rangle = \frac{1}{\sqrt{2}}\left(i|10\rangle + |01\rangle\right)$$

To je od zadnjič zamaknjen rezultat za fazo $(-i)$. To je matematično drugačen rezultat, ampak je fizikalno čisto enako.

[Kvanten ekvivalent precesije magnetnega momenta okoli mag. polja]



Klasično:
$$H = -\vec{\mu}\cdot\vec{B} \; ; \; \vec{\mu} = -\mu_B \frac{\vec{L}}{\hbar}$$
$$\Downarrow$$
$$H = \lambda \vec{L}\cdot\vec{B}$$

Za $l=1$ ustreza 3D Hilbertov prostor:

$$L^2 |lm\rangle = \hbar^2 l(l+1)|lm\rangle$$

$$L_z |lm\rangle = \hbar m |lm\rangle$$

Baza:
$|1\,1\rangle$
$|1\,0\rangle$
$|1\,-1\rangle$

$m=1$ $\quad L_z|1\,1\rangle = \hbar|1\,1\rangle$ "kaže v smeri z"

$$\vec{L}\,\hat{e}_z|1\,1\rangle = \hbar|1\,1\rangle \qquad \vec{L} = (L_x, L_y, L_z)$$

Torej Začetni pogoj

$$\vec{L}\cdot\hat{n}\,|\psi, 0\rangle = \hbar|\psi, 0\rangle \qquad |\psi, t\rangle = ?$$

$|\psi, 0\rangle = \alpha|1\,1\rangle + \beta|1\,0\rangle + \gamma|1\,-1\rangle$

Na začetku lahko zapišemo normalo ($\phi = 0°$)

$$\hat{n} = \begin{bmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{bmatrix} = \begin{bmatrix} \sin\theta \\ 0 \\ \cos\theta \end{bmatrix}$$

Izvrednotimo:

$$\vec{L}\cdot\vec{n} = L_x \sin\theta + L_z \cos\theta$$

Torej:

$$(L_x \sin\theta + L_z \cos\theta)(\alpha|1\,1\rangle + \beta|1\,0\rangle + \gamma|1\,-1\rangle) = \hbar(\alpha|1\,1\rangle + \beta|1\,0\rangle + \gamma|1\,-1\rangle)$$

To je spet problem lastnih vrednosti kjer je lastna vrednost ($\hbar m$) določena z začetnim pogojem.

Vpeljemo (s podobno vlogo kot a in $a^\dagger$ pri LHO):

$$L_\pm = L_x \pm i L_y$$

$$L_x = \frac{L_+ + L_-}{2}$$

$$L_y = \frac{L_+ - L_-}{2i}$$

$$L_\pm |l\,m\rangle = \hbar\sqrt{l(l+1) - m(m\pm 1)}\,|l, m\pm 1\rangle$$

To stanje ne obstaja

$l\,;\ m = -l, \dots, l$

$m \in \mathbb{Z}$

$$L_x|1\,1\rangle = \frac{L_+ + L_-}{2}|1\,1\rangle = \frac{1}{2}\hbar\left(\underbrace{\sqrt{2-2}}_{=0}\,|1\,2\rangle + \sqrt{2-0}\,|1\,0\rangle\right)$$

$$L_x|1\,0\rangle = \frac{\hbar}{2}\left(\sqrt{2}\,|1\,1\rangle + \sqrt{2}\,|1\,-1\rangle\right) = \frac{\sqrt{2}}{2}\hbar\left(|1\,1\rangle + |1\,-1\rangle\right)$$

$$L_x|1\,-1\rangle = \frac{\hbar}{2}\left(\sqrt{2}\,|1\,0\rangle\right) = \frac{\sqrt{2}}{2}|1\,0\rangle$$

$L_z|1\,1\rangle = \hbar|1\,1\rangle$

$L_z|1\,0\rangle = 0$

$L_z|1\,-1\rangle = -\hbar|1\,-1\rangle$

To je sedaj konec stranskega računa, vrnemo se nazaj

$$\sin\theta \frac{\sqrt{2}}{2}\hbar\left(\alpha|1\,0\rangle + \beta|1\,1\rangle + \beta|1\,-1\rangle + \gamma|1\,0\rangle\right) +$$

$$+\cos\theta\,\hbar\left(\alpha|1\,1\rangle - \gamma|1\,-1\rangle\right) = \hbar\left(\alpha|1\,1\rangle + \beta|1\,0\rangle + \gamma|1\,-1\rangle\right)$$

Iz tega lahko dobimo ven 3 enačbe (zaradi ortogonalnosti)

$|1\,0\rangle$: $\sin\theta \frac{\sqrt{2}}{2}\hbar(\alpha+\gamma) = \hbar\beta$

$|1\,1\rangle$: $\sin\theta \frac{\sqrt{2}}{2}\hbar\beta + \cos\hbar\alpha = \hbar\alpha$

$|1\,-1\rangle$: $\sin\theta \frac{\sqrt{2}}{2}\hbar\beta - \gamma\cos\theta\,\hbar = \hbar\gamma$

To so že enačbe za določitev lastnega vektorja

$$\beta = \sin\theta \frac{\sqrt{2}}{2}(\alpha+\gamma)$$

$$\alpha = \frac{-\sin\theta\frac{\sqrt{2}}{2}}{\cos\theta - 1}\beta = \beta\frac{\sqrt{2}\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}} = \frac{\sqrt{2}}{2}\,\mathrm{ctg}\frac{\theta}{2}\beta$$

$$\gamma = \frac{\sin\theta\frac{\sqrt{2}}{2}}{\cos\theta + 1}\beta = $$

$$= \frac{\sqrt{2}\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}} = \frac{\sqrt{2}}{2}\,\mathrm{tg}\frac{\theta}{2}\,\beta$$

$\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

$1+\cos\theta = 2\cos^2\frac{\theta}{2}$

$1-\cos\theta = 2\sin^2\frac{\theta}{2}$

Torej vmesno:

$$|\Psi, 0\rangle = \frac{\sqrt{2}}{2}\,\mathrm{ctg}\frac{\theta}{2}\beta|1\,1\rangle + \beta|1\,0\rangle + \frac{\sqrt{2}}{2}\,\mathrm{tg}\frac{\theta}{2}\beta|1\,-1\rangle$$

β pa dobimo iz normalizacije:

$$\left(\frac{1}{2}\mathrm{ctg}^2\frac{\theta}{2} + 1 + \frac{1}{2}\mathrm{tg}^2\frac{\theta}{2}\right)|\beta|^2 = 1$$

Torej:
$$\frac{\cos^4\frac{\theta}{2} + 2\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2} + \sin^4\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}} = \frac{(\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2})^2}{2\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}} = \frac{1}{2\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}}$$

$$\Rightarrow \beta = \sqrt{2}\sin\frac{\theta}{2}\cos\frac{\theta}{2}$$

Torej imamo na začetku:

$$|\Psi, 0\rangle = \cos^2\frac{\theta}{2}|1\ 1\rangle + \sqrt{2}\sin\frac{\theta}{2}\cos\frac{\theta}{2}|1\ 0\rangle + \sin^2\frac{\theta}{2}|1\ -1\rangle$$

Rabimo časovni razvoj. Najlažje ga dobimo kot razvoj po lastnih stanjih Hamiltoniana.

Torej so l. stanja L_z tudi l. stanja H

$$H = \lambda\vec{L}\cdot\vec{B} = \lambda B L_z$$

$\uparrow\ \vec{B} = (0, 0, B)$

$$L_z|l\,m\rangle = \hbar m|l\,m\rangle$$

$H|1\,1\rangle = \lambda B\hbar|1\,1\rangle \qquad H|1\,0\rangle = 0|1\,0\rangle \qquad H|1\,-1\rangle = -\lambda B\hbar|1\,-1\rangle$

Tako brez težav zapišemo časovni razvoj:

$$|\Psi, t\rangle = \cos^2\frac{\theta}{2}e^{-i\lambda Bt}|1\,1\rangle + \sqrt{2}\sin\frac{\theta}{2}\cos\frac{\theta}{2}|1\ 0\rangle + \sin^2\frac{\theta}{2}e^{i\lambda Bt}|1, -1\rangle$$

Poskusimo izluščiti, kaj pomeni ta valovna funkcija.

$L_z|l\,m\rangle = \hbar m|l\,m\rangle$

$$\langle\vec{L}, t\rangle = ?$$

$$\langle L_z, t\rangle = \langle\Psi, t|L_z|\Psi, t\rangle =$$

$$L_z|\Psi, t\rangle = \cos^2\frac{\theta}{2}e^{-i\lambda Bt}\hbar|1\,1\rangle + 0 - \hbar\sin^2\frac{\theta}{2}e^{i\lambda Bt}|1, -1\rangle$$

$$\langle L_z, t\rangle = \hbar\left(\cos^4\frac{\theta}{2} - \sin^4\frac{\theta}{2}\right) = \hbar\left(\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}\right)\left(\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2}\right) =$$

$$= \hbar\cos\theta$$

$$\langle L_x, t\rangle = \left\langle \frac{L_+ + L_-}{2}, t\right\rangle = \frac{1}{2}\langle L_+, t\rangle + \frac{1}{2}\langle L_-, t\rangle =$$

$\longleftarrow$ com. conj tega $\quad \langle L_-, t\rangle = \langle L_+, t\rangle^*$

$$= Re(\langle L_+, t\rangle)$$

$$\langle L_y, t\rangle = \left\langle \frac{L_+ - L_-}{2i}, t\right\rangle = \frac{1}{2i}\left[\langle L_+, t\rangle - \langle L_-, t\rangle\right] =$$

$$= Im(\langle L_+, t\rangle)$$

$L_\pm |lm\rangle = \hbar\sqrt{l(l+1) - m(m\pm 1)}\,|l, m\pm 1\rangle$

$$\langle L_+, t\rangle = \langle \Psi, t | L_+ | \Psi, t\rangle$$

$L_+|1,-1\rangle = \hbar\sqrt{2}\,|1 0\rangle$

$$L_+|\Psi, t\rangle = 0 + 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\,\hbar|11\rangle + \hbar\sqrt{2}\sin^2\frac{\theta}{2}e^{i\lambda Bt}|10\rangle$$

$L_+|1 0\rangle = \sqrt{2}\hbar|1 1\rangle$

$L_+|1 1\rangle = 0$

$$\Rightarrow \langle L_+, t\rangle = 2\cos^3\frac{\theta}{2}\sin\frac{\theta}{2}e^{i\lambda Bt}\hbar + 2\hbar\sin^3\frac{\theta}{2}\cos\frac{\theta}{2}e^{i\lambda Bt} =$$

$$= 2\hbar e^{i\lambda Bt}\cos\frac{\theta}{2}\sin\frac{\theta}{2}\left(\cos^2\theta/2 + \sin^2\theta/2\right) =$$

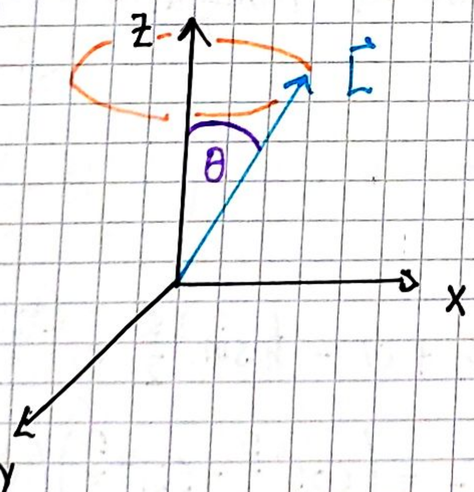
$$= \hbar\sin\theta\, e^{i\lambda Bt}$$

Tako lahko dobimo L_x in L_y

$$\langle L_x, t\rangle = Re\langle L_+, t\rangle = \underline{\hbar\cos(\lambda Bt)\sin\theta}$$

$$\langle L_y, t\rangle = Im\langle L_+, t\rangle = \underline{\hbar\sin(\lambda Bt)\sin\theta}$$

Vektor tog precesira po stožcu:



To je kot klasično za vrtilno količino. Temu se v kvantni reče Larmorjeva precesija. Vrtilna količina precesira z Larmorjevo frekvenco

$$\omega_L = \lambda B$$

Kaj pa bi nam dala meritev L_z?

Razvijemo VF po lastnih stanjih ~~meri~~ operatorja, kar že imamo. Možne vrednosti so lastne vrednosti operatorja. Verjetnost pa iz koeficientov pred.

L_z	$\|c_m\|^2$	Ψ ob $t+dt$
$\hbar$	$\cos^4\frac{\theta}{2}$	$\|11\rangle$
0	$2\cos^2\frac{\theta}{2}\sin^2\frac{\theta}{2}$	$\|10\rangle$
$-\hbar$	$\sin^4\frac{\theta}{2}$	$\|1-1\rangle$

Kaj se zgodi z delcem takoj po meritvi?

Meritev kolapsira VF torej je valovna funkcija le tisto stanje, ki ustreza lastni vrednosti, ki smo jo izmerili.

Če želimo izmeriti pričakovano vrednost to ne moremo ~~nar~~ narediti več meritev. Če naredimo več zaporednih meritev na enem delcu dobimo vedno tisto kar smo prvo izmerili zaradi kolapsa valovne funkcije.
Zato moramo vsaki začeti ob $t=0$ z novo generiranim začetnim stanjem.

$$\bar{L}_z = \hbar\cos^4\frac{\theta}{2} + 0 - \hbar\sin^4\frac{\theta}{2} = \dots = \hbar\cos\theta$$

Torej je pričakovana vrednost $\langle L_z, t\rangle$ povprečje neodvisnih meritev.

Spin

$$H = \frac{p^2}{2m} + \lambda\underbrace{(p_x S_y - p_y S_x)}_{\text{Rashba sklopitev}} \; ; \quad S = \frac{1}{2}$$

$\vec{p} = (p_x, p_y)$ 2D

Rešujemo

$$H|\psi\rangle = E|\psi\rangle$$

$[H, \vec{p}] = ?$

v drugem podprostoru torej vedno komutira

$$[H, p_x] = \left[\frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \lambda(p_x S_y - p_y S_x), p_x\right] = 0$$

Potem lahko iščemo stanje z lastnim stanji $\vec{p}$:

$$\vec{p}|\vec{u}\rangle = \hbar\vec{u}\,|\vec{u}\rangle$$

$$\downarrow$$

$$\psi_{\vec{u}}(\vec{r}) = e^{i\vec{u}\cdot\vec{r}}$$

$$\vec{p} = -i\hbar\nabla$$

Torej zaradi komutiranja H in $\vec{p}$ lahko iščemo rešitve z produktnim nastavkom:

$$|\psi\rangle = \underbrace{|\vec{u}\rangle}_{\text{krajevni del}}\underbrace{|\chi\rangle}_{\text{Spinski del}}$$

$$\left[\frac{p^2}{2m} + \lambda(p_x S_y - p_y S_x)\right]|\vec{u}\rangle|\chi\rangle = E|\vec{u}\rangle|\chi\rangle =$$

$$= \left(\frac{p^2}{2m}|\vec{u}\rangle\right)|\chi\rangle + \lambda(p_x|\vec{u}\rangle)(S_y|\chi\rangle) - \lambda(p_y|\vec{u}\rangle)(S_x|\chi\rangle) = \ldots$$

$$= \frac{\hbar^2 u^2}{2m}\cancel{|\vec{u}\rangle}|\chi\rangle + \lambda(\hbar u_x\cancel{|\vec{u}\rangle}S_y|\chi\rangle) - \lambda(\hbar u_y\cancel{|\vec{u}\rangle}S_x|\chi\rangle) = E\cancel{|\vec{u}\rangle}|\chi\rangle$$

Dobimo:

$$\frac{\hbar^2 k^2}{2m}|\chi\rangle + \lambda\hbar\left(k_x S_y|\chi\rangle - k_y S_x|\chi\rangle\right) = E|\chi\rangle$$

Torej zaradi $[H, \vec{p}] = 0$ smo problem iz ∞ dim na 2D problem v spinskem prostoru.

$S = \frac{1}{2}$

$$\left|\tfrac{1}{2}\ \tfrac{1}{2}\right\rangle = |\uparrow\rangle$$

$$\left|\tfrac{1}{2}, -\tfrac{1}{2}\right\rangle = |\downarrow\rangle$$

$$S_x = \frac{S_+ + S_-}{2}$$

$$S_y = \frac{S_+ - S_-}{2i}$$

Torej je nastavek za $|\chi\rangle$ lin komb.:

$$|\chi\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle$$

$$S_+|\uparrow\rangle = 0$$

$$S_+|\downarrow\rangle = \hbar\sqrt{\tfrac{1}{2}\cdot\tfrac{3}{2} + \tfrac{1}{2}\tfrac{1}{2}} = \hbar|\uparrow\rangle$$

$$S_-|\uparrow\rangle = \hbar|\downarrow\rangle$$

$$S_-|\downarrow\rangle = 0$$

Tako:

$$S_x|\uparrow\rangle = \frac{\hbar}{2}|\downarrow\rangle$$

$$S_x|\downarrow\rangle = \frac{\hbar}{2}|\uparrow\rangle$$

$$S_y|\uparrow\rangle = \frac{\hbar}{2i}|\downarrow\rangle$$

$$S_y|\downarrow\rangle = \frac{\hbar}{2i}|\uparrow\rangle$$

$$\Rightarrow \frac{\hbar^2 k^2}{2m}\left(\alpha|\uparrow\rangle + \beta|\downarrow\rangle\right) + \lambda\hbar\left(k_x\frac{\hbar}{2i}\left(-\alpha|\downarrow\rangle + \beta|\uparrow\rangle\right) - k_y\frac{\hbar}{2}\left(\alpha|\downarrow\rangle + \beta|\uparrow\rangle\right)\right) =$$

$$= E\left(\alpha|\uparrow\rangle + \beta|\downarrow\rangle\right)$$

Iz ujemanja koeficientov dobimo dve linearni enačbi:

$$\frac{\hbar^2 k^2}{2m}\alpha + \lambda\hbar\beta\left(k_x\frac{\hbar}{2i} - k_y\frac{\hbar}{2}\right) = E\alpha$$

$$\frac{\hbar^2 k^2}{2m}\beta + \lambda\hbar\alpha\left(-k_x\frac{\hbar}{2i} - k_y\frac{\hbar}{2}\right) = E\beta$$

To je problem lastnih vrednosti:

$$\begin{pmatrix} \frac{\hbar^2 k^2}{2m} & \frac{\lambda\hbar^2}{2}\left(\frac{k_x}{i} - k_y\right) \\ -\frac{\lambda\hbar^2}{2}\left(\frac{k_x}{i} + k_y\right) & \frac{\hbar^2 k^2}{2m} \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = E\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

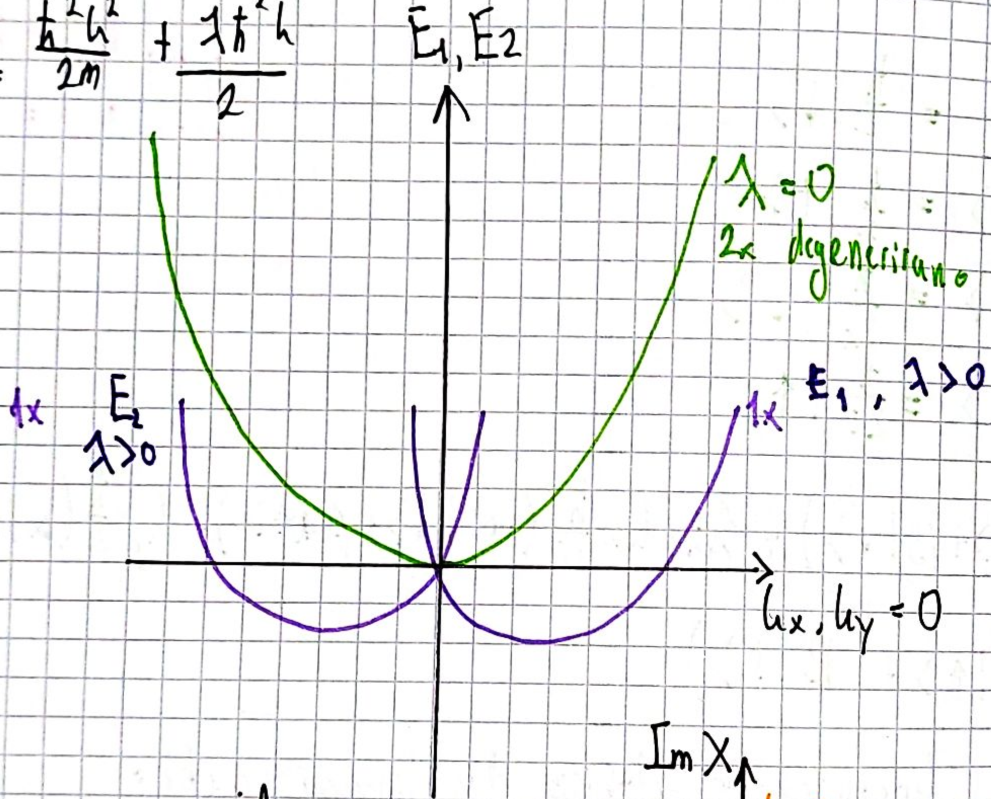
$$\begin{vmatrix} \frac{\hbar^2 k^2}{2m} - E & \frac{\lambda\hbar^2}{2}\left(\frac{k_x}{i} - k_y\right) \\ \frac{\lambda\hbar^2}{2}\left(-\frac{k_x}{i} - k_y\right) & \frac{\hbar^2 k^2}{2m} - E \end{vmatrix} = 0$$

$$\left(\frac{\hbar^2 k^2}{2m} - E\right)^2 + \frac{\lambda^2\hbar^4}{4}\underbrace{\left(\frac{k_x}{i} - k_y\right)\left(\frac{k_x}{i} + k_y\right)}_{-k_x^2 - k_y^2 = -k^2} =$$

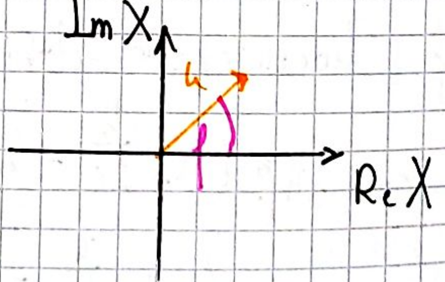
$$= \left(\frac{\hbar^2 k^2}{2m} - E - \frac{\lambda\hbar^2 k}{2}\right)\left(\frac{\hbar^2 k^2}{2m} + E + \frac{\lambda\hbar^2 k}{2}\right)$$

$$\Rightarrow \quad E_1 = \frac{\hbar^2 k^2}{2m} - \frac{\lambda\hbar^2 k}{2}$$

$$E_2 = \frac{\hbar^2 k^2}{2m} + \frac{\lambda\hbar^2 k}{2}$$



$$k_x + i k_y = X = k e^{i\varphi}$$



$$\frac{k_x}{i} - k_y = \frac{1}{i}(k_x - i k_y) = k e^{-i\varphi} \cdot \frac{1}{i}$$

$$-\frac{k_x}{i} - k_y = \frac{1}{-i} e^{i\varphi} k$$

Torej za E_1

$$\begin{pmatrix} \frac{\lambda\hbar^2 h}{2} & \frac{\lambda\hbar^2}{2}\frac{1}{i} h e^{-i\varphi} \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = 0$$

$$\lambda\hbar^2\left(\frac{h}{2}\alpha + \frac{h}{2i}e^{-i\varphi}\beta\right) = 0 \quad \Rightarrow \beta = -i\alpha e^{i\varphi}$$

in

$$|\alpha|^2 + |\beta|^2 = 1$$

$$\Rightarrow \quad E_1 \Rightarrow \frac{|\uparrow\rangle - ie^{i\varphi}|\downarrow\rangle}{\sqrt{2}}$$

Za E_2 pa na hitro

$$E_2 \Rightarrow \frac{|\uparrow\rangle + ie^{i\varphi}|\downarrow\rangle}{\sqrt{2}}$$

$$\alpha|\uparrow\rangle + \beta|\downarrow\rangle = \cos\frac{\theta}{2}|\uparrow\rangle + \sin\frac{\theta}{2}e^{i\phi}|\downarrow\rangle$$ Blochova sfera

$$|\alpha|^2 + |\beta|^2 = 1$$

$\alpha \in \mathbb{R}$ in $\alpha \geqslant 0$ (*) Razberimo α, β za naš primer

$$\cos\frac{\theta}{2} = \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{2}$$

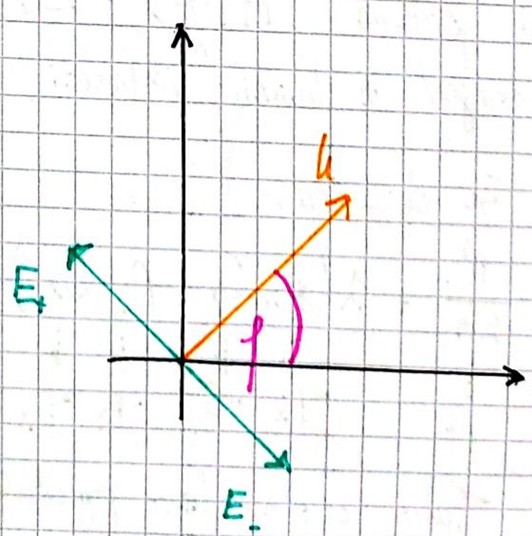
$$\frac{i}{\sqrt{2}}e^{i\varphi} = \sin\frac{\theta}{2}e^{i\phi}$$

$$ie^{i\varphi} = e^{i\phi} \Rightarrow \phi = \varphi + \frac{\pi}{2}$$

Za drugo stanje pa podobno:

$$\theta = \frac{\pi}{2}$$

$$\phi = \varphi - \frac{\pi}{2}$$



[Rešimo isto nalogo z Paulijevimi matrikami]

Dela samo za spin 1/2
prejšnji postopek pa za
splošen spin

$$\alpha|\uparrow\rangle + \beta|\downarrow\rangle \longrightarrow \begin{pmatrix}\alpha\\ \beta\end{pmatrix}$$

Dirac — Spinor

$$\vec{S} = (S_x, S_y, S_z) = \frac{\hbar}{2}(\sigma_x, \sigma_y, \sigma_z)$$

$$\sigma_x = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}, \quad \sigma_y = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}, \quad \sigma_z = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

Prepišimo Hamiltonian z Paulijevimi matrikami:

$$H = \frac{p^2}{2m}\underbrace{I_2}_{\text{2x2 identiteta ker } p \text{ ne deluje na spine}} + \frac{\lambda\hbar}{2}(p_x\sigma_y - p_y\sigma_x)$$

Kot zadnjič uporabimo nastavek $|\Psi\rangle = |\vec{k}\rangle|\chi\rangle$

$$\Rightarrow \quad H = \frac{\hbar^2 k^2}{2m} + \frac{\lambda\hbar}{2}(\hbar k_x\sigma_y - \hbar k_y\sigma_x) =$$

$$= \begin{pmatrix}\frac{\hbar^2k^2}{2m} & \frac{\lambda\hbar^2}{2}(-ik_x - k_y)\\ \frac{\lambda\hbar^2}{2m}(ik_x - k_y) & \frac{\hbar^2k^2}{2m}\end{pmatrix}$$

Reševali bi problem $H|\chi\rangle = E|\chi\rangle$ in dobimo spet problem lastnih vektorjev in lastnih vrednosti, ki smo ga tudi zadnjič dobili.

[Poglejmo še obrat časa]

$$H = \frac{p^2}{2m} + \lambda(p_x S_y - p_y S_x)$$

Obrat: $p^2 \to p^2$, $p_x \to -p_x$, $S_y \to -S_y$, $p_y \to -p_y$, $S_x \to -S_x$

Tako smo na hitro pogledali invarianco

Poglejmo si to invarianco še formalno:

$$H = \frac{p^2}{2m} + \frac{\lambda\hbar}{2}(p_x \sigma_y - p_y \sigma_x)$$

Operator obrata časa

$$T = i\sigma_y K$$

Operator kompleksne konjugacije

T je antiuniteren

$$\langle T\phi | T\psi \rangle = \langle \phi | \psi \rangle^*$$

$$K|\vec{k}\rangle = |-\vec{k}\rangle$$

$$K e^{i\vec{k}\cdot\vec{r}} = e^{-i\vec{k}\cdot\vec{r}}$$

$$K\begin{pmatrix}\alpha \\ \beta\end{pmatrix} = \begin{pmatrix}\alpha^* \\ \beta^*\end{pmatrix}$$

Da je H formalno invarianten na obrat časa mora veljati:

$$[H, T] = 0 \Rightarrow HT = TH$$

$$TH = i\sigma_y K\left(\frac{p^2}{2m} + \frac{\lambda\hbar}{2}(p_x\sigma_y - p_y\sigma_x)\right) = i\sigma_y\left(\frac{p^2}{2m} + \frac{\lambda\hbar}{2}(-p_x(-\sigma_y)K - (-p_y)(\sigma_x)K)\right)$$

$p_x = -i\hbar\frac{\partial}{\partial x}$, $p_y = -i\hbar\frac{\partial}{\partial y}$

$p^2 = -\frac{\hbar^2}{2m}\nabla^2 \in \mathbb{R}$

$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

$$= i\sigma_y\left(\frac{p^2}{2m} + \frac{\lambda\hbar}{2}(-p_x(-\sigma_y) - (-p_y)(\sigma_x))\right)K = (*)$$

Velja:

$$\{\sigma_\alpha, \sigma_\beta\} = 2\delta_{\alpha\beta}$$

$$\Rightarrow \sigma_x\sigma_y + \sigma_y\sigma_x = 0$$

$$K\sigma_y = \sigma_y^* K = (-\sigma_y)K$$

$$i\sigma_y \cdot \sigma_y = \sigma_y \cdot i\sigma_y$$

$$i\sigma_y \cdot \sigma_x = -\sigma_x \cdot i\sigma_y$$

$$\Rightarrow (*) = \left(\frac{p^2}{2m} + \frac{\lambda\hbar}{2}(p_x\sigma_y - (-p_y)(-\sigma_x)\right) i\sigma_y K = HT$$

Torej:

$$H|\psi\rangle = E|\psi\rangle$$

$$\underline{HT|\psi\rangle} = TH|\psi\rangle = TE|\psi\rangle = \underline{ET|\psi\rangle}$$

$T|\psi\rangle$ tudi lastno stanje

Ali sta ti dve lastni stanji isti?

$$T^2 = (i\sigma_y K)(i\sigma_y K) = i\sigma_y(-i)(-\sigma_y)\underbrace{K^2}_{=I} = -\underbrace{\sigma_y^2}_{=I} = -I$$

$$\langle\psi|T\psi\rangle = \langle T\psi|T^2\psi\rangle^* = \langle T\psi|-\psi\rangle^* =$$

$$= \langle -\psi|T\psi\rangle = -\langle\psi, T\psi\rangle = 0$$

Torej sta stanji ortogonalni. Torej če imamo H invarianten na T in $T^2 = -1$ dobimo lahko z apliciranjem T še drugo stanje. Temu se reče Kramersova degeneracija (Kramersov dublet).

$$\underset{|u\rangle}{T}\left(\frac{|\uparrow\rangle + ie^{i\varphi}|\downarrow\rangle}{\sqrt{2}}\right) \Rightarrow T e^{i\vec{k}\cdot\vec{r}}\begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{ie^{i\varphi}}{\sqrt{2}}\end{pmatrix} = i\sigma_y e^{-i\vec{k}\cdot\vec{r}}\begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{-ie^{-i\varphi}}{\sqrt{2}}\end{pmatrix} =$$

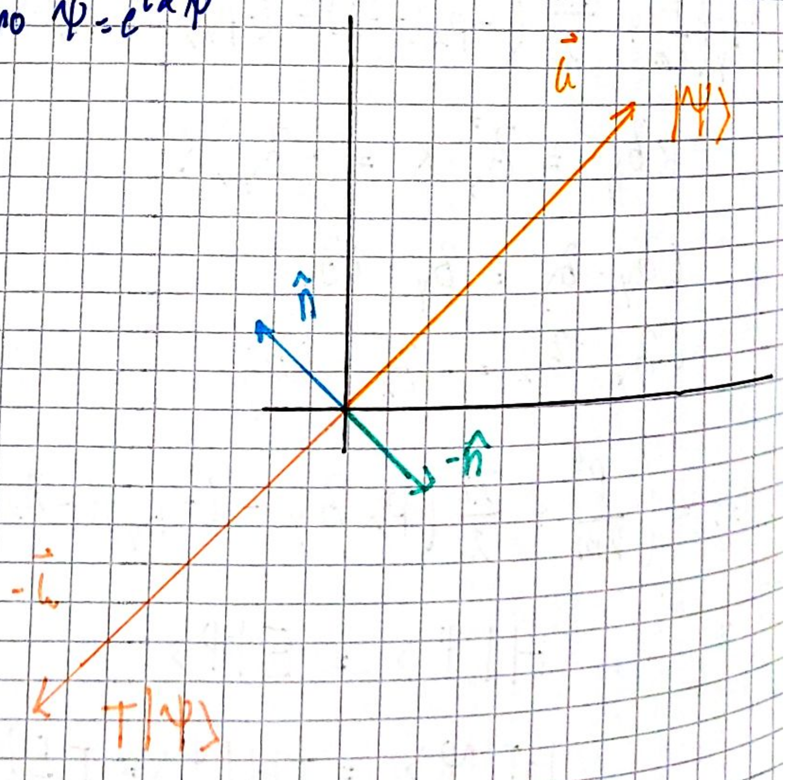
$$\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$$

$\alpha > 0 \in \mathbb{R}$

$$= e^{-i\vec{k}\cdot\vec{r}}\begin{pmatrix}-\frac{ie^{-i\varphi}}{\sqrt{2}}\\ -\frac{1}{\sqrt{2}}\end{pmatrix} = e^{-i\vec{k}\cdot\vec{r}}\begin{pmatrix}\frac{1}{\sqrt{2}}\\ -\frac{1}{\sqrt{2}}(+ie^{i\varphi})\end{pmatrix} =$$

To lahko ker fizikalno $\psi = e^{i\alpha}\psi$

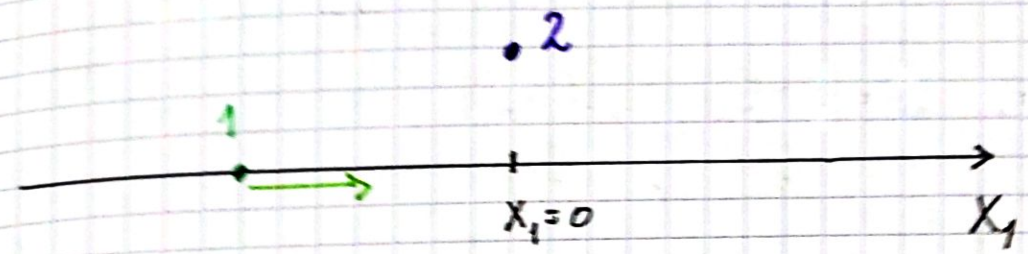
$$= e^{-i\vec{k}\cdot\vec{r}}\begin{pmatrix}\frac{1}{\sqrt{2}}\\ -\frac{ie^{i\varphi}}{\sqrt{2}}\end{pmatrix}$$



[Dva delca, ki se čutita]

1D: $H = \frac{p_1^2}{2m} - \frac{\lambda}{\hbar^2}\delta(x_1)\vec{S}_1\cdot\vec{S}_2$; $S_1 = \frac{1}{2}$

$S_2 = 1$



a) Poglejmo si vezana stanja

$\vec{S} = \vec{S}_1 + \vec{S}_2 \Rightarrow S^2 = S_1^2 + S_2^2 + 2S_1\cdot S_2$ $\quad S_z = S_{1z} + S_{2z}$

$\Rightarrow \vec{S}_1\cdot\vec{S}_2 = \frac{1}{2}(S^2 - S_1^2 - S_2^2)$

To vstavimo v H:

$H = \frac{p_1^2}{2m} - \frac{\lambda}{2\hbar^2}\delta(x_1)(S^2 - S_1^2 - S_2^2)$

$\Rightarrow [H, S^2] = [H, S_1^2] ; [H, S_2^2] = [H, S_z] = 0$

$[S^2, S_1^2] = [S^2, S_2^2] = [S^2, S_z] = 0$ $\quad H, S^2, S_1^2, S_2^2, S_z$ skupaj komutirajo

$[S_1^2, S_2^2] = [S_1^2, S_z] = 0$

$[S_2^2, S_z] = 0$

1. delec | 2. delec | 1. in 2. delec | dogovor ker S_1 konst.

$|\uparrow\rangle = |\frac{1}{2}\ \frac{1}{2}\rangle$ $\quad |1\,1\rangle$ $\quad |\uparrow\rangle|1\,1\rangle = |\uparrow\rangle|1\rangle$

$|\downarrow\rangle = |\frac{1}{2}\ {-\frac{1}{2}}\rangle$ $\quad |1\,0\rangle$ $\quad |\uparrow\rangle|1\,0\rangle = |\uparrow\rangle|0\rangle$

$|1\,{-1}\rangle$ $\quad |\uparrow\rangle|1\,{-1}\rangle = |\uparrow\rangle|{-1}\rangle$

$\vdots$

$S_1 \quad S_{1z}$ $\quad S_2 \quad S_{2z}$ $\quad |\downarrow\rangle|1\,1\rangle$

$|\downarrow\rangle|1\,0\rangle$

2D $\quad$ 3D $\quad |\downarrow\rangle|1\,{-1}\rangle$

Produktna baza: lastna za

$2\cdot 3 = 6D$ $\quad S_1^2, S_{1z}, S_2^2, S_{2z}$

$$S_1^2 |\uparrow\rangle |10\rangle = \hbar^2 \frac{1}{2}\left(\frac{1}{2}+1\right) |\uparrow\rangle|10\rangle = \frac{3}{4}\hbar^2 |\uparrow\rangle|10\rangle$$

Izberimo bazo z dobrim skupnim spinom

$$S^2, S_z, S_1^2, S_2^2$$

$$S_1, S_2 \Rightarrow S = |S_1 - S_2|, \ldots, |S_1 + S_2|$$

$$\frac{1}{2} \quad , \quad \frac{3}{2}$$

$|S_1 S_2 S S_z\rangle$ dogovor $|S S_z\rangle$ ker S_1 in S_2 konst.

$$\Rightarrow \quad \left|\tfrac{1}{2}\ 1\ \tfrac{1}{2}\ -\tfrac{1}{2}\right\rangle = \left|\tfrac{1}{2}\ -\tfrac{1}{2}\right\rangle$$
$$\left|\tfrac{1}{2}\ 1\ \tfrac{1}{2}\ \tfrac{1}{2}\right\rangle = \left|\tfrac{1}{2}\ \tfrac{1}{2}\right\rangle$$
$$\left|\tfrac{1}{2}\ 1\ \tfrac{3}{2}\ -\tfrac{3}{2}\right\rangle = \left|\tfrac{3}{2}\ -\tfrac{3}{2}\right\rangle$$
$$\left|\tfrac{1}{2}\ 1\ \tfrac{3}{2}\ -\tfrac{1}{2}\right\rangle = \left|\tfrac{3}{2}\ -\tfrac{1}{2}\right\rangle$$
$$\left|\tfrac{1}{2}\ 1\ \tfrac{3}{2}\ \tfrac{1}{2}\right\rangle = \left|\tfrac{3}{2}\ \tfrac{1}{2}\right\rangle$$
$$\left|\tfrac{1}{2}\ 1\ \tfrac{3}{2}\ \tfrac{3}{2}\right\rangle = \left|\tfrac{3}{2}\ \tfrac{3}{2}\right\rangle$$

Torej rešujemo $H|\psi\rangle = E|\psi\rangle$, z nastavkom $|\psi\rangle = |\phi\rangle|S S_z\rangle$

$$H|\phi\rangle|S S_z\rangle = E|\phi\rangle|S S_z\rangle$$

$$\left[\frac{p_1^2}{2m} - \frac{\lambda}{2\hbar^2}\delta(x_1)\left(S^2 - S_1^2 - S_2^2\right)\right]|\phi\rangle|S S_z\rangle = E|\phi\rangle|S S_z\rangle$$

$$\left(\frac{p_1^2}{2m}|\phi\rangle\right)|S S_z\rangle - \frac{\lambda}{2\hbar^2}\delta(x_1)|\phi\rangle \overbrace{|S S_z\rangle}^{(S^2 - S_1^2 - S_2^2)} = E|\phi\rangle|S S_z\rangle$$

$$S^2|S S_z\rangle = \hbar^2 S(S+1)|S S_z\rangle$$

$$S_1^2|S S_z\rangle = \frac{3}{4}\hbar^2|S S_z\rangle$$

$$S_2^2|S S_z\rangle = 2\hbar^2|S S_z\rangle$$

$$\frac{p_1^2}{2m}|\phi\rangle\Big)|SS_z\rangle - \frac{\lambda}{2\hbar^2}\delta(x_1)|\phi\rangle\Big[\hbar^2 S(S+1) - \underbrace{\hbar^2 S_1(S_1+1) - \hbar^2 S_2(S_2+1)}_{-11/4\,\hbar^2}\Big]|SS_z\rangle =$$

$$= E|SS_z\rangle|\phi\rangle$$

$$\Big(\frac{p_1^2}{2m}\Big)|\phi\rangle + \frac{\lambda}{2}\delta(x_1)\Big(\frac{11}{4} - S(S+1)\Big)|\phi\rangle = E|\phi\rangle$$

Rešujemo pravzaprav dva problema:

$$H_{3/2} = \frac{p_1^2}{2m} - \frac{\lambda}{2}\delta(x_1) \rightarrow$$ Vezano stanje (4x degeneracija)

$$H_{1/2} = \frac{p_1^2}{2m} + \lambda\delta(x_1)$$

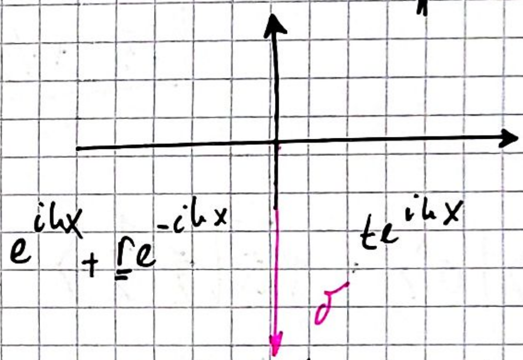
Od začetka semestra:

$$H = \frac{p^2}{2m} - \lambda\delta(x)$$

$$E_0 = -\frac{\hbar^2\lambda^2}{2m}\,;\quad \kappa = \frac{m\lambda}{\hbar^2}$$

$$\psi_0 = \sqrt{\kappa}\,e^{-\kappa|x|}$$

$$E>0:\ k = \frac{\sqrt{2mE}}{\hbar^2}$$



$$S = \frac{1}{\kappa + ik}\begin{pmatrix} -\kappa & ik \\ ik & -\kappa \end{pmatrix}$$

Poglejmo sipalna stanja:

$$E>0:\quad S_{3/2} = \begin{pmatrix} r_{3/2} & \cdot \\ t_{3/2} & \cdot \end{pmatrix}$$

$$S_{1/2} = \begin{pmatrix} r_{1/2} & \cdot \\ t_{1/2} & \cdot \end{pmatrix}$$

Torej so sipalna stanja (vpadni val iz leve) pri energiji $E>0$:

$S=\frac{3}{2}$: $\quad (e^{ikx_1} + r_{3/2}e^{-ikx_1})|\tfrac{3}{2}\,S_z\rangle \quad | \quad t_{3/2}e^{ikx_1}|\tfrac{3}{2}\,S_z\rangle$

$\lambda/2$

$S=\frac{1}{2}$: $\quad (e^{ikx_1} + r_{1/2}e^{-ikx_1})|\tfrac{1}{2}\,S_z\rangle \quad |^{\lambda} \quad t_{1/2}e^{ikx_1}|\tfrac{1}{2}\,S_z\rangle$

Torej imamo za $E>0$ 12 sipalnih stanj (še 6 za val iz desne) in $E<0$ imamo 4 vezana stanja. To je celotna ~~energija~~ baza našega problema.

Rešujemo problem sipanja ko delec 1 prihaja z $E>0$ iz leve mimo delca z $S_{2z}=0$

$$\Rightarrow \quad e^{ikx_1}|\uparrow\rangle|0\rangle \quad ; \quad k=\sqrt{\frac{2mE}{\hbar^2}}$$

Rabimo pretvoriti med bazama. To naredimo z Clebsch-Gordanovimi koeficienti.

$1\times 1/2$		3/2			
		3/2	3/2	1/2	
+1	+1/2	1	+1/2	+1/2	
+1	−1/2	1/3	2/3	3/2	1/2
0	+1/2	2/3	−1/3	−1/2	−1/2
0	−1/2	2/3	1/3	3/2	
−1	+1/2	1/3	−2/3	−3/2	
−1	−1/2	1			

$$|\uparrow\rangle|0\rangle = \sqrt{\frac{2}{3}}\left|\frac{3}{2}\ \frac{1}{2}\right\rangle - \sqrt{\frac{1}{3}}\left|\frac{1}{2}\ \frac{1}{2}\right\rangle$$

$$\sqrt{\frac{2}{3}}\left(e^{ikx_1}+r_{3/2}e^{-ikx_1}\right)\left|\frac{3}{2}\ \frac{1}{2}\right\rangle - \sqrt{\frac{1}{3}}\left(e^{ikx_1}+r_{1/2}\right)\left|\frac{1}{2}\ \frac{1}{2}\right\rangle \leftarrow \text{Vpadni in odbiti}$$

$$\sqrt{\frac{2}{3}}\,t_{3/2}\,e^{ikx_1}\left|\frac{3}{2}\ \frac{1}{2}\right\rangle - \sqrt{\frac{1}{3}}\,t_{1/2}\,e^{ikx_1}\left|\frac{1}{2}\ \frac{1}{2}\right\rangle \leftarrow \text{Prepuščeni}$$

Vprašanja in meritve delamo bolj v produktni bazi kot v bazi z dobrim skupnim spinom. Spet uporabimo tabele, da transformiramo nazaj

$$\left|\frac{3}{2}\ \frac{1}{2}\right\rangle = \sqrt{\frac{1}{3}}|\downarrow\rangle|1\rangle + \sqrt{\frac{2}{3}}|\uparrow\rangle|0\rangle$$

$$\left|\frac{1}{2}\ \frac{1}{2}\right\rangle = \sqrt{\frac{2}{3}}|\downarrow\rangle|1\rangle - \sqrt{\frac{1}{3}}|\uparrow\rangle|0\rangle$$

$$\sqrt{\tfrac{2}{3}}\left(e^{ikx_1}+r_{3/2}e^{-ikx_1}\right)\left[\sqrt{\tfrac{1}{3}}\,|\downarrow\rangle|1\rangle+\sqrt{\tfrac{2}{3}}\,|\uparrow\rangle|0\rangle\right]-\sqrt{\tfrac{1}{3}}\left(e^{ikx_1}+r_{1/2}e^{-ikx_1}\right)\cdot$$

$$\cdot\left[\sqrt{\tfrac{2}{3}}\,|\downarrow\rangle|1\rangle-\sqrt{\tfrac{1}{3}}\,|\uparrow\rangle|0\rangle\right]\Big]$$

$$\downarrow$$

$$\left[\frac{\sqrt{2}}{3}\left(e^{ikx_1}+r_{3/2}e^{-ikx_1}\right)-\frac{\sqrt{2}}{3}\left(e^{ikx_1}+r_{1/2}e^{-ikx_1}\right)\right]|\downarrow\rangle|1\rangle+$$

$$+\left[\frac{2}{3}\left(e^{ikx_1}+r_{3/2}e^{-ikx_1}\right)+\frac{1}{3}\left(e^{ikx_1}+r_{1/2}e^{-ikx_1}\right)\right]|\uparrow\rangle|0\rangle$$

$$\downarrow$$

$$e^{ikx_1}|\uparrow\rangle|0\rangle+e^{-ikx_1}\underbrace{\left(\frac{\sqrt{2}}{3}r_{3/2}-\frac{\sqrt{2}}{3}r_{1/2}\right)}_{r_{\downarrow\uparrow}}|\downarrow\rangle|1\rangle+e^{-ikx_1}\overbrace{\left(\frac{2}{3}r_{3/2}+\frac{1}{3}r_{1/2}\right)}^{r_{\uparrow\uparrow}}|\uparrow\rangle|0\rangle$$

za $x_1<0$

$$e^{ikx_1}\underbrace{\left(\frac{\sqrt{2}}{3}t_{3/2}-\frac{\sqrt{2}}{3}t_{1/2}\right)}_{t_{\downarrow\uparrow}}|\downarrow\rangle|1\rangle+e^{ikx_1}\underbrace{\left(\frac{2}{3}t_{3/2}+\frac{1}{3}t_{1/3}\right)}_{t_{\uparrow\uparrow}}$$

Ubistu elementi 12×12 matrike

Vpeljemo lahko 4 različne verjetnosti:

$$R_{\downarrow\uparrow\uparrow}=|r_{\downarrow\uparrow\uparrow}|^2=\left|\frac{\sqrt{2}}{3}r_{3/2}-\frac{\sqrt{2}}{3}r_{1/2}\right|^2=\frac{2}{9}|r_{3/2}|^2+\frac{2}{9}|r_{1/2}|^2-2\mathrm{Re}\left(r_{3/2}^{*}r_{1/2}\right)\cdot\frac{2}{9}$$

$$R_{\uparrow\uparrow\uparrow}=|r_{\uparrow\uparrow\uparrow}|^2=\left|\frac{2}{3}r_{3/2}+\frac{1}{3}r_{1/2}\right|^2=\frac{4}{9}|r_{3/2}|^2+\frac{1}{9}|r_{1/2}|^2+\frac{4}{9}\mathrm{Re}\left(r_{3/2}^{*}r_{1/2}\right)$$

$$T_{\uparrow\uparrow}=|t_{\uparrow\uparrow}|^2=\ldots$$

$$T_{\downarrow\uparrow}=|t_{\downarrow\uparrow}|^2=\ldots$$

Interferenčni členi

Če bi gledali celotno odbojnost: $R=R_{\downarrow\uparrow}+R_{\uparrow\uparrow}=\frac{2}{3}|r_{3/2}|^2+\frac{1}{3}|r_{1/2}|^2$

Podobno za celotno prepustnost: $T=\frac{2}{3}|t_{3/2}|^2+\frac{1}{3}|t_{1/2}|^2$

Preverimo če res velja $T + R = 1$

$$T + R = \frac{2}{3}\Big(\underbrace{|r_{3/2}|^2 + |\overset{t}{\cancel{r}}_{3/2}|^2}_{1}\Big) + \frac{1}{3}\Big(\underbrace{|r_{1/2}|^2 + |t_{1/2}|^2}_{1}\Big) = 1$$

Iz pogoja za unitarnost sipalne matrike.

To smo izpeljali brez da bi kje upoštevali delto, torej čisto splošno.

Teorija Motnje

["Anharmonski" linearni oscilator]

$$H = \underbrace{\frac{p^2}{2m} + \frac{1}{2}kx^2}_{H_0} + \underbrace{\lambda x^4}_{H'}$$

$$\left.\begin{aligned} H_0 |n\rangle^0 &= E_n^0 |n\rangle^0 \\ E_n^0 &= \hbar\omega\left(\tfrac{1}{2} + n\right) \; ; \quad n = 0, 1, 2, \ldots \end{aligned}\right\} \text{Nemoten oscilator}$$

$$E_n + E_n^0 + \underbrace{{}^0\langle n | H' | n \rangle^0}_{} + \ldots$$

1\. red teorije motnje za nedegenerirana stanja

Dogovor:

$|n\rangle^0 \Rightarrow |n\rangle$

~~Nemoten~~ Lastna stanja nemotenega.

$$x = \frac{x_0}{\sqrt{2}}\left(a^\dagger + a\right)$$

$$x^2 = \frac{x_0^2}{2}\left(a^\dagger + a\right)^2$$

$\langle n|\lambda x^4|n\rangle = \lambda\langle x^2 n|x^2 n\rangle$

$$x^2 = \frac{x_0^2}{2}\left(a^{+2} + aa^+ + a^+a + a^2\right) = \frac{x_0^2}{2}\left(a^{+2} + 1 + 2a^+a + a^2\right)$$

$|x^2 n\rangle = \cancel{4|n\rangle}$

$$= \frac{x_0^2}{2}\left(|a^{+2}|\,|n\rangle + 1|n\rangle + 2a^+a|n\rangle + a^2|n\rangle\right) =$$

$$= \frac{x_0^2}{2}\left(\sqrt{(n+1)(n+2)}\,|n+2\rangle + |n\rangle + 2n|n\rangle + \sqrt{n(n-1)}\,|n-2\rangle\right)$$

$$\Rightarrow \lambda\langle x^2 n|x^2 n\rangle = \lambda\frac{x_0^4}{4}\left[(n+1)(n+2) + (2n+1)^2 + n(n-1)\right] =$$

$$= \lambda\frac{x_0^4}{4}\left[n^2 + 3n + 2 + 4n^2 + 4n + 1 + n^2 - n\right] =$$

$$= \lambda\frac{x_0^4}{4}\left[6n^2 + 6n + 3\right] = \frac{3}{4}x_0^4\lambda\left[2n^2 + 2n + 1\right] =$$

$$= \frac{3x_0^4\lambda}{4}\left[2n^2 + 2n + 1\right]$$

Torej so energije v prvem redu popravka:

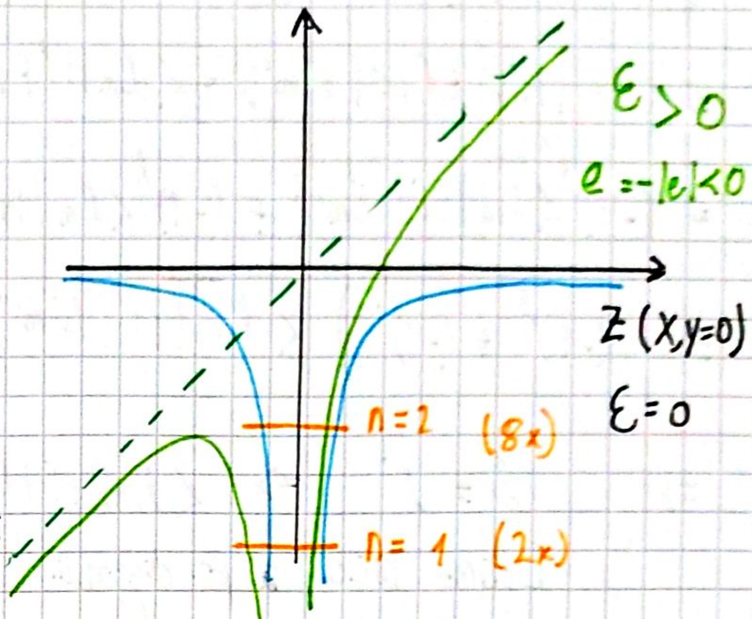
$$E_n = \hbar\omega\left(n + \frac{1}{2}\right) + \frac{3x_0^4}{4}\lambda\left[2n^2 + 2n + 1\right]$$

$$E_{n+1} - E_n = \hbar\omega + \alpha n$$

Nadaljevanje Vodikov atom v $\vec{E}$

$H = H_0 + H'$

$= \frac{p^2}{2m} - \frac{e^2}{4\pi\varepsilon_0 r} - e\mathcal{E}Z$



Uvedli bazo

$|\ell m\rangle |n s\rangle$

$\downarrow$

$|00\rangle$

$|11\rangle$

$|10\rangle$

$|1-1\rangle$

Naredimo perturbacijski popravek 1. reda

$$\begin{array}{c|cccc} & |00\rangle & |11\rangle & |10\rangle & |1-1\rangle \\ \hline \langle 00| & 0 & 0 & M & 0 \\ \langle 11| & 0 & 0 & 0 & 0 \\ \langle 10| & M^* & 0 & 0 & 0 \\ \langle 1-1| & 0 & 0 & 0 & 0 \end{array}$$

Hočemo računati čim manj matričnih elementov zato uporabimo nekaj fundamentalnih dejstev in lahko hitro ugotovimo, da jih je veliko enakih 0.

1) $[H', L_z] = 0$, $[H, L_z] = 0$ $\Rightarrow$ $\langle \ell m | H' | \ell' m' \rangle = 0$ za $m \neq m'$

2) $P: \vec{r} \to -\vec{r}$

$P^2 \psi(\vec{r}) = \lambda^2 \psi(\vec{r}) = \psi(\vec{r})$; $\lambda = \pm 1$

$[H_0, P] = 0$

$\{H', P\} = 0$

3) $\psi_{n\ell m}(\vec{r}) = R_{n\ell}(r)\, Y_{\ell m}(\theta, \varphi)$

$P Y_{\ell m}(\theta, \varphi) = (-1)^\ell\, Y_{\ell m}(\theta, \varphi)$

$$\langle lm|\{H',P\}|l'm'\rangle = 0$$

$$= \langle lm|H'P + PH'|l'm'\rangle = \langle lm|H'P|l'm'\rangle + \langle lm|PH'|l'm'\rangle =$$

$$= (-1)^{l'}\langle lm|H'|l'm'\rangle + (-1)^{l}\langle lm|H'|l'm'\rangle =$$

$$= \left((-1)^{l'} + (-1)^{l}\right)\langle lm|H'|l'm'\rangle$$

$$\Rightarrow \quad l = l' \quad \Rightarrow \quad \langle lm|H'|l'm'\rangle = 0$$

Dobimo ničle ~~z~~ po diagonali.

Zračunajmo tisto kar ostane:

$$M = \langle 00|H'|10\rangle = \int_{\mathbb{R}^3} dx\,dy\,dz\, R_{20}^* Y_{00}^*(\theta,\varphi) R_{21}(r) Y_{10}(\theta,\varphi)(-e\mathcal{E}z)$$

$$= \int_0^\infty r^2 dr \int_0^\pi d\theta \int_0^{2\pi} d\varphi \, \sin\theta \frac{2}{(2r_B)^{3/2}}\left(1 - \frac{r}{2r_B}\right)e^{-r/2r_B} \frac{1}{\sqrt{4\pi}} \frac{1}{\sqrt{3}} \frac{1}{(2r_B)^{3/2}} \frac{r}{r_B} e^{-r/2r_B}$$

$$\cdot \sqrt{\frac{3}{4\pi}} \cos\theta (-e\mathcal{E} r\cos\theta) =$$

$$= -\frac{e\mathcal{E}}{(2r_B)^3}\frac{2}{4\pi}\int_0^\infty r^2 dr\left(1 - \frac{r}{2r_B}\right)e^{-r/2r_B}\frac{r^2}{r_B}e^{-r/2r_B}\underbrace{\int_{-1}^{1} d(\cos\theta)\cos^2\theta}_{2/3}\underbrace{\int_0^{2\pi} d\varphi}_{2\pi} =$$

$2\cdot 1/2$

$$= -\frac{2}{3}\frac{e\mathcal{E}}{(2r_B)^3}\int_0^\infty d\left(\frac{r}{r_B}\right) r_B \left(\frac{r}{r_B}\right)^2\left(1 - \frac{r}{2r_B}\right)e^{-\frac{r}{r_B}}\left(\frac{r}{r_B}\right)^2 r_B =$$

$$= -\frac{e\mathcal{E}}{12} r_B \int_0^\infty du\left(u^4 - \frac{1}{2}u^5\right)e^{-u} = -\frac{e\mathcal{E}}{12} r_B\left(\underbrace{\Gamma(3)}_{4!} - \frac{1}{2}\underbrace{\Gamma(4)}_{5!}\right)$$

$$= -e\mathcal{E} r_B\, 2\cdot\left(-\frac{3}{2}\right) = 3e\mathcal{E} r_B$$

...smo zapisali matriko motnje, ki jo moramo sedaj diagonalizirati.
...lahko naredimo tudi bločno

$$
\begin{array}{c|cccc}
 & |00\rangle & |10\rangle & |11\rangle & |1-1\rangle \\
\hline
\langle 00| & 0 & M & & \\
\langle 10| & M^* & 0 & & \\
\langle 11| & & & 0 & \\
\langle 1-1| & & & & 0
\end{array}
$$

Že diag $\lambda_{1,2} = 0$

lastna vektorja sta $|11\rangle$, $|1-1\rangle$

Te dve stanji torej ne čutita motnje.

$$\det\begin{pmatrix} -\lambda & M \\ M & -\lambda \end{pmatrix} = \lambda^2 - M^2 = 0 \quad \Rightarrow \quad \lambda_{3,4} = \pm M$$

1.) $\lambda = -M = 3|e|\varepsilon r_B$

$$\begin{pmatrix} M & M \\ M & M \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \quad \alpha + \beta = 0 \Rightarrow \alpha = -\beta$$

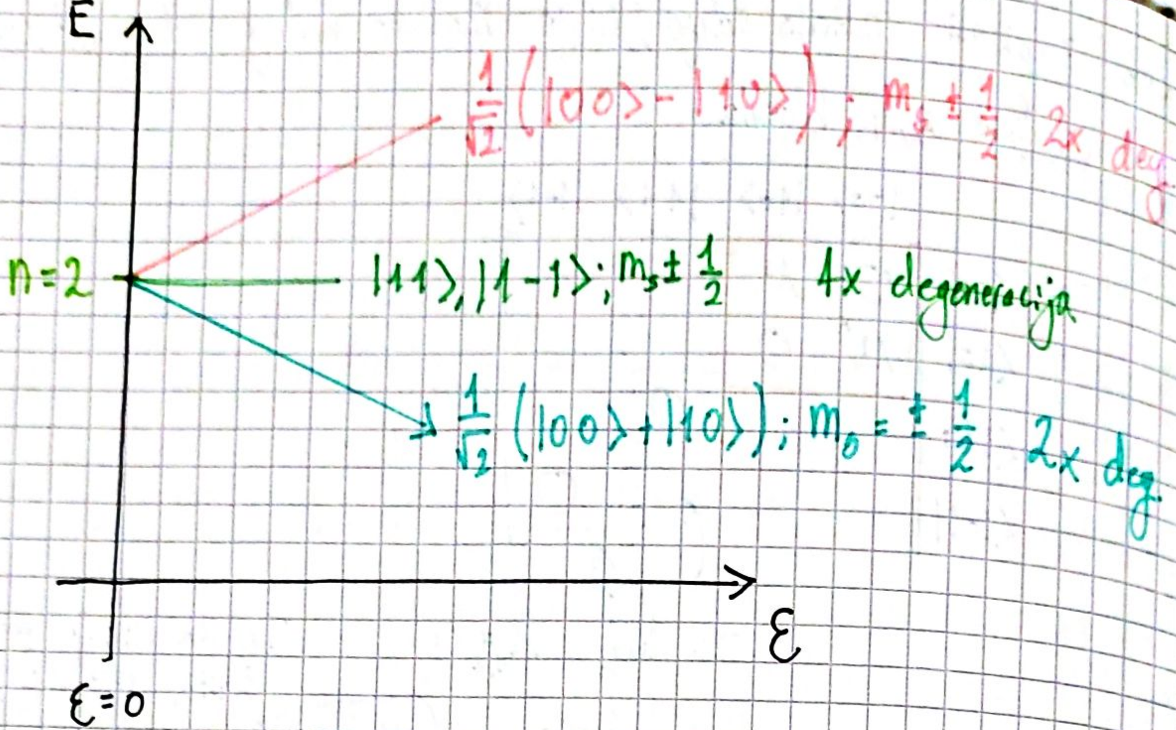
$$\begin{pmatrix} \alpha \\ -\alpha \end{pmatrix} \Rightarrow \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Torej je lastno stanje $\frac{1}{\sqrt{2}}\left(|00\rangle - |10\rangle\right)$

2.) $\lambda = M = -3|e|\varepsilon r_B$

$$\begin{pmatrix} -M & M \\ M & -M \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \quad \alpha = \beta$$

Torej je lastno stanje $\frac{1}{\sqrt{2}}\left(|00\rangle + |10\rangle\right)$



Recimo: $\frac{1}{\sqrt{2}}(|00\rangle - |10\rangle) \equiv |-\rangle$

$\frac{1}{\sqrt{2}}(|00\rangle + |10\rangle) \equiv |+\rangle$

Izračunajmo:

$$\langle -|z|-\rangle = \frac{1}{\sqrt{2}}(\langle 00| - \langle 10|)\, z\, \frac{1}{\sqrt{2}}(|00\rangle - |10\rangle) =$$

Samo mešani členi ostanejo (Zaradi parnosti bra ket in lihi z)

$$= \frac{1}{2}(-\langle 00|z|10\rangle - \langle 10|z|00\rangle) =$$

$$= \frac{1}{2}(-\langle 00|z|10\rangle - \langle 00|z|10\rangle^*) = -Re(\langle 00|z|10\rangle) =$$

$\langle 00|H'|10\rangle = 3e\varepsilon r_B$

$\langle 00|z|10\rangle = \frac{1}{-e\varepsilon}\langle 00|H'|10\rangle = -3r_B$

$= +r_B$

Drugi se pa premakne ravno v nasprotno smer

$\langle +|r_B|+\rangle = -3r_B$